to my mother iluminada,
my wife imelda,
and my children kim deunice,
ken dainiel, and
karla denise

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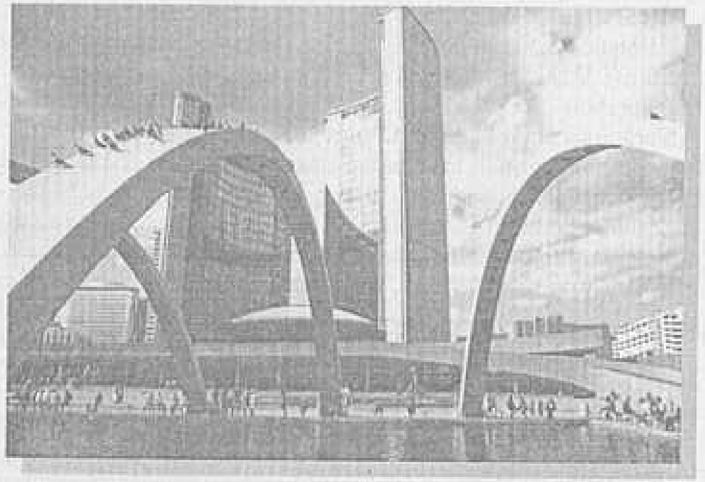


Part 1
ALGEBRA & ADVANCED MATH

BASIC LAW OF NATURAL NUMBERS

Let a, b, and c be any number.

1. Law of closure for addition:
   $a + b$
2. Commutative law for addition:
   $a + b = b + a$
3. Associative law for addition:
   $a + (b + c) = (a + b) + c$
4. Law of closure for multiplication:
   $a \times b$
5. Commutative law for multiplication
   $a \times b = b \times a$
6. Associative law for multiplication
   $a(bc) = (ab)c$
7. Distributive Law
   $a(b + c) = ab + ac$

BASIC LAWS OF EQUALITY

1. Reflexive property
   $a = a$
2. Symmetric property
   If $a = b$, then $b = a$
3. Transitive property
   If $a = b$ and $b = c$, then $a = c$. That is, things equal to the same thing are equal to each other.
4. If $a = b$ and $c = d$, then $a + c = b + d$. That is, if equals are added to equals, the results are equal.
5. If $a = b$ and $c = d$, then $ac = bd$. That is, if equals are multiplied to equals, the results are equal.

INEQUALITY

A statement that one quantity is greater than or less than another quantity

Symbols used in inequality

$a > b$ — a is greater than b
$a < b$ — a is less than b
$a \le b$ — a is less than or equal to b
$a \ge b$ — a is greater than or equal to b

Theorems on Inequalities

1. $a > b$ if and only if $-a < -b$
2. If $a > 0$, then $-a < 0$
3. If $-a < 0$, then $a < 0$
4. If $a > b$, $c < 0$, then $ac < bc$
5. If $a > b$, $c > d$, then $(a + c) > (b + d)$
6. If $a > b$, $c > d$, and $a, b, c, d > 0$, then $ac > bd$
7. If $a > 0$, $b > 0$, $a > b$, then $\frac{1}{a} < \frac{1}{b}$

OTHER IMPORTANT PROPERTIES IN ALGEBRA

1. $a \times 0 = 0$
2. If $a \times b = 0$, then either $a =$ or $b = 0$ or both a and b are zero.
3. $\frac{0}{a} = 0$ if $a \neq 0$
4. $\frac{a}{0}$ = undefined
5. $\frac{a}{\infty} = 0$

LAWS OF EXPONENTS (INDEX LAW)

1. $a^n = a \times a \times a \ldots$ (n factors)
2. $a^m \times a^n = a^{m+n}$
3. $\frac{a^m}{a^n} = a^{m-n}$
4. $(a^m)^n = a^{mn}$
5. $(abc)^n = a^n b^n c^n$
6. $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$
7. $a^{\frac{m}{n}} = \sqrt[n]{a^m}$
8. $a^{-m} = \frac{1}{a^m}$ and $\frac{1}{a^{-m}} = a^m$
9. $a^0 = 1$
10. If $a^m = a^n$, then $m = n$ (provided $a \neq 0$)

PROPERTIES OF RADICALS

1. $a^{\frac{1}{n}} = \sqrt[n]{a}$
2. $a^{\frac{m}{n}} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$
3. $\left(\sqrt[n]{a}\right)^n = a$
4. $\sqrt[n]{a} \times \sqrt[n]{b} = \sqrt[n]{ab}$
5. $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$ provided that $b \neq 0$

PROPERTIES OF LOGARITHM

1. $\log_a MN = \log_a M + \log_a N$
2. $\log_a \frac{M}{N} = \log_a M - \log_a N$
3. $\log_a M^n = n \log_a M$
4. $\log_a a = 1$
5. $\log_a a^x = x \log_a a = x$
6. $\log_a 1 = 0$
7. If $\log_a M = N$, then $a^N = M$
8. If $\log_a M = \log_a N$, then $M = N$
9. $\log_e M = \ln M$
   $e = 2.71828\ldots$ (Naperian logarithm)
10. $\log_{10} M = \log M$ (Common logarithm)
11. $\log_n M = \log M / \log n = \ln M / \ln n$
12. If $\log_b x = a$ then $x =$ antilog$_b$ a
13. $a^x =$ antilog$_a$ x
14. $\log_{10} 4250 = \log_{10} (1000 \times 4.25)$
    $= \log 1000 + \log 4.25$
    $\log_{10} 4250 = 3 + 0.6284 = 3.6284$

    3, the *integral part*, is called the characteristic
    0.6284, a *non-negative decimal* fraction part, is called the mantissa

POLYNOMIALS

Expanding Brackets

By multiplying two brackets together, each term in one bracket is multiplied by each term of the other bracket.

$$(a + b + c)(d + e) = ad + ae + bd + be + cd + ce$$

Factorization

Factorization is the opposite process of expanding brackets. The usual process includes changing a long expression without any brackets to a shorter expression that includes brackets.

$$2x^2 - 6x + 4 = 2(x^2 - 3x + 2) = 2(x - 2)(x - 1)$$

Special Products and Factoring

1. $(x + y)(x - y) = x^2 - y^2$
2. $(x + y)^2 = x^2 + 2xy + y^2$
3. $(x - y)^2 = x^2 - 2xy + y^2$
4. $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2xz + 2yz$
5. $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$
6. $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$
7. $x^6 - y^6 = (x^2)^3 - (y^2)^3 = (x^2 - y^2)[(x^2)^2 + (x^2)(y^2) + (y^2)^2]$
   $= (x + y)(x - y)(x^4 + x^2 y^2 + y^4)$

Division of Polynomials

Carrying out the division of polynomials is no different, in principle, to numerical division. Consider the following example.

Example Divide $x^4 - 10x^2 - 9x - 20$ by $x - 4$.

Solution A By long division

$$\begin{array}{rrl}
 & x^3 + 4x^2 + 6x + 15 \text{ remainder } 40 & \\
x - 4 \,\big) & x^4 - 10x^2 - 9x - 20 & 1.\ x^4 \div x = x^3 \\
- & x^4 - 4x^3 & \\
\hline
 & 4x^3 - 10x^2 & 2.\ 4x^3 \div x = 4x^2 \\
- & 4x^3 - 16x^2 & \\
\hline
 & 6x^2 - 9x & 3.\ 6x^2 \div x = 6x \\
- & 6x^2 - 24x & \\
\hline
 & 15x - 20 & 4.\ 15x \div x = 15 \\
- & 15x - 60 & \\
\hline
\text{remainder} \rightarrow & 40 &
\end{array}$$

Solution B BY SYNTHETIC DIVISION

Write the coefficients of the terms, supplying zero as the coefficient of the missing power of x.

$$\begin{array}{rrrrr|l}
1 & 0 & -10 & -9 & -20 & \underline{4} \\
 & 4 & 16 & 24 & 60 & \\
\hline
1 & 4 & 6 & 15 & 40 &
\end{array}$$

The quotient is $x^3 + 4x^2 + 6x + 15$ remainder 40

Factor Theorem

Consider a function $f(x)$. If $f(1) = 0$ then $(x - 1)$ is a factor of $f(x)$. If $f(-3) = 0$ then $(x + 3)$ is a factor of $f(x)$. Use of factor theorem can produce the factors of an expression in a *trial and error* manner.

Example Factorize $2x^3 + 5x^2 - x - 6$

Solution

$f(x) = 2x^3 + 5x^2 - x - 6$

$f(1) = 2(1)^3 + 5(1)^2 - (1) - 6 = 0,$
hence $(x - 1)$ is a factor
$f(-1) = 2(-1)^3 + 5(-1)^2 - (-1) - 6 = -2,$
hence $(x + 1)$ is not a factor
$f(2) = 2(2)^3 + 5(2)^2 - (2) - 6 = 28,$
hence $(x - 2)$ is not a factor
$f(-2) = 2(-2)^3 + 5(-2)^2 - (-2) - 6 = 0,$
hence $(x + 2)$ is a factor
$f(-3/2) = 2(-3/2)^3 + 5(-3/2)^2 - (-3/2) - 6 = 0,$
hence $2x + 3$ is a factor.

Thus, $2x^3 + 5x^2 - x - 6 = (x - 1)(x + 2)(2x + 3)$

Remainder Theorem

If a polynomial $f(x)$ is divided by $(x - r)$ until a remainder which is free of x is obtained, the remainder is $f(r)$.
If $f(r) = 0$ then $(x - r)$ is a factor of $f(x)$.

Example Find the remainder when $x^4 - 10x^2 - 9x - 20$ is divided by $x - 4$.

Solution

$f(x) = x^4 - 10x^2 - 9x - 20$
$x - r = x - 4$
$r = 4$
Remainder $= f(4) = 4^4 - 10(4)^2 - 9(4) - 20$
Remainder $= 40$

Example Find k such that $x - 3$ is a factor of $kx^3 - 6x^2 + 2kx - 12$.

Solution

Remainder $= f(3) = k(3)^3 - 6(3)^2 + 2k(3) - 12 = 0$
$k = 2$

BINOMIAL THEOREM

Expansion of $(a+b)^n$

Properties

1. The number of terms in the expansion $n + 1$,
2. The first term is a^n & the last term is b^n,
3. The exponent of a descends linearly from n to 0,

4. The exponent of b ascends linearly from 0 to n,
5. The sum of the exponents of a and b in any of the terms is equal to n,
6. The coefficient of the second term and the second from the last term is n,

Pascal's Triangle

Used to determine the coefficients of the terms in a binomial expansion.

$$\begin{array}{lccccccccccc}
(a+b)^0 & & & & & & 1 & & & & & \\
(a+b)^1 & & & & & 1 & & 1 & & & & \\
(a+b)^2 & & & & 1 & & 2 & & 1 & & & \\
(a+b)^3 & & & 1 & & 3 & & 3 & & 1 & & \\
(a+b)^4 & & 1 & & 4 & & 6 & & 4 & & 1 & \\
(a+b)^5 & 1 & & 5 & & 10 & & 10 & & 5 & & 1
\end{array}$$

r^{th} term of $(a + b)^n$

$$r^{th}\text{ term} = \frac{n!}{(n-r+1)!(r-1)!}\,a^{n-r+1}\,b^{r-1}$$

To get the middle term (for even value of n),

set $r = \dfrac{n}{2} + 1$

Example

Find the 3rd term in the expansion of $(x^2 + y)^5$.

Solution A

Using the properties and Pascal's triangle:

$$\begin{aligned}(x^2+y)^5 &= (x^2)^5 + 5(x^2)^4\,y + 10(x^2)^3\,y^2\\ &= x^{10} + 5x^8 y + 10x^6 y^2\end{aligned}$$

Solution B

Using the formula:

$$r^{th}\text{ term} = \frac{n!}{(n-r+1)!(r-1)!}\,a^{n-r+1}\,b^{r-1}$$

$r = 3$ $\quad n = 5$

$a = x^2$ $\quad b = y$

$$\begin{aligned}3^{rd}\text{ term} &= \frac{5!}{(5-3+1)!(3-1)!}\,(x^2)^{5-3+1}\,(y)^{3-1}\\ &= 10\,x^6\,y^2\end{aligned}$$

Coefficient of Next Term

$$C = \frac{(\text{Coefficient of previous term})(\text{exponent of } x)}{(\text{exponent of } y) + 1}$$

Example

Expand completely the expression $(x + y)^8$.

Solution

By principle, the first term is x^8, the second term is $8x^7 y$. The variable part of the third term is $x^6 y^2$.

The coefficient of the third term is, $C_3 = \dfrac{8(7)}{1+1} = 28$

3rd term = $28\,x^6 y^2$

4th term = $\dfrac{28(6)}{2+1}\,x^5 y^3 = 56\,x^5 y^3$

5th term = $\dfrac{56(5)}{3+1}\,x^4 y^4 = 70\,x^4 y^4$

$$\begin{aligned}(x+y)^8 = {} & x^8 + 8\,x^7 y + 28\,x^6 y^2 + 56\,x^5 y^3 + 70\,x^4 y^4\\ & + 56\,x^3 y^5 + 28\,x^2 y^6 + 8\,x\,y^7 + y^8\end{aligned}$$

Sum of Coefficient of Variables

To get the sum of the coefficients in the expansion of $(ax + by + \ldots)^n$, substitute 1 to each of the variables $x, y, \ldots$.

Example

Find the sum of the coefficient of the variables in the expansion of $(2x + 3y - z)^8$.

Solution

Sum = $[2(1) + 3(1) - 1]^8 = 4^8 =$ **65,536**

Example

Find the sum of the coefficient of the variables in the expansion of $(3x - 5)^6$.

Solution

Note that the last term in the expansion of $(3x - 5)^6$ is constant and is equal to $(-5)^6$ = 15,625. This value must be subtracted from the result after substituting 1 for the variable x.

Sum of coefficients = $[3(1) - 5]^6 - (-5)^6 =$ **-15,561**

PROPORTION

Proportion is a statement of equality between two ratios.

In the following proportion:

$$a : b = c : d \quad \text{or} \quad \frac{a}{b} = \frac{c}{d}$$

b and c are called the means
a and d are called the extremes
d is the fourth proportional to a, b, and c.

In the ratio a/b, a is called the *antecedent* and b is called the *consequent*.

Mean Proportional

The mean proportional to two terms a and $b = \sqrt{ab}$

Properties of Proportion

1. Proportion by inversion
   $$\text{If } \frac{a}{b} = \frac{c}{d} \text{ then } \frac{b}{a} = \frac{d}{c}$$
2. Proportion by alteration
   $$\text{If } \frac{a}{b} = \frac{c}{d} \text{ then } \frac{a}{c} = \frac{b}{d}$$
3. Proportion by composition
   $$\text{If } \frac{a}{b} = \frac{c}{d} \text{ then } \frac{a+b}{b} = \frac{c+d}{d}$$
4. Proportion by division
   $$\text{If } \frac{a}{b} = \frac{c}{d} \text{ then } \frac{a-b}{b} = \frac{c-d}{d}$$
5. Proportion by composition and division
   $$\text{If } \frac{a}{b} = \frac{c}{d} \text{ then } \frac{a+b}{a-b} = \frac{c+d}{c-d}$$

QUADRATIC FORMULA

For the quadratic equation $Ax^2 + Bx + C = 0$

$$x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

where $B^2 - 4AC$ is called the discriminant.

If $B^2 = 4AC$, the roots are equal.
If $B^2 > 4AC$, the roots are real, unequal.
If $B^2 < 4AC$, the roots are imaginary.

Properties of Roots

If the roots of the quadratic equation $Ax^2 + Bx + C = 0$ are x_1 and x_2, then;

Sum of roots, $x_1 + x_2 = -\dfrac{B}{A}$

Product of roots, $x_1 x_2 = \dfrac{C}{A}$

PARTIAL FRACTION

Functions of x that can be expressed in the form $P(x)/Q(x)$, where both $P(x)$ and $Q(x)$ are polynomials of x, is known as rational functions.

Improper Fraction

A rational function is said to be an **improper fraction** if the degree of $P(x)$ is greater than or equal to the degree of $Q(x)$.

Improper Fractions: $\dfrac{3x^2 - 2x + 1}{2x^2 + 6}$, $\dfrac{4x^2 - 2x + 3}{3x + 2}$

Improper fractions may be expressed as the sum of a polynomial and a proper fraction.

For example: $\dfrac{12x^2 - 13x - 9}{4x - 7} = 3x + 2 + \dfrac{5}{4x - 7}$

Proper Fraction

A rational function is known as a **proper fraction** if the degree of $P(x)$ is less than the degree of $Q(x)$.

Proper Fraction: $\dfrac{2x^2 + 4x - 5}{5x^3 + 6x^2 - 2x - 1}$

Proper fractions such as $\frac{x-4}{2x^2-4x}$ can be expressed as the sum of **partial fraction**, provided that the denominator will *factorize*. Consider the following examples:

Example

$$\frac{2}{x}-\frac{3}{2x-4}=\frac{2(2x-4)-3x}{x(2x-4)}=\frac{x-8}{x(2x-4)}$$

If we reverse the process:

$$\frac{x-8}{x(2x-4)}=\frac{2}{x}-\frac{3}{2x-4}$$

Thus, the fraction $\frac{x-8}{x(2x-4)}$ can be expressed or resolved into **partial fractions** $\frac{2}{x}-\frac{3}{2x-4}$.

Method of Resolving Proper Fraction into Partial Fraction

The method of resolving to partial fraction is more easily understood from the following examples:

$$\frac{3}{(x-1)(x+2)}=\frac{A}{x-1}+\frac{B}{x+2}$$

$$\frac{3x^2+2x+1}{(x-1)^3}=\frac{A}{x-1}+\frac{B}{(x-1)^2}+\frac{C}{(x-1)^3}$$

$$\frac{3x-5}{(x-1)(x^2+x+1)^2}=\frac{A}{x-1}+\frac{Bx+C}{x^2+x+1}+\frac{Dx+E}{(x^2+x+1)^2}$$

$$\frac{3x-2}{(x-1)^2(x^2+x+1)}=\frac{A}{x-1}+\frac{B}{(x-1)^2}+\frac{Cx+D}{x^2+x+1}$$

The following are the different cases of fractions that can be resolved into partial fraction.

Case I

Factors of the denominator all linear, none repeated

$$\frac{3x^2+32x-51}{(x-1)(x-2)(x+3)}=\frac{A}{x-1}+\frac{B}{x-2}+\frac{C}{x+3}$$

$$3x^2+32x-51=A(x-2)(x+3)+B(x-1)(x+3)+C(x-1)(x-2)$$

This equation is an *identity*; hence it is true for any value of x.

To solve for A, set $x = 1$; $A = 4$
To solve for B, set $x = 2$; $B = 5$
To solve for C, set $x = -3$; $C = -6$

Case II

Factors of the denominator all linear, some repeated.

$$\frac{4x^2+7x+8}{x(x+2)^3}=\frac{A}{x}+\frac{B}{x+2}+\frac{C}{(x+2)^2}+\frac{D}{(x+2)^3}$$

$$4x^2+7x+8=A(x+2)^3+Bx(x+2)^2+Cx(x+2)+Dx$$

Expand and equate the coefficients of like powers to solve for A, B, C, and D

Case III

Some factors of the denominator quadratic, none repeated

$$\frac{x^4-x^3+14x^2-2x+22}{(x+1)(x^2+4)(x^2-2x+5)}=\frac{A}{x+1}+\frac{Bx+C}{x^2+4}+\frac{Dx+E}{x^2-2x+5}$$

$$x^4-x^3+14x^2-2x+22=A(x^2+4)(x^2-2x+5)+(Bx+C)(x+1)(x^2-2x+5)+(Dx+E)(x+1)(x^2+4)$$

Expand and equate the coefficients of like powers to solve for A, B, C, D, and E

Case IV

Some factors of the denominator quadratic some repeated

$$\frac{3x^4-19x^3+60x^2-91x+64}{x(x^2-3x+4)^2}=\frac{A}{x}+\frac{Bx+C}{(x^2-3x+4)}+\frac{Dx+E}{(x^2-3x+4)^2}$$

$$3x^4-19x^3+60x^2-91x+64=A(x^2-3x+4)^2+(Bx+C)x(x^2-3x+4)+(Dx+E)x$$

Expand and equate the coefficients of like powers to solve for A, B, C, D, and E

Partial fractions are often used to help simplify a separate problem such as one involving integration.

$$\int \frac{4x+1}{2x^2+5x-3}dx = \int \frac{1}{2x-1}dx + \frac{2}{x+3}dx$$

VARIATION

A mathematical function that relates the values of one variable to those of other variables.

Direct Variation

If x is directly proportional to y, then,

$$x \propto y \text{ or } x = ky$$

k = constant of proportionality

Inverse Variation

If x is inversely proportional to y, then,

$$x \propto \frac{1}{y} \text{ or } x = \frac{k}{y}$$

Joint Variation

If x is directly proportional to y and inversely proportional to the square of z, then,

$$x \propto \frac{y}{z^2} \text{ or } x = k\frac{y}{z^2}$$

ARITHMETIC PROGRESSION A.P.

A sequence of numbers in which the difference of any two adjacent terms is constant.

Ex. 4, 7, 10, 13, 16,… (common difference = 3)

Elements:

a_1 = first term
a_n = nth term
a_m = any term before a_n
d = common difference
$d = a_2 - a_1 = a_4 - a_3 = a_7 - a_6$, etc.
S = sum of all the terms

n^{th} term of A.P.

$$a_n = a_1 + (n-1)\,d$$

$$\text{or } a_n = a_m + (n-m)d$$

Sum of n terms of A.P.

$$S = \frac{n}{2}(a_1 + a_n)$$

$$\text{or } S = \frac{n}{2}\left[2a_1 + (n-1)d\right]$$

GEOMETRIC PROGRESSION G.P.

A sequence of numbers in which the ratio of any two adjacent terms is constant.

Ex. 2, 6, 18, 54, … (common ratio, $r = 3$)

n^{th} term of G.P.

$$a_n = a_1 r^{n-1} \text{ or } a_n = a_m r^{n-m}$$

$$\text{Common ratio, } r = \frac{a_2}{a_1} = \frac{a_5}{a_4} = \ldots$$

Sum of n terms of G.P.

$$S = \frac{a_1(r^n - 1)}{r-1} \text{ when } r > 1$$

$$S = \frac{a_1(1 - r^n)}{1-r} \text{ when } r < 1$$

Sum of Infinite Geometric Progression (I.G.P.)

For a geometric progression where $-1 < r < 1$ and $n = \infty$ "infinity":

$$\text{Sum of I.G.P.} = \frac{a_1}{1-r}$$

HARMONIC PROGRESSION

A sequence of numbers in which their reciprocals forms an Arithmetic Progression.

Example Find the 12th term of the series 6, 3, 2.

Solution Their reciprocals are: 1/6; 1/3; ½ which forms an A.P. with a common difference d of 1/6.

In A.P. the 12th term is:

$$a_{12} = 1/6 + (12-1)(1/6) = 2$$

Therefore, in H.P. the 12th term is ½

WORK PROBLEM

$$\text{Rate} = \frac{1}{\text{time to finish the work}}$$

$$\text{Work done} = \text{Rate} \times \text{time}$$

Example If A can do a job in 4 hours and B can do the same job in 8 hours, working together from start (a) what part of the job have they done in 2 hours, and (b) how many hours can they finish the job?

Solution

Rate of $A = 1/4$
Rate of $B = 1/8$

Work done in 2 hours:
$1/4(2) + (1/8)(2) = 3/4$

This means 3/4 or 75% of the work was done.

Time to finish the job:
$1/4\,t + (1/8)t = 1$ (one complete job)
$t = 2.667$ **hours**

Work problem with n persons with the same rate doing the job

Principle:

- If 8 persons can do a job in 6 days, the number of man-days to finish the job is 8(6) = **48 man-days**
- Thus, if 12 persons will do the job, it will take them 48/12 = 4 days to finish it.

Example

A job could be done by eleven workers in 15 days. Five workers started the job. They were reinforced with four more workers at the beginning of the 6th day. Find the total number of days it will take them to finish the job.

Solution

Let t = number of days the four workers has to work with the five workers to finish the job.

Number of man-days to finish the job = 11(15) = 165 man-days

Five workers started the job for 5 days and (5 + 4) workers continued the job at the beginning of the 6th day for t days until completion.

$5(5) + (5 + 4)t = 165$; $t = 15.56$
Total number of days = 15.56 + 5 = **20.56 days**

PRINCIPLES IN AGE PROBLEMS

The difference of the ages of two persons is constant.

If x is the age of Peter now:
His age 5 years ago is $x - 5$
His age 7 years hence is $x + 7$.

PRINCIPLES IN DIGIT PROBLEMS

For a three-digit number:

Let: h = hundred's digit
t = ten's digit
u = unit's digit

The number is: $100h + 10t + u$

The reversed number is: $100u + 10t + h$

The sum of the digits is: $h + t + u$

PRINCIPLES IN NUMBER PROBLEMS

Let x be the first number and y be the second number
First number is twice the other: $x = 2y$
First number is five (5) more than thrice the other:
$x = 3y + 5$
First number is six (6) less than one half of the other:
$x = y/2 - 6$

The sum of their squares = $x^2 + y^2$
The cube of their difference = $(x - y)^3$

CLOCK PROBLEM

If the minute hand moves a distance of x, the hour hand moves $x/12$.

If the second hand move a distance x, the minute-hand moves $x/60$ and the hour hand moves $x/720$.

In 12 hours, the minute-hand and the hour-hand of the clock overlap each other for 11 times (not 12 times). So in one day, they will be together for 22 times.

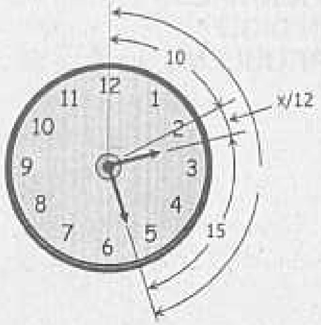
Each five-minute mark is subtends an angle of 30° from the center of the clock.

Example

How many minutes after 2 o'clock will the hands of the clock be perpendicular for the first time?

Solution

$x = 10 + x/12 + 15$
$x = 27.273$ min

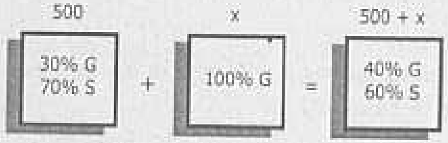


MIXTURE PROBLEM

Example

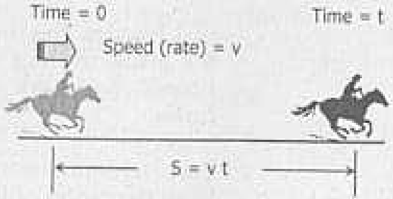
How many grams of gold must be added with 500 grams of an alloy containing 30% gold and 70% silver in order to produce another alloy analyzing 40% gold and 60% silver?

Solution



$500(30\%) + x\,(100\%) = (500 + x)(40\%)$
$1500 + 10x = 2000 + 4x$
$x = 83.33$ grams

MOTION PROBLEM (Uniform Motion or Constant Speed)

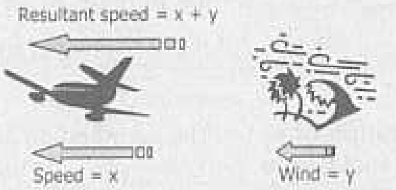


$$S = v\,t$$

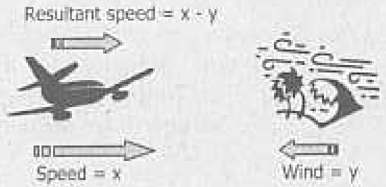
where S = distance, t = time, v = speed

If x = speed of airplane (or boat) in still air (or still water) and y = speed of wind (or water current) in the same direction then the speed of the airplane (or boat) with the wind (or current) is $x + y$ and its speed against the wind (current) is $x - y$.

Airplane Traveling with the Wind



Airplane Traveling against the Wind



PERMUTATION

Permutation refers to an arrangement of objects in a definite order.

Permutation of n different things taken r at a time

The permutation of n different things taken r at a time is:

$$P(n, r) = \frac{n!}{(n-r)!} \text{ and } P(n, n) = n!$$

Note: $0! = 1$

Example

How many permutations can be made out of the letters in the word DIEGO taken 3 at a time?

Solution

$n = 5;\ r = 3$

$$P(5, 3) = \frac{5!}{(5-3)!} = 60 \text{ ways}$$

Permutation of n objects with some objects identical

The permutation of n things of which q are alike, r are alike, and so on is:

$$P = \frac{n!}{q!\,r!\ldots}$$

Example

How many permutations can be made out of the letters in the word G I L L E S A N I A?

Solution

$n = 10$; (2-I, 2-L, 2-A)

$$P = \frac{10!}{2!2!2!} = 453{,}600 \text{ ways}$$

Permutation of n Things in a Circle (Cyclical Permutation)

The permutation of n different objects in a circle is:

$$P = (n-1)!$$

COMBINATION

Combination refers to a collection of objects without regard to sequence or order in which they were chosen.

Combination of n things taken r at a time

The combination of n things taken r at a time is:

$$C(n, r) = \frac{P(n,r)}{r!} = \frac{n!}{(n-r)!r!}$$

$$\text{and } C(n, n) = 1$$

Example

How many ways can you draw 3 QUEENs and 2 KINGs from a deck of 52 cards?

Solution

A deck of 52 cards has 4 QUEENs and 4 KINGs, thus:

$C = C(4, 3) \times C(4, 2) = 24$ ways

Combination of n things taken 1, 2, 3, ... n at a time

The combination of n object taken one, two, three, ... n at a time is:

$$C = 2^n - 1$$

Example

How many ways can you invite any one or more of your five friends to your birthday party?

Solution

In this case, it was not specified if how many of your friend you will invite at a time. Thus you may invite only one, or only two, or only three, or only four, or all five of them.

$C = 2^5 - 1 = 31$ ways

PROBABILITY

Probability is the likelihood that an event will occur expressed as the ratio of the number of favorable outcomes in the set of outcomes divided by the total number of possible outcomes

Single Event

$$\text{Probability} = \frac{\text{number of favorable ways}}{\text{total number of ways}}$$

If an event can happen in h ways and can fail in f ways, then the probability that the event will happen is:

$$p = \frac{h}{h+f}$$

and the probability that the event will fail is:

$$q = \frac{f}{h+f}$$

$$\text{and } p + q = 1$$

Example

For a single question in the board exams, there are four choices and only one of which is correct. By guessing, what is the probability that you will get the correct answer?

Solution

The event here is to get the correct answer and there are four trials. Out of four trials, the event (correct answer) can happen only once, and can fail three times. Thus, the probability that the event can happen is:

$$p = \frac{1}{1+3} = \frac{1}{4}$$

Multiple Events

Dependent and Independent Events

Two or more events are said to be dependent if the happening of one affects the probability of the happening of the others, and independent if the happening of one does not affect the probability of the happening of the others.

The probability of happening of two or more independent of dependent events is the product of their individual probabilities.

$$P = P_1 \times P_2 \times P_3 \times \ldots$$

Mutually Exclusive Events

Two or more events are said to be Mutually Exclusive if it is impossible for more than one of them to happen in a single trial.

The probability that some one, two, or more mutually exclusive events to happen is the sum of their individual probabilities.

$$P = P_1 + P_2 + P_3 + \ldots$$

Example

A box contains 4 blue chips and 5 red chips.

a) If one chip is drawn at random what is the probability that it is blue?
b) If two chips is drawn at random, what is the probability that both are red?
c) If two chips are drawn at random, what is the probability that one is blue and the other is red?

Solution

a) Single Event: There are four blue chips out of nine chips.

$P = 4/9$

b) Multiple Events: The events (getting red) are to occur twice.

First draw red: There are five red chips out of nine chips.

$P_1 = 5/9$

Second draw red: There are now only four chips out of eight chips.

$P_2 = 4/8 = ½$

Thus;

$P = P_1 \times P_2 = (5/9) \times (1/2) = \mathbf{15/18}$

c) Mutually Exclusive: The event here is to get a red and a blue ball in two draws. This can happen in two ways: (first draw red and second draw blue) and another is (first draw blue and second draw red), but these two cannot happen at the same time, hence they are mutually exclusive events.

First draw RED second draw BLUE

$$P_1 = (5/9) \times (4/8) = \mathbf{5/18}$$

First draw BLUE second draw RED

$$P_2 = (4/9) \times (5/8) = \mathbf{5/18}$$

Thus; $P = P_1 + P_2 = (5/18) + (5/18) = \mathbf{10/18}$

Repeated Trials

The probability that an event can occur exactly r times in n trials is:

$$P_{(n,r)} = C(n,r)p^r q^{n-r}$$

Where p is the probability that the event happen and q is the probability that the event can fail.

Example

There are ten questions in an examination. The probability that an examinee will get the correct answer is 0.25. What is the probability that he will get (*a*) exactly 7, and (*b*) at least 7 correct answers?

Solution

a) There are 10 questions, $n = 10$ with $p = 0.25$ and $q = 0.75$. The probability of getting exactly 7 is:

$P_{(10,7)} = C(10, 7)(0.25)^7(0.75)^{10-7}$
$P_{(10,7)} = \mathbf{0.00309}$ **or** $\mathbf{405/131072}$

b) "At least seven" means can be exactly 7, or 8, or 9, or 10.

$$P_{\text{at least }7} = P_{(10,7)} + P_{(10,8)} + P_{(10,9)} + P_{(10,10)}$$
$$\begin{aligned} P_{\text{at least }7} = {} & C(10, 7)(0.25)^7(0.75)^{10-7} \\ & + C(10, 8)(0.25)^8(0.75)^{10-8} \\ & + C(10, 9)(0.25)^9(0.75)^{10-9} \\ & + C(10, 10)(0.25)^{10}(0.75)^{10-10} \end{aligned}$$
$$\begin{aligned} P_{\text{at least }7} = {} & 0.00309 + 0.000386 + 0.0000286 \\ & + 0.000000953 \end{aligned}$$

$$P_{\text{at least }7} = \mathbf{0.00351}$$

The "At least one" Condition

The probability that the event can happen at lest once in n trials is:

$$P = 1 - Q$$

where Q is the probability that the event will totally fail.

Example

The probability of getting a credit in each of three examinations is 0.65. What is the probability of getting at least one credit?

Solution

There are three trials ($n = 3$) with $p = 0.65$ and $q = 0.35$. The probability of getting no credit at all is $Q = (0.35)(0.35)(0.35) = 0.042875$. Thus,

$$P = 1 - Q = 1 - 0.042875 = \mathbf{0.957125}$$

Example In a shooting game, the probability that Kim, Ken, and Karla can hit the target is 1/3, 1/4, and 1/6, respectively. What is the probability that the target will be hit if they all shoot at it once?

Solution In this problem, it is not necessary that the three of them must hit the target. The target may be hit by any one, or two, or all of them. Thus the "at least one" condition holds.

Then;

$P = 1 - Q = 1 - (2/3)(3/4)(5/6)$
$P = 7/12$

MATRICES AND DETERMINANTS

A matrix is a rectangular collection of variables or scalars contained within a set of square [ ] or round ( ) brackets. a matrix consist of m rows and n columns.

Classification of Matrices

Square Matrix A matrix whose number of rows m is equal to the number of columns n.

Diagonal Matrix A diagonal matrix is a square matrix will all zero values except for the a_{ij} value for all $i = j$.

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Identity Matrix An identity matrix is a diagonal matrix with all non-zero entries equal to 1.

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Scalar Matrix A scalar matrix is a diagonal matrix with all non-zero entries equal to some other constant.

$$\begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{bmatrix}$$

Triangular Matrix A triangular matrix has zeros in all positions above or below the diagonal.

Important Algebraic Operations of Matrices

1. Equality of Matrices Two matrices are equal if they have the same number of rows and columns and their corresponding entries are also equal.

2. Addition and Subtraction of Matrices Addition (or subtraction) of two matrices can be accomplished by adding (or subtracting) the corresponding entries of two matrices which have the same shape.

Ex: Add: $\begin{bmatrix} 1 & 4 & 1 \\ 7 & 1 & 6 \\ -3 & 0 & 4 \end{bmatrix} + \begin{bmatrix} 3 & 0 & 2 \\ 2 & 5 & 6 \\ 9 & 1 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 1+3 & 4+0 & 1+2 \\ 7+2 & 1+5 & 6+6 \\ -3+9 & 0+1 & 4+1 \end{bmatrix} = \begin{bmatrix} 4 & 4 & 3 \\ 9 & 6 & 12 \\ 6 & 1 & 5 \end{bmatrix}$$

3. Multiplication of Matrices Multiplication of matrix can be done only if the number of columns of the left-hand matrix is equal to the number of rows of the right-hand matrix. Multiplication is accomplished by multiplying the elements in each right-hand matrix column, adding the products, and then placing the sum at the intersection point of the involved row and column.

Ex:

If $A = \begin{bmatrix} 2 & 3 \\ 3 & 10 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 3 \\ 5 & -2 \end{bmatrix}$, find $A \times B$.

Sol:

$$A \times B = \begin{bmatrix} 2 & 3 \\ 3 & 10 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 5 & -2 \end{bmatrix}$$

$$A \times B = \begin{bmatrix} 2(1)+3(5) & 2(3)+3(-2) \\ 3(1)+10(5) & 3(3)+10(-2) \end{bmatrix} = \begin{bmatrix} 17 & 0 \\ 53 & -11 \end{bmatrix}$$

Ex: $$\begin{bmatrix} 2 & 1 & 5 \\ 1 & 4 & 7 \end{bmatrix} \times \begin{bmatrix} 2 \\ 4 \\ 1 \end{bmatrix} = \begin{bmatrix} 2(2)+1(4)+5(1) \\ 1(2)+4(4)+7(1) \end{bmatrix} = \begin{bmatrix} 13 \\ 25 \end{bmatrix}$$

Ex: If $\begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 6 \end{bmatrix}$ then:

$1x + 3y = 5$

$2x + 4y = 6$

4. Division of Matrices

Division of matrices can be accomplished only by multiplying the inverse of the denominator matrix.

Other Operations on a Matrix

1. The Transpose of a Matrix

The transpose is an $(n \times m)$ matrix formed from the original $(m \times n)$ matrix by taking the i^{th} row and making it the i^{th} column. The diagonal is unchanged in this operation. The transpose of a matrix is indicated as A'.

Ex: Determine the transpose of $A = \begin{bmatrix} 1 & 6 & 9 \\ 2 & 3 & 4 \\ 7 & 1 & 5 \end{bmatrix}$

Sol: $$A' = \begin{bmatrix} 1 & 2 & 7 \\ 6 & 3 & 1 \\ 9 & 4 & 5 \end{bmatrix}$$

2. The Determinant of a Matrix

The determinant D, is a scalar calculated from a square matrix. The determinant of a matrix is indicated by enclosing the matrix by vertical lines.

Properties of Determinant

A. If a matrix has a row or column of zeros, the determinant is zero.

$$\begin{vmatrix} 1 & 4 & 0 \\ 4 & 5 & 0 \\ 1 & 7 & 0 \end{vmatrix} = 0 \qquad \begin{vmatrix} 2 & 1 & 5 \\ 0 & 0 & 0 \\ 4 & 1 & 9 \end{vmatrix} = 0$$

B. If a matrix has two identical rows or columns, the determinant is zero.

$$\begin{vmatrix} 1 & 2 & 5 \\ 4 & 6 & 1 \\ 1 & 2 & 5 \end{vmatrix} = 0 \qquad \begin{vmatrix} 3 & 3 & 6 \\ 3 & 3 & 1 \\ 6 & 6 & 2 \end{vmatrix} = 0$$

C. If a matrix is triangular, the determinant is equal to the product of the diagonal entries.

$$\begin{vmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{vmatrix} = (2)(3)(5) = 30$$

D. The value of the determinant is not changed if corresponding rows and columns are interchanged.

$$\begin{vmatrix} 1 & 4 & 6 \\ 2 & 5 & 2 \\ 1 & 7 & 9 \end{vmatrix} = \begin{vmatrix} 1 & 2 & 1 \\ 4 & 5 & 7 \\ 6 & 2 & 9 \end{vmatrix}$$

E. If each of a column or row of a determinant is multiplied by m, the value of the determinant is multiplied by m.

$$\begin{vmatrix} 1 & 4 & 5 \\ 4 & 6 & 1 \\ 2 & 8 & 4 \end{vmatrix} = \begin{vmatrix} 1 & 2\times2 & 5 \\ 4 & 3\times2 & 1 \\ 2 & 4\times2 & 4 \end{vmatrix} = 2 \begin{vmatrix} 1 & 2 & 5 \\ 4 & 3 & 1 \\ 2 & 4 & 4 \end{vmatrix}$$

By properties defined in *B* & *E*, the following can be applied:

$$\begin{vmatrix} 1 & 4 & 2 \\ 5 & 1 & 10 \\ 3 & 6 & 6 \end{vmatrix} = 0$$

(since the elements of columns 1 and 3 are exact multiples)

F. If two columns or rows of a determinant are interchanged the, sign is changed.

$$\begin{vmatrix} 2 & 1 & 6 \\ 5 & 4 & 7 \\ 1 & 3 & 9 \end{vmatrix} = - \begin{vmatrix} 2 & 6 & 1 \\ 5 & 7 & 4 \\ 1 & 9 & 3 \end{vmatrix}$$

G. The value of a determinant is not changed if each element of a column (or row) is multiplied by a number k and added (or subtracted) to the corresponding elements of a column (or row).

$$\begin{vmatrix} 1 & 4 & 5 \\ 4 & 6 & 1 \\ 2 & 8 & 4 \end{vmatrix} = \begin{vmatrix} 1 & 4 & 5+1(3) \\ 4 & 6 & 1+4(3) \\ 2 & 8 & 4+2(3) \end{vmatrix} = \begin{vmatrix} 1 & 4 & 8 \\ 4 & 6 & 13 \\ 2 & 8 & 10 \end{vmatrix}$$

H. If each element of a column (say the k^{th} column) of a matrix is expressed as the sum of two terms, the determinant is equal to the sum of the two determinants, where (*a*) the elements of each of the two determinants are identical to the corresponding elements of the given determinant except for the elements of the k^{th} column, and (*b*) the first term of the k^{th} column of the given determinant form the k^{th} column of one of the two determinants and the second term form the k^{th} column of the other determinant..

$$\begin{vmatrix} 4 & 1 & 2 \\ 6 & 2 & 3 \\ 2 & 5 & 1 \end{vmatrix} = \begin{vmatrix} 2 & 1 & 2 \\ 5 & 2 & 3 \\ 1 & 5 & 1 \end{vmatrix} + \begin{vmatrix} 2 & 1 & 2 \\ 1 & 2 & 3 \\ 1 & 5 & 1 \end{vmatrix}$$

Example

Solve for x: (2^{nd} order)

$$x = \begin{vmatrix} 4 & 5 \\ 2 & 3 \end{vmatrix} = (4)(3) - (2)(5) = 2$$

Example

Solve for x: (3^{rd} order)

$$x = \begin{vmatrix} 1 & -3 & 2 \\ 2 & 1 & -4 \\ 3 & -2 & 1 \end{vmatrix}$$

$$x = 1\begin{vmatrix} 1 & -4 \\ -2 & 1 \end{vmatrix} - 2\begin{vmatrix} -3 & 2 \\ -2 & 1 \end{vmatrix} + 3\begin{vmatrix} -3 & 2 \\ 1 & -4 \end{vmatrix}$$

$$x = 1(1 - 8) - 2(-3 + 4) + 3(12 - 2) = 21$$

Use of sign: $\begin{vmatrix} + & - & + \\ - & + & - \\ + & - & + \end{vmatrix}$

Another Solution:

$$x = \begin{vmatrix} 1 & -3 & 2 \\ 2 & 1 & -4 \\ 3 & -2 & 1 \end{vmatrix}$$

$$x = \left|\begin{array}{ccc|cc} 1 & -3 & 2 & 1 & -3 \\ 2 & 1 & -4 & 2 & 1 \\ 3 & -2 & 1 & 3 & -2 \end{array}\right|$$

$$x = [\,(1)(1)(1) + (-3)(-4)(3) + (2)(2)(-2)\,]$$
$$-\,[(3)(1)(2) + (-2)(-4)(1) + (1)(2)(-3)]$$

$x = \mathbf{21}$

Example

Solve for x: (4^{th} order)

$$x = \begin{vmatrix} 2 & 0 & 1 & -1 \\ 1 & 2 & -2 & 3 \\ 3 & -2 & 3 & -2 \\ 4 & -4 & -2 & -3 \end{vmatrix}$$

By pivotal element method, select the element in the second row, first column: (since it's a unity)

$$x = 1(-1)^{2+1}\begin{vmatrix} 0-(2)(2) & 1-(2)(-2) & -1-(2)(3) \\ -2-(3)(2) & 3-(3)(-2) & -2-(3)(3) \\ -4-(4)(2) & -2-(4)(-2) & -3-(4)(3) \end{vmatrix}$$

$$x = -1\left|\begin{array}{ccc|cc} -4 & 5 & -7 & -4 & 5 \\ -8 & 9 & -11 & -8 & 9 \\ -12 & 6 & -15 & -12 & 6 \end{array}\right|$$

$$x = -1\{[540 + 660 + 336\,] - [\,756 + 264 + 600\,]\} = 84$$

3. The Cofactor of an Entry in a Matrix

The cofactor of an entry in a matrix is the determinant of the matrix formed by omitting the entry's row and column in the original matrix. The sign of the cofactor is determined from the following positional matrices:

$$\begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix}$$

or, the sign of the cofactor can be determined by the relation $(-1)^{i+j}$, where i is column number and j is the row number.

Example

Find the cofactor of -2 in the following matrix.

$$\begin{bmatrix} 2 & 7 & 3 \\ -2 & 5 & 6 \\ 3 & 4 & 7 \end{bmatrix}$$

Solution

-2 is at column 1 row 2. The resulting matrix is:

$$(-1)^{1+2}\begin{bmatrix} 7 & 3 \\ 4 & 7 \end{bmatrix}$$

The cofactor is:

$$-1\begin{vmatrix} 7 & 3 \\ 4 & 7 \end{vmatrix} = -[(7)(7) - (4)(3)] = -37$$

4. Classical Adjoint

The classical adjoint is a matrix formed from the transposed cofactor matrix with the conventional sign arrangement. The resulting matrix is represented as A_{adj}.

Example

Determine the classical adjoint of

$$\begin{bmatrix} 2 & 3 & -4 \\ 0 & -4 & 2 \\ 1 & 1 & 5 \end{bmatrix}$$

Solution

After solving the cofactors of each entry, the matrix of cofactors is:

$$\begin{bmatrix} -18 & 2 & 4 \\ -11 & 14 & 5 \\ -10 & -4 & -8 \end{bmatrix}$$

The classical adjoint is:

$$A_{adj} = \begin{bmatrix} -18 & -11 & -10 \\ 2 & 14 & -4 \\ 4 & 5 & -8 \end{bmatrix}$$

5. The Inverse Matrix

The inverse, A^{-1}, of a matrix A is a matrix such that $(A)(A^{-1}) = I$, where I is a square matrix with ones along the left-to-right diagonal and zeros elsewhere.

Example

Determine the inverse of $\begin{bmatrix} 4 & 5 \\ 2 & 3 \end{bmatrix}$

Solution

The determinant is:

$$D = \begin{vmatrix} 4 & 5 \\ 2 & 3 \end{vmatrix} = 4(3) - 2(5) = 2$$

The inverse is:

$$\frac{1}{2}\begin{bmatrix} 3 & -5 \\ -2 & 4 \end{bmatrix} = \begin{bmatrix} 3/2 & -5/2 \\ -1 & 2 \end{bmatrix}$$

SOLUTION TO SYSTEMS OF LINEAR EQUATION USING DETERMINANTS (CRAMER'S RULE)

For a system of linear equations:

$$x = \frac{N_x}{D};\; y = \frac{N_y}{D};\ldots$$

where:

D = determinant of the coefficient of the variables

N_x = determinant taken form D replacing the coefficients of x by their corresponding constant terms leaving all other terms unchanged.

N_y = determinant taken form D replacing the coefficients of y by their corresponding constant terms leaving all other terms unchanged.

Example

Solve for x, y, and w in the following equations:

$3x - 2y + w = 11,$
$x + 5y - 2w = -9,$
$2x + y - 3w = -6.$

Solution

$$D = \left|\begin{array}{ccc|cc} 3 & -2 & 1 & 3 & -2 \\ 1 & 5 & -2 & 1 & 5 \\ 2 & 1 & -3 & 2 & 1 \end{array}\right|$$

$$D = (-45 + 8 + 1) - (10 - 6 + 6) = -46$$

$$N_x = \left|\begin{array}{ccc|cc} 11 & -2 & 1 & 11 & -2 \\ -9 & 5 & -2 & -9 & 5 \\ -6 & 1 & -3 & -6 & 1 \end{array}\right|$$

$$N_x = (-165 - 24 - 9) - (-30 - 22 - 54) = -92$$

$$x = \frac{N_x}{D} = \frac{-92}{-46} = 2$$

$$N_y = \left|\begin{array}{ccc|cc} 3 & 11 & 1 & 3 & 11 \\ 1 & -9 & -2 & 1 & -9 \\ 2 & -6 & -3 & 2 & -6 \end{array}\right|$$

$$N_y = (81 - 44 - 6) - (-18 + 36 - 33) = 46$$

$$y = \frac{N_y}{D} = \frac{46}{-46} = -1$$

$$N_w = \left|\begin{array}{ccc|cc} 3 & -2 & 11 & 3 & -2 \\ 1 & 5 & -9 & 1 & 5 \\ 2 & 1 & -6 & 2 & 1 \end{array}\right|$$

$$N_w = (-90 + 36 + 11) - (110 - 27 + 12) = -138$$

$$w = \frac{N_w}{D} = \frac{-138}{-46} = 3$$

COMPLEX NUMBERS

Complex number is a number combining real and imaginary parts: a number in the form $a + bi$, where a and b are real numbers and $i = \sqrt{-1}$, so that bi is imaginary unless b = 0.

Algebraic or Rectangular Form

The algebraic of rectangular form of a complex number is:

$$a + bi$$

where:

a = real part
bi = imaginary part
$i = \sqrt{-1}$ and $i^2 = -1$

Ex:

$i^7 = i^6\,\mathrm{i} = (i^2)^3\,i = (-1)^3 i = -i$
$i^{245} = i^{244}\,i = (i^2)^{122}\,i = (-1)^{122} i = i$

Addition or Subtraction of Complex Numbers

Addition or subtraction of complex numbers is obtained by combining similar terms and applying $i^2 = -1$

Simplify $i^{30} - 2i^{25} + 3i^{17}$

$$\begin{aligned} i^{30} - 2i^{25} + 3i^{17} &= (i^2)^{15} - 2i^{24}\,i + 3i^{16}\,i \\ &= (-1)^{15} - 2(i^2)^{12}\,i + 3(i^2)^8\,i \\ &= -1 - 2(-1)^{12}\,i + 3(-1)^8\,i \\ &= -1 - 2i + 3i = -1 + i \end{aligned}$$

Multiplication of Complex Numbers

Multiplication of complex numbers is similar to multiplication of polynomials.

Example

$$\begin{aligned} (3 + 2i)(4 - 3i) &= 12 - 9i + 8i - 6i^2 \\ &= 12 - i - 6(-1) = 18 - i \end{aligned}$$

Example

$$(3 + i)^2 = 9 + 6i + i^2 = 9 + 6i - 1 = 8 + 6i$$

Conjugate of Complex Number

The conjugate of a complex number is obtained by changing the sign of the imaginary part.

Number	Conjugate
$2 + 3i$	$2 - 3i$
$3 - 5i$	$3 + 5i$
$-5 + 2i$	$-5 - 2i$

The product of a complex number and its conjugate is always a real number.

Example

$$(2 + 3i)(2 - 3i) = 4 - 9i^2 = 4 - 9(-1) = 13$$

Division of Complex Numbers

Division of a complex number is obtained by multiplying the numerator and denominator by the conjugate of the denominator.

Example

$$\begin{aligned} \frac{3+4i}{2-i} \times \frac{2+i}{2+i} &= \frac{6+3i+8i+4i^2}{4-i^2} \\ &= \frac{6+11i-4}{4-(-1)} = \frac{2+11i}{5} = \frac{2}{5} + \frac{11}{5}i \end{aligned}$$

Complex Equation

Two complex numbers are equal if their real parts are equal and their imaginary parts are equal.

$(a + b\,i)$ is equal to $(c + d\,i)$ if $a = c$ and $b = d$

Example

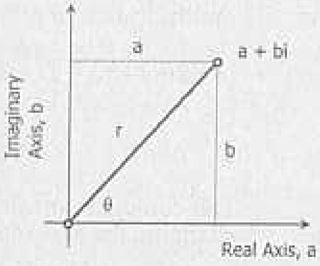
Solve for x & y if $3x - 2y\,i = 6 + 8i$

$3x = 6;\quad x = 2$
$-2y = 8;\quad y = -4$

Polar Form or Trigonometric Form

The polar form of a complex number is used to find the roots of a complex number.

Argand Diagram (Complex Plane)



In the Argand chart shown:
r = absolute value or modulus
θ = argument or amplitude
$a = r\cos\theta$
$b = r\sin\theta$

$$r = \sqrt{a^2 + b^2}$$

$$\tan\theta = \frac{b}{a}$$

$$a + bi = r\cos\theta + r\sin\theta\, i$$

$$a + bi = r(\cos\theta + i\sin\theta) = r\,\text{cis}\,\theta$$

$$a + bi = r\angle\theta$$

Multiplication

$$r_1\angle\theta_1 \times r_2\angle\theta_2 = r_1 r_2 \angle(\theta_1 + \theta_2)$$

Example

$5\angle 30^\circ \times 6\angle 45^\circ = \mathbf{30\angle 75^\circ}$

Division

$$\frac{r_1\angle\theta_1}{r_2\angle\theta_2} = \frac{r_1}{r_2}\angle(\theta_1 - \theta_2)$$

Example

$\frac{45\angle 67^\circ}{15\angle 17^\circ} = 3\angle 50^\circ$

De Moivre's Theorem

$$[r\angle\theta]^n = r^n \angle n\theta$$

True for all values of n.

Example

$[5\angle 15^\circ]^3 = 5^3\angle 3(15^\circ) = \mathbf{125\angle 45^\circ}$

To obtain the m^{th} root of a complex number, use De Moivre's theorem with $n = 1/m$.

In finding the m^{th} root of a complex number, there are m solutions. The modulus r is always the same and the argument θ are symmetrically spaced at $(360^\circ/m)$ apart, where m is the number of root required.

Example

Find $\sqrt{5 + 12i}$

Solution

There will be two roots, each are 360/2 = 180° apart.

First convert $5 + 12i$ to polar form:
$(a = 5, b = 12)$
$r = \sqrt{5^2 + 12^2} = 13$
$\tan\theta = 12/5;\ \theta = 67.38^\circ$
Thus; $\sqrt{5 + 12i} = [13\angle 67.38^\circ]^{1/2}$
$= 13^{1/2}\angle \tfrac{1}{2}(67.38)$
$= 3.61\angle 33.69^\circ$ and $3.61\angle 213.69^\circ$

Change to rectangular form:
$3.61\angle 33.69^\circ = 3.61(\cos 33.69^\circ + i\sin 33.69^\circ)$
$= 3 + 2i$
$3.61\angle 213.69^\circ = 3.61(\cos 213.69^\circ + i\sin 213.69^\circ)$
$= -3 - 2i$

Thus, the roots are $\mathbf{3 + 2i}$ and $\mathbf{-3 - 2i}$

Exponential Form

The exponential form of a complex number is used when finding the logarithms of a complex number.
$a + bi = r(\cos\theta + i\sin\theta)$
$e^{i\theta} = \cos\theta + i\sin\theta$
$e^{-i\theta} = \cos\theta - i\sin\theta$

$$a + bi = r e^{i\theta}$$

Example

Find $\ln(3 + 4i)$

Solution

Convert $3 + 4i$ to polar form $(a = 3, b = 4)$
$r = \sqrt{3^2 + 4^2} = 5$ $\tan\theta = 4/3$
$\theta = 53.13^\circ = 0.9273$ radians

$3 + 4i = 5\angle 53.13^\circ = 5\,e^{0.9273i}$

$$\begin{aligned}\ln(3+4i) &= \ln(5e^{0.9273i})\\ &= \ln 5 + \ln e^{0.9273i}\\ &= \ln 5 + 0.9273\,i \ln e \qquad \text{(but } \ln e = 1\text{)}\\ \ln(3+4i) &= \mathbf{1.609 + 0.9273}\boldsymbol{i}\end{aligned}$$

VENN DIAGRAM

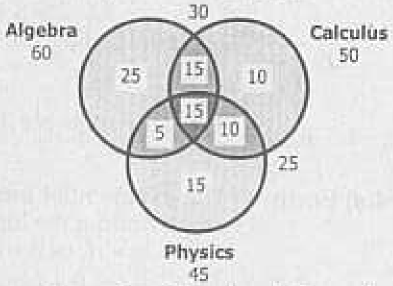
A mathematical diagram representing sets as circles, with their relationships to each other expressed through their overlapping positions, so that all possible relationships between the sets are shown

Example

An engineering professor conducted a survey regarding the favorite subjects of the students. The following data were gathered: 60 students like the subject algebra, 50 like the subject calculus, and 45 likes the subject physics. Thirty students like both algebra and calculus subjects, 25 students likes both calculus and physics subjects, and 20 students like both algebra and physics subjects. Only 15 students like all the three subjects. How many students were surveyed?

Solution

Represent each subject with a circle.



The diagram shows the following information:

25 like the subject algebra only
15 like the subject physics only
10 like the subject calculus only
15 likes both algebra and calculus subjects
10 likes both calculus and physics subjects
5 likes both algebra and physics subjects
15 likes all the three subjects

Thus, the total number of students surveyed is 25 + 15 + 10 + 15 + 10 + 5 + 15

Number of students surveyed = **95**

Problems - Set 1
Conversion

PROBLEM 1 - 1

What is the temperature in degree Celsius of absolute zero?

A. -32
B. 0
C. 273
D. -273

PROBLEM 1 - 2
ME April 1996

How many degrees Celsius is 100 degrees Fahrenheit?

A. 37.8 °C
B. 2.667 °C
C. 1.334 °C
D. 13.34 °C

PROBLEM 1 - 3
ECE Nov. 1997

A comfortable room temperature is 72 °F. What is this temperature, expressed in degrees Kelvin?

A. 263
B. 290
C. 295
D. 275

PROBLEM 1 - 4

255°C is equivalent to:

A. 491 °F
B. 427 °F
C. 173.67 °F
D. 109.67 °F

PROBLEM 1 - 5

At what temperature will the °C and °F readings be equal?

A. 40°
B. -40°
C. 32°
D. 0°

PROBLEM 1 - 6
ME Oct. 1994

How many degree Celcius is 80 degrees Fahrenheit?

A. 26.67
B. 86.4
C. 33.33
D. 16.33

PROBLEM 1 - 7
EE Oct. 1990

What is the absolute temperature of the freezing point of water in degree Rankine?

A. -32
B. 0
C. 428
D. 492

PROBLEM 1 - 8

The angle of inclination of the road is 32°. What is the angle of inclination in mils?

A. 456.23
B. 568.89
C. 125.36
D. 284.44

PROBLEM 1 - 9

An angle measures x degrees. What is its measure in radians?

A. $180^\circ x/\pi$
B. $\pi x/180^\circ$
C. $180^\circ \pi/x$
D. $180\pi x$

PROBLEM 1 - 10
ECE Nov. 1995

Express 45° in mils.
A. 80 mils
B. 800 mils
C. 8000 mils
D. 80000 mils

PROBLEM 1 - 11
ME April 1997

What is the value in degrees of π radians?
A. 90°
B. 57.3°
C. 180°
D. 45°

PROBLEM 1 - 12
CE May 1993

How many degrees is 3200 mils?
A. 360°
B. 270°
C. 180°
D. 90°

PROBLEM 1 - 13
ECE Nov. 1995

An angular unit equivalent to 1/400 of the circumference of a circle is called:
A. mil
B. grad
C. radian
D. degree

PROBLEM 1 - 14
EE Oct. 1994

Carry out the following multiplication and express your answer in cubic meters: 3cm × 5mm × 2m.
A. 3×10^{-3}
B. 3×10^{-4}
C. 8×10^{-2}
D. 8×10^{2}

PROBLEM 1 - 15
ME April 1994

Add the following and express in meters: 3m + 2 cm + 70 mm
A. 2.90 m
B. 3.14 m
C. 3.12 m
D. 3.09 m

PROBLEM 1 - 16

One nautical mile is equivalent to:
A. 5280 ft.
B. 6280 ft.
C. 1.256 statute mile
D. 1.854 km

PROBLEM 1 - 17
ME Oct. 1994

How many square feet is 100 square meters?
A. 328.10
B. 956.36
C. 1075.84
D. 1563.25

PROBLEM 1 - 18

A tank contains 1500 gallons of water. What is the equivalent in cubic meters?
A. 4.256
B. 5.865
C. 6
D. 5.685

PROBLEM 1 - 19
ME Oct. 1994

How many cubic feet is equivalent to 100 gallons of water?

A. 74.80
B. 1.337
C. 13.37
D. 133.7

PROBLEM 1 - 20
ME April 1998

How many cubic meters is 100 gallons of liquid?
A. 0.1638 cu. meter
B. 1.638 cu. meters
C. 0.3785 cu. meter
D. 3.7850 cu. meters

PROBLEM 1 - 21
ME Oct. 1995

The number of board feet in a plank 3 inches thick, 1 foot wide, and 20 feet long is:
A. 30
B. 60
C. 120
D. 90

PROBLEM 1 - 22

Which of the following is correct?
A. 1 horsepower = 746 kW
B. 1 horsepower = 0.746 watts
C. 1 horsepower = 0.746 kW
D. 1 horsepower = 546 watts

PROBLEM 1 - 23
ME Oct. 1996

The acceleration due to gravity in English unit is equivalent to:
A. 32.2 ft/sec^2
B. 3.22 ft/sec^2
C. 9.81 ft/sec^2
D. 98.1 ft/sec^2

PROBLEM 1 - 24
ME April 1999

The prefix nano is opposite to:
A. mega
B. tera
C. hexa
D. giga

PROBLEM 1 - 25
ME Oct. 1996

10 to the 12th power is the value of the prefix:
A. giga
B. pico
C. tera
D. peta

PROBLEM 1 - 26
CE Nov. 2003

Convert 630 degrees to its value in centesimal system.
A. 700 grade
B. 800 grade
C. 750 grade
D. 850 grade

PROBLEM 1 - 27
CE Nov. 2003

When rounded-off to four significant figures, 102.48886 becomes:
A. 102.4
B. 102.4889
C. 102.4888
D. 102.5

PROBLEM 1 - 28

Which of the following is a prime number?
A. 221
B. 63
C. 523
D. 703

PROBLEM 1 - 29

The number of significant figures in the value 500.00 is:
A. one
B. three
C. five
D. six

PROBLEM 1 - 30

A line on a map was drawn at a scale of 5:100,000. If a line in the map is 290 mm long, the actual length of the line is:

A. 4.8 km
B. 5.8 km
C. 2.9 km
D. 6.4 km

PROBLEM 1 - 31

Which of the following is correct:

A. 0.001 has three significant figures
B. 100 has three significant figures
C. 100.00 has five significant figures
D. 0.000249 has six significant figures

PROBLEM 1 - 32

The scale on the map is 1:*x*. A lot having an area of 640 sq. m. is represented by an area of 25.6 cm² on the map. What is the value of *x*?

A. 500
B. 1000
C. 50
D. 100

ANSWER SHEET

1.	A	B	C	D	E	11.	A	B	C	D	E	21.	A	B	C	D	E
2.	A	B	C	D	E	12.	A	B	C	D	E	22.	A	B	C	D	E
3.	A	B	C	D	E	13.	A	B	C	D	E	23.	A	B	C	D	E
4.	A	B	C	D	E	14.	A	B	C	D	E	24.	A	B	C	D	E
5.	A	B	C	D	E	15.	A	B	C	D	E	25.	A	B	C	D	E
6.	A	B	C	D	E	16.	A	B	C	D	E	26.	A	B	C	D	E
7.	A	B	C	D	E	17.	A	B	C	D	E	27.	A	B	C	D	E
8.	A	B	C	D	E	18.	A	B	C	D	E	28.	A	B	C	D	E
9.	A	B	C	D	E	19.	A	B	C	D	E	29.	A	B	C	D	E
10.	A	B	C	D	E	20.	A	B	C	D	E	30.	A	B	C	D	E

Solutions to Set 1
Conversion

SOLUTION 1 - 1
Ans: D

Absolute temperature = 273 + °C = 0
°C = **-273**

SOLUTION 1 - 2
Ans: A

°C = (°F - 32)(5/9)
°C = (100 - 32)(5/9) = **37.8**

SOLUTION 1 - 3
Ans: C

°K = °C + 273
°C = (°F - 32)(5/9) = (72 - 32)(5/9) = 22.22
°K = 22.22 + 273 = **295.22 °K**

SOLUTION 1 - 4
Ans: A

°F = $\frac{9}{5}$ °C + 32 = $\frac{9}{5}$ (255) + 32
°F = **491 °F**

SOLUTION 1 - 5
Ans: B

The relationship between °F and °C is given by the equation:
°F = $\frac{9}{5}$ °C + 32
since °F = °C, then
°F = $\frac{9}{5}$ °F + 32
- (4/5)°F = 32; °F = - 40°

SOLUTION 1 - 6
Ans: A

°C = $\frac{5}{9}$ (°F – 32) = $\frac{5}{9}$ (80 – 32)
°C = **26.67°C**

SOLUTION 1 - 7
Ans: D

The freezing point of water is 0°C or 32°F.
°R = °F + 460 = 32 + 460
= **492°R**

SOLUTION 1 - 8
Ans: B

Angle = 32° × (6400 mils/360°)
Angle = **568.89 mils**

SOLUTION 1 - 9
Ans: B

x° (π radians/180°) = **π*x*/180°**

SOLUTION 1 - 10
Ans: B

Note: 90° = 2π radians = 100 grads = 1600 mils

$$\text{Angle} = 45° \times \frac{1600 \text{ mils}}{90°}$$

Angle = **800 mils**

SOLUTION 1 - 11
Answer: C

π radians = 180°

SOLUTION 1 - 12
Ans: C

3200 mils × (360°/6400 mils) = **180°**

SOLUTION 1 - 13
Ans: B

One circumference of a circle = 360°
One circumference of a circle = **400 grads**

SOLUTION 1 - 14
Ans: B

$$P = \frac{3}{100} \times \frac{5}{1000} \times 2 = \mathbf{3 \times 10^{-4}}$$

SOLUTION 1 - 15
Ans: D

3m + 2cm + 70mm = 3 + 2/100 + 70/1000
= **3.09 m**

SOLUTION 1 - 16
Ans: D

1 nautical mile = 6080 ft × (1 m/3.28 ft)
= **1.854 km**

SOLUTION 1 - 17
Answer: C

$$100\text{ m}^2 \times \left(\frac{3.28\text{ ft}}{1\text{ m}}\right)^2 = \mathbf{1075.84\text{ ft}^2}$$

SOLUTION 1 - 18
Ans: D

1500 gal × (3.79 lit/gal) × (1 m³/1000 lit) = **5.685 m³**

SOLUTION 1 - 19
Ans: C

$$100\text{ gal} \times \frac{1\text{ ft}^3}{7.48\text{ gal}} = \mathbf{13.37\text{ ft}^3}$$

SOLUTION 1 - 20
Ans: C

$$100\text{ gal} \times \frac{3.79\text{ lit.}}{\text{gal}} \times \frac{1\text{ cu. m.}}{1000\text{ lit.}} = \mathbf{0.379\text{ cu. m.}}$$

SOLUTION 1 - 21
Ans: B

$$\text{Board ft.} = \frac{\text{width in inches x thickness in inches x length in feet}}{12}$$

$$\text{Board ft.} = \frac{3(12)(20)}{12} = 60\text{ board ft.}$$

SOLUTION 1 - 22
Ans: C

1 hp = 746 watts = 0.746 kW

SOLUTION 1 - 23
Ans: A

Acceleration due to gravity, $g = 9.81\text{ m/s}^2$
Acceleration due to gravity, $g = 32.2\text{ ft/s}^2$

SOLUTION 1 - 24
Ans: D

The prefix nano represent 10^{-9} which is opposite to giga (10^9)

SOLUTION 1 - 25
Ans: C

10^{12} is the value of the prefix tera.

SOLUTION 1 - 26
Ans: D

$$630° \times \frac{100\text{ grads}}{90°} = \mathbf{700\text{ grade}}$$

SOLUTION 1 - 27
Ans: D

102.48889 rounded-off to four significant figures = **102.5**

SOLUTION 1 - 28
Ans: C

523 is a prime number

SOLUTION 1 - 29
Ans: C

Five

SOLUTION 1 - 30
Ans: B

Scale = 5:100,000 = 1: 20000
Actual dimension = 290(20,000) = 5,800,000 mm
Actual dimension = 5800 m = **5.8 km**

SOLUTION 1 - 31
Ans: C

0.001 has one significant figure
100 has one significant figures
100.00 has five significant figures
0.000249 has three significant figures

Note:
300; 8000; 9; 500 has one significant figure

SOLUTION 1 - 32
Ans: A

Since the map is similar actual, by similar areas:

$$\frac{\text{Map Area}}{\text{Actual Area}} = \left(\frac{1}{x}\right)^2$$

$$\frac{25.6}{640 \times 100^2} = \left(\frac{1}{x}\right)^2$$

$$x = \mathbf{500}$$

Problems - Set 2
Exponents and Radicals

PROBLEM 2 - 1

Solve for x: $x = -(1/-27)^{-2/3}$

A. 9
B. 1/9
C. -9
D. -1/9

PROBLEM 2 - 2

Solve for a in the equation: $a = 64^x 4^y$.

A. 4^{x+3y}
B. 4^{3xy}
C. 256^{xy}
D. 4^{3x+y}

PROBLEM 2 - 3

Simplify $3^x - 3^{x-1} - 3^{x-2}$.

A. 3^{x-2}
B. 3^{3x-3}
C. $5 \times 3^{x-2}$
D. 13×3^x

PROBLEM 2 - 4

Which of the following is true?

A. $\sqrt{-2} \times \sqrt{-2} = 2$
B. $24 = 4\sqrt{6}$
C. $\sqrt{10} = \sqrt{5} + \sqrt{2}$
D. $5^5 + 5^5 + 5^5 + 5^5 + 5^5 = 5^6$

PROBLEM 2 - 5

Solve for x: $x = \sqrt{18} - \sqrt{72} + \sqrt{50}$.

A. $-2\sqrt{2}$
B. $2\sqrt{2}$
C. 4
D. $4\sqrt{3}$

PROBLEM 2 - 6

Solve for x: $\sqrt{x - \sqrt{1-x}} = 1 - \sqrt{x}$.

A. -16/25 & 0
B. 25/16 & 0
C. -25/16 & 0
D. 16/25 & 0

PROBLEM 2 - 7

Simplify $\sqrt[3]{2x^4} - \sqrt[3]{16x^4} + 2\sqrt[3]{54x^4}$.

A. $5\sqrt[3]{x^4}$
B. $2\sqrt[3]{5x^4}$
C. $5\sqrt[3]{2x^4}$
D. $2\sqrt[3]{x^4}$

PROBLEM 2 - 8

Solve for x: $3^x 5^{x+1} = 6^{x+2}$.

A. 2.1455
B. 2.1445
C. 2.4154
D. 2.1544

PROBLEM 2 - 9

Simplify $\frac{(a^{-2} b^3)^2}{a^2 b^{-1}}$.

A. $a^2 b^7$
B. $a^{-2} b^5$
C. $a^{-6} b^7$
D. $a^{-6} b^5$

PROBLEM 2 - 10

$(3^x)^x$ is equal to:

A. 3^{x^2}
B. $3^x x^x$
C. $3\ x^x$
D. 3^{2x}

PROBLEM 2 - 11

Solve for x: $3^{7x+1} = 6561$.

A. 1
B. 2
C. 3
D. 4

PROBLEM 2 - 12

If $3a = 7b$, then $3a^2 / 7b^2 =$

A. 1
B. 3/7
C. 7/3
D. 49/9

PROBLEM 2 - 13

Solve for U if $U = \sqrt{1 - \sqrt{1 - \sqrt{1 - \ldots}}}$

A. 0.723
B. 0.618
C. 0.852
D. 0.453

PROBLEM 2 - 14
ME April 1996

If x to the 3/4 power equals 8, then x equals:

A. -9
B. 6
C. 9
D. 16

PROBLEM 2 - 15

If $33^y = 1$, what is the value of $y/33$?

A. 0
B. 1
C. infinity
D. 1/33

PROBLEM 2 - 16
ME April 1998

Find the value of x that will satisfy the following expression:

$$\sqrt{x-2} = -\sqrt{x} + 2$$

A. $x = 3/2$
B. $x = 18/6$
C. $x = 9/4$
D. none of these

PROBLEM 2 - 17
ME April 1998

e^{-3} is equal to:

A. 0.048787
B. 0.049001
C. 0.049787
D. 0.048902

PROBLEM 2 - 18

b to the m/n^{th} power is equal to:

A. n^{th} root of b to the m power
B. b to the m + n power
C. $1/n$ square root of b to the m power
D. b to the m power over n

PROBLEM 2 - 19
ECE April 1993

Find x from the following equations:

$$27^x = 9^y$$
$$81^y\,3^{-x} = 243$$

A. 2.5 C. 1
B. 2 D. 1.5

PROBLEM 2 - 20
ECE April 1990

Solve for a from the following equations:

$$\left(a^m\right)\left(a^n\right) = 100{,}000 \qquad a^{mn} = 100{,}000$$
$$\frac{a^n}{a^m} = 10$$

A. 15.85 C. 12
B. 10 D. 12.56

PROBLEM 2 - 21
ECE April 1991

Simplify $\dfrac{\left(x^2 y^3 z^{-2}\right)^{-3}\left(x^{-3} y z^3\right)^{-\frac{1}{2}}}{\left(x y z^{-3}\right)^{-\frac{5}{2}}}$.

A. $\dfrac{1}{x^2 y^7 z^3}$ C. $\dfrac{1}{x^2 y^5 z^3}$
B. $\dfrac{1}{x^2 y^7 z^5}$ D. $\dfrac{1}{x^5 y^7 z^3}$

PROBLEM 2 - 22
ECE April 1991

Simplify the following: $7^{a+2} - 8(7^{a+1}) + 5(7^a) + 49(7^{a-2})$.

A. -5^a C. -7^a
B. 3 D. 7^a

PROBLEM 2 - 23

Simplify: $\left(\dfrac{x y^{-3}}{x^{-3} y^3}\right)^4 \div \left(\dfrac{x^2 y^{-2}}{x^{-3} y^3}\right)^3$

A. $x y^3$ C. $x^3 y$
B. $\dfrac{y}{x^3}$ D. $\dfrac{1}{x^3 y}$

PROBLEM 2 - 24

Simplify the following: $\dfrac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}$

A. $4 + \sqrt{15}$ C. $8 + \sqrt{8}$
B. $4 - \sqrt{15}$ D. $8 - \sqrt{8}$

PROBLEM 2 - 25

Which of the following is equivalent to $\sqrt[m]{\sqrt[n]{a}}$

A. $\sqrt[m]{a^n}$ C. $\sqrt{a^{mn}}$
B. $\sqrt[n]{a^m}$ D. $\sqrt[mn]{a}$

PROBLEM 2 - 26
CE Nov. 2002

Find the value of x in (3^5)(9^6) = 3^(2x)
Note: The expression ^ means power.

A. 8.5 C. 9.5
B. 9 D. 8

PROBLEM 2 - 27

Solve for x:

$$x^{x^{x^{x^{\cdot^{\cdot^{\cdot}}}}}} = 10$$

A. 1.2589 C. 1.1745
B. 2.4156 D. Cannot be solved

ANSWER SHEET

1.	A	B	C	D	E	11.	A	B	C	D	E	21.	A	B	C	D	E
2.	A	B	C	D	E	12.	A	B	C	D	E	22.	A	B	C	D	E
3.	A	B	C	D	E	13.	A	B	C	D	E	23.	A	B	C	D	E
4.	A	B	C	D	E	14.	A	B	C	D	E	24.	A	B	C	D	E
5.	A	B	C	D	E	15.	A	B	C	D	E	25.	A	B	C	D	E
6.	A	B	C	D	E	16.	A	B	C	D	E	26.	A	B	C	D	E
7.	A	B	C	D	E	17.	A	B	C	D	E	27.	A	B	C	D	E
8.	A	B	C	D	E	18.	A	B	C	D	E	28.	A	B	C	D	E
9.	A	B	C	D	E	19.	A	B	C	D	E	29.	A	B	C	D	E
10.	A	B	C	D	E	20.	A	B	C	D	E	30.	A	B	C	D	E

Solutions to Set 2
Exponents and Radicals

SOLUTION 2 - 1
Ans: C

$x = -(1/-27)^{-2/3} = -(1/-3)^{-2} = -9$

SOLUTION 2 - 2
Ans: D

$a = 64^x 4^y = \left(4^3\right)^x 4^y = 4^{3x} 4^y = 4^{3x+y}$

SOLUTION 2 - 3
Ans: C

$$3^x - 3^{x-1} - 3^{x-2} = 3^x - \frac{3^x}{3^1} - \frac{3^x}{3^2} = 3^x\left(1 - \frac{1}{3} - \frac{1}{9}\right)$$
$$= 3^x\left(\frac{5}{9}\right) = 3^x\left(\frac{5}{3^2}\right) = 5 \times 3^{x-2}$$

SOLUTION 2 - 4
Ans: D

Only choice D is true in the given equations:
$5^5 + 5^5 + 5^5 + 5^5 + 5^5 = 5 \times 5^5 = 5^6$

SOLUTION 2 - 5
Ans: B

$x = \sqrt{18} - \sqrt{72} + \sqrt{50}$
$x = \sqrt{(9)(2)} - \sqrt{(36)(2)} + \sqrt{(25)(2)}$
$x = 3\sqrt{2} - 6\sqrt{2} + 5\sqrt{2} = \sqrt{2}\,(3 - 6 + 5) = 2\sqrt{2}$

SOLUTION 2 - 6
Ans: D

$\sqrt{x - \sqrt{1-x}} = 1 - \sqrt{x}$ square both sides
$x - \sqrt{1-x} = 1 - 2\sqrt{x} + x$ cancel x
$-\sqrt{1-x} = 1 - 2\sqrt{x}$ square both sides
$1 - x = 1 - 4\sqrt{x} + 4x$
$4\sqrt{x} = 5x$ square both sides
$16x = 25x^2$ or $25x^2 - 16x = 0$
$x(25x - 16) = 0$; $x =$ **0 and 16/25**

Note: this problem can be solved by trial and error using the choices

SOLUTION 2 - 7
Ans: C

$$\sqrt[3]{2x^4} - \sqrt[3]{16x^4} + 2\sqrt[3]{54x^4} = \sqrt[3]{2x^4} - \sqrt[3]{8 \times 2x^4}$$
$$+ 2\sqrt[3]{27 \times 2x^4}$$
$$= \sqrt[3]{2x^4}\,(1 - 2 + 2 \times 3)$$
$$= 5\sqrt[3]{2x^4}$$

SOLUTION 2 - 8
Ans: D

$3^x 5^{x+1} = 6^{x+2}$; $3^x 5^x 5^1 = 6^x 6^2$
$(3 \times 5)^x 5 = 15^x 5 = 6^x 36$
$\frac{15^x}{6^x} = (15/6)^x = 36/5$ taking the logarithms of both sides
$x \ln(15/6) = \ln(36/5)$; $x =$ **2.1544**

Note: This problem can be solved by trial and error using the choices.

SOLUTION 2 - 9
Ans: C

$$\frac{\left(a^{-2}b^3\right)^2}{a^2 b^{-1}} = \frac{a^{-4}b^6}{a^2 b^{-1}} = a^{-6}b^7$$

SOLUTION 2 - 10
Ans: A

$\left(3^x\right)^x = 3^{x^2}$

SOLUTION 2 - 11
Ans: A

$3^{7x+1} = 6561$ taking the logarithms of both sides
$\ln 3^{7x+1} = \ln 6561$
$(7x + 1)\ln 3 = \ln 6561$; $x =$ **1**

Note: This can be solved by trial and error using the choices

SOLUTION 2 - 12
Ans: C

$3a = 7b$
$\frac{3a}{7b} = 1$ square both sides
$\frac{9a^2}{49b^2} = \frac{3 \times 3a^2}{7 \times 7b^2} = 1$; $\frac{3a^2}{7b^2} = \frac{7}{3}$

SOLUTION 2 - 13
Ans: B

$U = \sqrt{1 - \sqrt{1 - \sqrt{1 - \ldots}}}$ Square both sides:
$U^2 = 1 - \sqrt{1 - \sqrt{1 - \sqrt{1 - \ldots}}}$ but $\sqrt{1 - \sqrt{1 - \sqrt{1 - \ldots}}} = U$

$U^2 = 1 - U$ or $U^2 + U - 1 = 0$
$$U = \frac{-1 \pm \sqrt{1^2 - 4(1)(-1)}}{2(1)} = -1.618 \text{ (absurd) and } 0.618$$

SOLUTION 2 - 14
Ans: D

$x^{3/4} = 8$ (raise both side to 4/3)
$x = 8^{4/3} = 16$

SOLUTION 2 - 15
Ans: A

If $33^y = 1$ then $y = 0$ and $0/33 = 0$

SOLUTION 2 - 16

$\sqrt{x-2} = -\sqrt{x} + 2$ Square both side:

Ans: C

$x - 2 = x - 4\sqrt{x} + 4$

$-6 = -4\sqrt{x}$ Square both side:

$36 = 16x; \quad x = 36/16 = 9/4$

SOLUTION 2 - 17
Ans: C

Using the calculator:

$e^{-3} = 2.71828^{-3} = 0.049787$

SOLUTION 2 - 18
Ans: A

b to the m/n^{th} power is equal to n^{th} root of b to the m power

SOLUTION 2 - 19
Ans: C

$27^{x} = 9^{y}; \quad 3^{3x} = 3^{2y}$, thus $3x = 2y$ and $y = 3x/2$

$81^{y}3^{-x} = 243$

$81^{3x/2}\ 3^{-x} = \left(3^{4}\right)^{3x/2}\ 3^{-x} = 243$

$81^{3x/2}\ 3^{-x} = 3^{6x}\ 3^{-x} = 3^{5x} = 3^{5}$

$5x = 5; \quad x = 1$

SOLUTION 2 - 20
Ans: A

$a^{m}a^{n} = 100{,}000 \rightarrow (1) \qquad \frac{a^{n}}{a^{m}} = 10 \qquad \rightarrow (2)$

Multiply: (1) × (2)

$$a^{m}a^{n} \times \frac{a^{n}}{a^{m}} = 100{,}000 \times 10$$

$$\left(a^{n}\right)^{2} = 1{,}000{,}000$$

$$a^{n} = 1{,}000 \text{ and } a^{m} = 100$$

$a^{mn} = \left(a^{m}\right)^{n} = 100{,}000$

$(100)^{n} = 100{,}000$

$n = \log(100{,}000) / \log(100); \quad n = 2.5$

$a^{n} = a^{2.5} = 1{,}000; \quad a = (1{,}000)^{\frac{1}{2.5}} = \mathbf{15.849}$

SOLUTION 2 - 21
Ans: A

$$\frac{\left(x^{2}y^{3}z^{-2}\right)^{-3}\left(x^{-3}yz^{3}\right)^{-\frac{1}{2}}}{\left(xyz^{-3}\right)^{-\frac{5}{2}}} = \frac{\left(x^{-6}y^{-9}z^{6}\right)\left(x^{3/2}y^{-1/2}z^{-3/2}\right)}{x^{-5/2}y^{-5/2}z^{15/2}}$$

$$= \frac{x^{-6+3/2}y^{-9-1/2}z^{6-3/2}}{x^{-5/2}y^{-5/2}z^{15/2}} = \frac{x^{-9/2}y^{-19/2}z^{9/2}}{x^{-5/2}y^{-5/2}z^{15/2}}$$

$$= \frac{1}{x^{-5/2+9/2}y^{-5/2+19/2}x^{15/2-9/2}} = \frac{1}{x^{2}y^{7}z^{3}}$$

SOLUTION 2 - 22
Ans: C

$$7^{a+2} - 8(7^{a+1}) + 5(7^{a}) + 49(7^{a-2})$$

$$= 7^{a}7^{2} - 8(7^{a}7) + 5(7^{a}) + (7^{2})(7^{a}7^{-2})$$

$$= 7^{a}\left[7^{2} - 8(7) + 5 + 7^{2}7^{-2}\right]$$

$$= 7^{a}(49 - 56 + 5 + 1) = 7^{a}(-1) = -7^{a}$$

SOLUTION 2 - 23
Ans: D

$$\left(\frac{xy^{-1}}{x^{-2}y^{3}}\right)^{4} \div \left(\frac{x^{2}y^{-2}}{x^{-3}y^{3}}\right)^{3} = \left(\frac{xy^{-1}}{x^{-2}y^{3}}\right)^{4} \times \left(\frac{x^{-3}y^{3}}{x^{2}y^{-2}}\right)^{3}$$

$$= \frac{x^{4}y^{-4}}{x^{-8}y^{12}} \times \frac{x^{-9}y^{9}}{x^{6}y^{-6}} = x^{4-9-(-8)-6}\,y^{-4+9-12-(-6)}$$

$$= x^{-3}y^{-1} = \frac{1}{x^{3}y}$$

SOLUTION 2 - 24
Ans: B

$$\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}} \times \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}-\sqrt{3}} = \frac{5-2\sqrt{5}\sqrt{3}+3}{5-3} = \frac{8-2\sqrt{5(3)}}{2}$$

$$= 4 - \sqrt{15}$$

SOLUTION 2 - 25
Ans: D

$$\sqrt[m]{\sqrt[n]{a}} = \left(a^{1/n}\right)^{1/m} = a^{\frac{1}{n}\times\frac{1}{m}} = a^{\frac{1}{mn}} = \sqrt[mn]{a}$$

SOLUTION 2 - 26
Ans: A

$3^{2x} = 3^{5}\,9^{6} = 3^{5}\,(3^{2})^{6}$

$3^{2x} = 3^{5}\,3^{12} = 3^{17}$

$2x = 17; x = 8.5$

SOLUTION 2 - 27
Ans: A

$x^{x^{x^{x^{\cdot}}}} = 10$

$(x)^{x^{x^{x^{\cdot}}}} = 10 \qquad \text{But } x^{x^{x^{x^{\cdot}}}} = 10$

$(x)^{10} = 10; x = 1.2589$

Problems - Set 3
Fundamentals in Algebra

PROBLEM 3 - 1
ME Board

Change 0.2272727... to a common fraction.
A. 7/44 C. 5/22
B. 5/48 D. 9/34

PROBLEM 3 - 2
ME Board

What is the value of 7! or 7 factorial?
A. 5040 C. 5020
B. 2540 D. 2520

PROBLEM 3 - 3
ME October 1994

The reciprocal of 20 is:
A. 0.50 C. 0.20
B. 20 D. 0.05

PROBLEM 3 - 4

If p is an odd number and q is an even number, which of the following expressions must be even?
A. $p + q$ C. pq
B. $p - q$ D. p / q

PROBLEM 3 - 5
ECE March 1996

MCMXCIV is a Roman Numeral equivalent to:
A. 2974 C. 2174
B. 3974 D. 1994

PROBLEM 3 - 6
ECE April 1998

What is the lowest common factor of 10 & 32?
A. 320 C. 180
B. 2 D. 90

PROBLEM 3 - 7

$4xy - 4x^2 - y^2$ is equal to:
A. $(2x - y)^2$ C. $(-2x + y)^2$
B. $(-2x - y)^2$ D. $-(2x - y)^2$

PROBLEM 3 - 8

Factor $x^4 - y^2 + y - x^2$ as completely as possible.
A. $(x^2 + y)(x^2 + y - 1)$ C. $(x^2 - y)(x^2 - y - 1)$
B. $(x^2 + y)(x^2 - y - 1)$ D. $(x^2 - y)(x^2 + y - 1)$

PROBLEM 3 - 9
ME April 1996

Factor the expression $x^2 + 6x + 8$ as completely as possible.
A. $(x + 8)(x - 2)$ C. $(x + 4)(x - 2)$
B. $(x + 4)(x + 2)$ D. $(x - 8)(x - 2)$

PROBLEM 3 - 10
ME Oct. 1997

Factor the expression $x^3 + 8$ as completely as possible:
A. $(x - 2)(x^2 + 2x + 4)$ C. $(-x + 2)(-x^2 + 2x + 2)$
B. $(x + 4)(x^2 + 2x + 2)$ D. $(x + 2)(x^2 - 2x + 4)$

PROBLEM 3 - 11
ME Oct. 1997

Factor the expression $(x^4 - y^4)$ as completely as possible:
A. $(x + y)(x^2 + 2xy + y^2)$ C. $(x^2 + y^2)(x + y)(x - y)$
B. $(x^2 + y^2)(x^2 - y^2)$ D. $(1 + x^2)(1 + y)(1 - y^2)$

PROBLEM 3 - 12
ME Oct. 1997

Factor the expression $3x^3 + 3x^2 - 18x$ as completely as possible:
A. $3x(x + 2)(x - 3)$ C. $3x(x - 3)(x + 6)$
B. $3x(x - 2)(x + 3)$ D. $(3x^2 - 6x)(x - 1)$

PROBLEM 3 - 13
ME April 1998

Factor the expression $16 - 10x + x^2$:
A. $(x + 8)(x - 2)$ C. $(x - 8)(x + 2)$
B. $(x - 8)(x - 2)$ D. $(x + 8)(x + 2)$

PROBLEM 3 - 14

Factor the expression $x^6 - 1$ as completely as possible.
A. $(x + 1)(x - 1)(x^4 + x^2 - 1)$
B. $(x + 1)(x - 1)(x^4 + 2x^2 + 1)$
C. $(x + 1)(x - 1)(x^4 - x^2 + 1)$
D. $(x + 1)(x - 1)(x^4 + x^2 + 1)$

PROBLEM 3 - 15

The roots of the equation $(x - 4)^2 (x + 2) = (x + 2)^2 (x - 4)$ are:
A. 4 and -2 only C. -2 and 4 only
B. 1 only D. 1, -2, and 4 only

PROBLEM 3 - 16

If $f(x) = x^2 + x + 1$, then $f(x) - f(x - 1) =$
A. 0 C. $2x$
B. x D. 3

PROBLEM 3 - 17

Which of the following is not an identity?
A. $(x - 1)^2 = x^2 - 2x + 1$
B. $(x + 3)(2x - 2) = 2(x^2 + 2x - 3)$
C. $x^2 - (x - 1)^2 = 2x - 1$
D. $2(x - 1) + 3(x + 1) = 5x + 4$

PROBLEM 3 - 18
ME Oct. 1997

Solve for x: $4 + \frac{x+3}{x-3} - \frac{4x^2}{x^2-9} = \frac{x+9}{x+3}$
A. $-18 = -18$ C. Any value
B. $12 = 12$ or $-3 = -3$ D. $-27 = -27$ or $0 = 0$

PROBLEM 3 - 19
ME Oct. 1997

Solve the simultaneous equations: $3x - y = 6$; $9x - y = 12$.
A. $x = 3$; $y = 1$
B. $x = 1$; $y = -3$
C. $x = 2$; $y = 2$
D. $x = 4$; $y = 2$

PROBLEM 3 - 20
ME April 1998

Solve algebraically: $4x^2 + 7y^2 = 32$; $11y^2 - 3x^2 = 41$
A. $y = 4$, $x = \pm 1$, and $y = -4$, $x = \pm 1$
B. $y = +2$, $x = \pm 1$, and $y = -2$, $x = \pm 1$
C. $x = 2$, $y = 3$, and $x = -2$, $y = -3$
D. $x = 2$, $y = -2$, and $x = 2$, $y = -2$

PROBLEM 3 - 21
CE May 1997

Solve for w from the following equations:
$3x - 2y + w = 11$ $\quad x + 5y - 2w = -9$
$2x + y - 3w = -6$
A. 1
B. 2
C. 3
D. 4

PROBLEM 3 - 22

When $(x + 3)(x - 4) + 4$ is divided by $x - k$, the remainder is k. Find the value of k.
A. 4 or 2
B. 2 or -4
C. 4 or -2
D. -4 or -2

PROBLEM 3 - 23

Find k in the equation $4x^2 + kx + 1 = 0$ so that it will only have one real root.
A. 1
B. 2
C. 3
D. 4

PROBLEM 3 - 24

Find the remainder when $(x^{12} + 2)$ is divided by $(x - \sqrt{3})$.
A. 652
B. 731
C. 231
D. 851

PROBLEM 3 - 25
CE Nov. 1997

If $3x^3 - 4x^2y + 5xy^2 + 6y^3$ is divided by $(x^2 - 2xy + 3y^2)$, the remainder is:
A. 0
B. 1
C. 2
D. 3

PROBLEM 3 - 26
CE Nov. 1997
& May 1999

If $(4y^3 + 8y + 18y^2 - 4)$ is divided by $(2y + 3)$, the remainder is:
A. 10
B. 11
C. 12
D. 13

PROBLEM 3 - 27
ECE April 1999

Given: $f(x) = (x + 3)(x - 4) + 4$ when divided by $(x - k)$, the remainder is k. Find k.
A. 2
B. 3
C. 4
D. -3

PROBLEM 3 - 28
EE March 1998

The polynomial $x^3 + 4x^2 - 3x + 8$ is divided by $x - 5$. What is the remainder?
A. 281
B. 812
C. 218
D. 182

PROBLEM 3 - 29

Find the quotient of $3x^5 - 4x^3 + 2x^2 + 36x + 48$ divided by $x^3 - 2x^2 + 6$.
A. $-3x^2 - 4x + 8$
B. $3x^2 + 4x + 8$
C. $3x^2 - 4x - 8$
D. $3x^2 + 6x + 8$

PROBLEM 3 - 30

If $1/x = a + b$ and $1/y = a - b$ then $x - y$ is equal to:
A. $1/2a$
B. $1/2b$
C. $2a / (a^2 - b^2)$
D. $2b / (a^2 - b^2)$

PROBLEM 3 - 31

If $x - 1/x = 1$, find the value of $x^3 - 1/x^3$.
A. 1
B. 2
C. 3
D. 4

PROBLEM 3 - 32

If $1/x + 1/y = 3$ and $2/x - 1/y = 1$, then x is equal to:
A. ½
B. 2/3
C. 3/4
D. 4/3

PROBLEM 3 - 33

Simplify the following expression:

$$\frac{5x}{2x^2+7x+3} - \frac{x+3}{2x^2-3x-2} + \frac{2x+1}{x^2+x-6}$$

A. $2 / (x - 3)$
B. $(x - 3) / 5$
C. $(x + 3)/(x - 1)$
D. $4 / (x + 3)$

PROBLEM 3 - 34

If $3x = 4y$ then $\dfrac{3x^2}{4y^2}$ is equal to:
A. 3/4
B. 4/3
C. 2/3
D. 3/2

PROBLEM 3 - 35

Simplify: $(a+1/a)^2 - (a-1/a)^2$.
A. -4
B. 0
C. 4
D. $-2/a^2$

PROBLEM 3 - 36
ECE Nov 1996

The quotient of $(x^5 + 32)$ by $(x + 2)$ is:
A. $x^4 - x^3 + 8$
B. $x^3 + 2x^2 - 8x + 4$
C. $x^4 - 2x^3 + 4x^2 - 8x + 16$
D. $x^4 + 2x^3 + x^2 + 16x + 8$

PROBLEM 3 - 37
ME April 1996

Solve the simultaneous equations:

$$y - 3x + 4 = 0$$
$$y + x^2/y = 24/y$$

A. $x = (-6 + 2\sqrt{14})/5$ or $(-6 - 2\sqrt{14})/5$
   $y = (2 + 6\sqrt{14})/5$ or $(-2 + 6\sqrt{14})/5$
B. $x = (6 + 2\sqrt{15})/5$ or $(6 - 2\sqrt{15})/5$
   $y = (-2 + 6\sqrt{14})/5$ or $(-2 - 6\sqrt{15})/5$
C. $x = (6 + 2\sqrt{14})/5$ or $(6 - 2\sqrt{14})/5$
   $y = (-2 + 6\sqrt{14})/5$ or $(-2 - 6\sqrt{14})/5$
D. $x = (6 + 2\sqrt{14})/5$ or $(6 - 2\sqrt{14})/5$
   $y = (-6\ 2\sqrt{14})/5$ or $(-6 + 2\sqrt{14})/5$

PROBLEM 3 - 38
CE May 1996

Find the value of A in the equation

$$\frac{(x^2+4x+10)}{(x^3+2x^2+5x)} = \frac{A}{x} + \frac{B(2x+2)}{(x^2+2x+5)} + \frac{C}{(x^2+2x+5)}$$

A. 2
B. -2
C. -1/2
D. ½

PROBLEM 3 - 39

Find A and B such that $\frac{x+10}{x^2-4} = \frac{A}{x-2} + \frac{B}{x+2}$.

A. $A = -3$; $B = 2$
B. $A = -3$; $B = -2$
C. $A = 3$; $B = -2$
D. $A = 3$; $B = 2$

PROBLEM 3 - 40
ME Oct. 1996

Resolve $\frac{x+2}{x^2-7x+12}$ into partial fraction.

A. $6/(x-4) - 2/(x-3)$
B. $6/(x-4) + 7/(x-3)$
C. $6/(x-4) - 5/(x-3)$
D. $6/(x-4) + 5/(x-3)$

PROBLEM 3 - 41
ECE April 1998

The arithmetic mean of 80 numbers is 55. If two numbers namely 250 and 850 are removed, what is the arithmetic mean of the remaining numbers?

A. 42.31
B. 57.12
C. 50
D. 38.62

PROBLEM 3 - 42
ECE April 1998

The arithmetic mean of 6 numbers is 17. If two numbers are added to the progression, the new set of number will have an arithmetic mean of 19. What are the two numbers if their difference is 4?

A. 21, 29
B. 23, 27
C. 24, 26
D. 22, 28

PROBLEM 3 - 43

If $2x - 3y = x + y$, then $x^2 : y^2 =$

A. 1 : 4
B. 4 : 1
C. 1 : 16
D. 16 : 1

PROBLEM 3 - 44

If $1/a : 1/b : 1/c = 2 : 3 : 4$, then $(a + b + c) : (b + c)$ is equal to:

A. 13 : 7
B. 15 : 6
C. 10 : 3
D. 7 : 9

PROBLEM 3 - 45

Find the mean proportional to 5 and 20.

A. 8
B. 10
C. 12
D. 14

PROBLEM 3 - 46

Find the fourth proportional of 7, 12, and 21.

A. 36
B. 34
C. 32
D. 40

PROBLEM 3 - 47
ECE Nov. 1997

If $(x + 3) : 10 = (3x - 2) : 8$, find $(2x - 1)$

A. 1
B. 2
C. 3
D. 4

PROBLEM 3 - 48

Solve for x: $-4 < 3x - 1 < 11$

A. $1 < x < -4$
B. $-1 < x < 4$
C. $1 < x < 4$
D. $-1 < x < -4$

PROBLEM 3 - 49

Solve for x: $x^2 + 4x > 12$.

A. $-6 > x > 2$
B. $6 > x > -2$
C. $-6 > x > -2$
D. $6 > x > 2$

PROBLEM 3 - 50

When the expression $x^4 + ax^3 + 5x^2 + bx + 6$ is divided by $(x - 2)$, the remainder is 16. When it is divided by $(x + 1)$ the remainder is 10. What is the value of the constant a?

A. -5
B. -9
C. 7
D. 8

ANSWER SHEET

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Solutions to Set 3 Fundamentals in Algebra

SOLUTION 3 - 1
Ans: C

$0.2272727\ldots = \dfrac{5}{22}$

SOLUTION 3 - 2
Ans: A

$7! = 7(6)(5)(4)(3)(2)(1) = \mathbf{5{,}040}$

SOLUTION 3 - 3
Ans: D

Reciprocal of 20 is 1/20 = **0.05**

SOLUTION 3 - 4
Ans: C

The product of any two integers with one of them "even" is always an even number.

Thus, among the choices "*pq*" is always even.

SOLUTION 3 - 5
Ans: D

M = 1000 V = 5; C = 100; I = 1; X = 10
MCMXCIV = M + CM + XC + IV
= 1000 + 900 + 90 + 4 = **1994**

SOLUTION 3 - 6
Ans: B

The lowest common factor (LCF) of a given numbers is the lowest number, which is a factor of all the given numbers.

The LCF of 10 and 32 is therefore **2**.

SOLUTION 3 - 7
Ans: D

$4xy - 4x^2 - y^2 = -(4x^2 - 4xy + y^2) = \mathbf{-(2x - y)^2}$

SOLUTION 3 - 8
Ans: D

$$x^4 - y^2 + y - x^2 = (x^4 - y^2) + (y - x^2) = (x^2 + y)(x^2 - y) - (x^2 - y)$$
$$x^4 - y^2 + y - x^2 = \mathbf{(x^2 - y)(x^2 + y - 1)}$$

SOLUTION 3 - 9
Ans: B

$x^2 + 6x + 8 = \mathbf{(x + 4)(x + 2)}$

SOLUTION 3 - 10
Ans: D

$$\begin{aligned} x^3 + 8 &= x^3 + 2^3 \\ &= (x + 2)[x^2 - x(2) + 2^2] \\ &= \mathbf{(x + 2)(x^2 - 2x + 4)} \end{aligned}$$

Note: The factor of a given expression must be identical to that expression (identity). Thus, this problem may be solved by assuming any value of the variable/s. The choice that gives the same result with the given expression is the correct answer.

Assume $x = 3$: $x^3 + 8 = 3^3 + 8 = 35$

Choice A. $(3 - 2)(3^2 + 2\times3 + 4) = 1(19) = 19 \neq 35$
Choice B. $(3 + 4)(3^2 + 2\times3 + 2) = 7(17) = 119 \neq 35$
Choice C. $(-3 + 2)(-3^2 + 2\times3 + 2) = (-1)(-1) = 1 \neq 35$
Choice D. $(3 + 2)(3^2 - 2\times3 + 4) = (5)(7) = 35$ (OK)

SOLUTION 3 - 11
Ans: C

$$x^4 - y^4 = (x^2 + y^2)(x^2 - y^2)$$
$$= (x^2 + y^2)(x + y)(x - y)$$

SOLUTION 3 - 12
Ans: A

$$3x^3 + 3x^2 - 18x = 3x(x^2 + x - 6)$$
$$= \mathbf{3x(x + 2)(x - 3)}$$

SOLUTION 3 - 13
Ans: B

$$16 - 10x + x^2 = x^2 - 10x + 16$$
$$= \mathbf{(x - 8)(x - 2)}$$

SOLUTION 3 - 14
Ans: D

$$x^6 - 1 = (x^2)^3 - 1^3 = (x^2 - 1)[(x^2)^2 + (x^2)(1) + (1)^2]$$
$$= \mathbf{(x + 1)(x - 1)(x^4 + x^2 + 1)}$$

SOLUTION 3 - 15
Ans: A

$$(x - 4)^2 (x + 2) = (x + 2)^2 (x - 4)$$
$$(x - 4)^2 (x + 2) - (x + 2)^2 (x - 4) = 0$$
$$(x - 4)(x + 2)\,[(x - 4) - (x + 2)] = 0$$
$(x - 4)(x + 2)(-6) = 0$; **$x = 4$ and -2**

SOLUTION 3 - 16
Ans: C

$$f(x) = x^2 + x + 1$$
$$f(x) - f(x - 1) = [x^2 + x + 1] - [(x - 1)^2 + (x - 1) + 1]$$
$$= x^2 + x + 1 - (x^2 - 2x + 1 + x - 1 + 1)$$
$$= x^2 + x + 1 - x^2 + x - 1 = \mathbf{2x}$$

SOLUTION 3 - 17
Ans: D

Choice A: $x^2 - 2x + 1 = x^2 - 2x + 1$ (identity)
Choice B: $2x^2 - 2x + 6x - 6 = 2x^2 + 4x - 6$
$2x^2 + 4x - 6 = 2x^2 + 4x - 6$ (identity)
Choice C: $x^2 - (x^2 - 2x + 1) = 2x - 1$
$2x - 1 = 2x - 1$ (identity)
Choice D: $2x - 2 + 3x + 3 = 5x + 4$
$5x + 1 \neq 5x + 4$ (not an identity)

SOLUTION 3 - 18
Ans: C

$$4 + \frac{x+3}{x-3} - \frac{4x^2}{x^2-9} = \frac{x+9}{x+3}$$

Simplify:

$$\frac{4(x^2-9) + (x+3)(x+3) - 4x^2}{(x+3)(x-3)} = \frac{x+9}{x+3}$$
$$\frac{4x^2 - 36 + x^2 + 6x + 9 - 4x^2}{x-3} = x + 9$$
$$x^2 + 6x - 27 = x^2 + 6x - 27$$

The given equation is an identity. Therefore, x can have any value.

SOLUTION 3 - 19
Ans: B

$3x - y = 6$; $y = 3x - 6$ → (1)
$9x - y = 12$; $y = 9x - 12$ → (2)

Comparing equations (1) and (2)

$$3x - 6 = 9x - 12$$
$6x = 6$; $\mathbf{x = 1}$

In Eq. (1):

$3(1) - y = 6$; $y = -3$

Note: This problem may be solved by substituting the choices to the given equations.

SOLUTION 3 - 20
Ans: B

Given: $4x^2 + 7y^2 = 32$ → (1)
$-3x^2 + 11y^2 = 41$ → (2)

Add: (1) × 3 + (2) × 4

$$\begin{array}{r} 12x^2 + 21y^2 = 96 \\ -12x^2 + 44y^2 = 164 \\ \hline 65y^2 = 260 \end{array}$$

$y^2 = 4$; $y = \pm2$

When $y = +2$, $x = \pm1$ and when $y = -2$, $x = \pm1$

SOLUTION 3 - 21
Ans: C

By Cramer's Rule, $w = N_w / D$

$$D = \left|\begin{array}{ccc|cc} 3 & -2 & 1 & 3 & -2 \\ 1 & 5 & -2 & 1 & 5 \\ 2 & 1 & -3 & 2 & 1 \end{array}\right|$$
$$D = [(3)(5)(-3) + (-2)(-2)(2) + (1)(1)(1)]$$
$$- [(2)(5)(1) + (1)(-2)(3) + (-3)(1)(-2)] = -46$$

$$N_w = \left|\begin{array}{ccc|cc} 3 & -2 & 11 & 3 & -2 \\ 1 & 5 & -9 & 1 & 5 \\ 2 & 1 & -6 & 2 & 1 \end{array}\right|$$

$$N_w = [(3)(5)(-6) + (-2)(-9)(2) + (11)(1)(1)]$$
$$- [(2)(5)(11) + (1)(-9)(3) + (-6)(1)(-2)] = -138$$
$$w = \frac{-138}{-46} = 3$$

SOLUTION 3 - 22
Ans: C

$$f(x) = (x + 3)(x - 4) + 4$$
$$x - r = x - k$$
$$r = k$$

By remainder theorem,

$$\begin{aligned} \text{Remainder} &= f(r) = k = f(k) = (k+3)(k-4)+4 = k \\ &= k^2 - k - 12 + 4 = k \end{aligned}$$

$k^2 - 2k - 8 = 0;\ (k-4)(k+2) = 0$

$k = 4$ **or** $\mathbf{-2}$

SOLUTION 3 - 23
Ans: D

For the quadratic equation $ax^2 + by^2 + c = 0$ to have only one real root, $b^2 = 4ac$

For the equation $4x^2 + kx + 1 = 0$, with $a = 4$, $b = k$, and $c = 1$,

$k^2 = 4(4)(1) = 16;\ k = \mathbf{\pm 4}$

SOLUTION 3 - 24
Ans: B

By remainder theorem:

$f(x) = x^{12} + 2$

$x - r = x - \sqrt{3};\ r = \sqrt{3}$

$f(\sqrt{3}) = (\sqrt{3})^{12} + 2 = \mathbf{731}$

SOLUTION 3 - 25
Ans: A

By long division:

$$\begin{array}{r l} & 3x+2y \\ x^2-2xy+3y^2 & \overline{)3x^3-4x^2y+5xy^2+6y^3} \\ - & 3x^3-6x^2y+9xy^2 \\ \hline & 0 \quad 2x^2y-4xy^2+6y^3 \\ - & 2x^2y-4xy^2+6y^3 \\ \hline & 0 \rightarrow \text{remainder} \end{array}$$

SOLUTION 3 - 26
Ans: B

By long division:

$$\begin{array}{r l} & 2y^2+6y-5 \\ 2y+3 & \overline{)4y^3+18y^2+8y-4} \\ - & 4y^3+6y^2 \\ \hline & 12y^2+8y \\ - & 12y^2+18y \\ \hline & -10y-4 \\ - & -10y-15 \\ \hline & 11 \rightarrow \text{remainder} \end{array}$$

By remainder theorem:

$f(y) = 4y^3 + 18y^2 + 8y - 4$

$r = -3/2$

Remainder $= f(r) = 4(-3/2)^3 + 18(-3/2)^2 + 8(-3/2) - 4$

Remainder $= 11$

SOLUTION 3 - 27
Ans: C

$f(x) = (x+3)(x-4) + 4$

$x - r = x - k;\ r = k$

Remainder $= f(k) = (k+3)(k-4) + 4 = k$

$k^2 - k - 12 + 4 = k$

$k^2 - 2k - 8 = 0; \quad (k-4)(k+2) = 0$

$k = \mathbf{4\ \&\ -2}$

SOLUTION 3 - 28
Ans: C

$f(x) = x^3 + 4x^2 - 3x + 8$

$x - r = x - 5;\ r = 5$

Remainder $= f(r) = f(5) = (5)^3 + 4(5)^2 - 3(5) + 8$

Remainder $= \mathbf{218}$

SOLUTION 3 - 29
Ans: D

$$\begin{array}{r l} & 3x^2+6x+8 \\ x^3-2x^2+6 & \overline{)3x^5-4x^3+2x^2+36x+48} \\ - & 3x^5-6x^4+18x^2 \\ \hline & 6x^4-4x^3-16x^2+36x \\ - & 6x^4-12x^3+36x \\ \hline & 8x^3-16x^2+48 \\ - & 8x^3-16x^2+48 \\ \hline & 0 \end{array}$$

The quotient is $\mathbf{3x^2 + 6x + 8}$

SOLUTION 3 - 30
Ans: D

$\dfrac{1}{x} = a + b; \qquad x = \dfrac{1}{a+b} \qquad \rightarrow (1)$

$\dfrac{1}{y} = a - b; \qquad y = \dfrac{1}{a-b} \qquad \rightarrow (2)$

$$x - y = \frac{1}{a+b} - \frac{1}{a-b}$$

$$x - y = \frac{(a-b)-(a+b)}{(a+b)(a-b)}$$

$$x - y = -\frac{2b}{a^2-b^2}$$

SOLUTION 3 - 31
Ans: D

$x - 1/x = 1$ raise both sides to 3

$(x - 1/x)^3 = 1^3$

$x^3 - 3x^2(1/x) + 3x(1/x)^2 - (1/x)^3 = 1$

$x^3 - 3x + 3/x - 1/x^3 = 1$

$x^3 - 3(x - 1/x) - 1/x^3 = 1$, but $x - 1/x = 1$

$x^3 - 3(1) - 1/x^3 = 1$

$x^3 - 1/x^3 = 1 + 3$

$x^3 - 1/x^3 = \mathbf{4}$

SOLUTION 3 - 32
Ans: C

$1/x + 1/y = 3 \quad \rightarrow (1)$

$2/x - 1/y = 1$ or $1/y = 2/x - 1 \quad \rightarrow (2)$

Substitute $1/y$ from Eq. (2) to Eq. (1):

$1/x + 2/x - 1 = 3$

$3/x = 4$, then $x = 3/4$

SOLUTION 3 - 33
Ans: D

$$\frac{5x}{(2x+1)(x+3)} - \frac{x+3}{(2x+1)(x-2)} + \frac{2x+1}{(x-2)(x+3)}$$

$$= \frac{5x(x-2)-(x+3)(x+3)+(2x+1)(2x+1)}{(2x+1)(x+3)(x-2)}$$

$$= \frac{5x^2-10x-(x^2+6x+9)+(4x^2+4x+1)}{(2x+1)(x+3)(x-2)}$$

$$= \frac{8x^2-12x-8}{(2x+1)(x+3)(x-2)} = \frac{4(2x^2-3x-2)}{(2x+1)(x+3)(x-2)}$$

$$= \frac{4(2x+1)(x-2)}{(2x+1)(x+3)(x-2)} = \frac{4}{x+3}$$

SOLUTION 3 - 34
Ans: B

$$3x = 4y;\ \frac{x}{y} = \frac{4}{3},\ \text{and}\ \frac{x^2}{y^2} = \frac{16}{9}$$

$$\frac{3x^2}{4y^2} = \frac{3}{4} \times \frac{x^2}{y^2} = \frac{3}{4}\times\frac{16}{9} = \frac{4}{3}$$

SOLUTION 3 - 35
Ans: C

$(a + 1/a)^2 - (a - 1/a)^2$

$= [(a + 1/a) + (a - 1/a)][(a + 1/a) - (a - 1/a)]$

$= [2a][2/a] = 4$

SOLUTION 3 - 36
Ans: C

By synthetic division:

$f(x) = x^5 + 0x^4 + 0x^3 + 0x^2 + 0x + 32$

$x - r = x + 2;\ r = -2$

$$\begin{array}{cccccc|l} 1 & 0 & 0 & 0 & 0 & 32 & \underline{-2} \\ & -2 & 4 & -8 & 16 & -32 & \\ \hline 1 & -2 & 4 & -8 & 16 & 0 & \rightarrow \text{Coefficients} \end{array}$$

The quotient is: $x^4 - 2x^3 + 4x^2 - 8x + 16$

SOLUTION 3 - 37
Ans: C

$y - 3x + 4 = 0 \quad \rightarrow (1)$

$y + x^2/y = 24/y \quad \rightarrow (2)$

Multiply Eq. (2) by y:

$y^2 + x^2 = 24$

From Eq. (1): $y = 3x - 4$

then;

$(3x - 4)^2 + x^2 = 24$

$9x^2 - 24x + 16 + x^2 = 24$

$10x^2 - 24x - 8 = 0;\ \ 5x^2 - 12x - 4 = 0$

$$x = \frac{-(-12) \pm \sqrt{(-12)^2 - 4(5)(-4)}}{2(5)} = \frac{12 \pm \sqrt{224}}{10}$$

$$x = \frac{12 \pm 4\sqrt{14}}{10} = \frac{6 \pm 2\sqrt{14}}{5}$$

$$y = 3\frac{6 \pm 2\sqrt{14}}{5} - 4 = \frac{18 \pm 6\sqrt{14} - 20}{5} = \frac{-2 \pm 6\sqrt{14}}{5}$$

SOLUTION 3 - 38
Ans: A

$$\frac{(x^2+4x+10)}{(x^3+2x^2+5x)} = \frac{A}{x} + \frac{B(2x+2)}{(x^2+2x+5)} + \frac{C}{(x^2+2x+5)}$$

$x^2 + 4x + 10 = A(x^2 + 2x + 5) + B(2x + 2)x + Cx \quad \rightarrow (1)$

Eq. (1) is an identity. Set $x = 0$ to eliminate B and C:

$10 = A(0 + 0 + 5)$

$A = 2$

SOLUTION 3 - 39
Ans: C

$$\frac{x+10}{x^2-4} = \frac{A}{x-2} + \frac{B}{x+2}$$

$x + 10 = A(x + 2) + B(x - 2)$

Set $x = 2$: $\quad 2 + 10 = A(2 + 2) + B(2 - 2);\ A = 3$

Set $x = -2$: $\quad -2 + 10 = A(-2 + 2) + B(-2 - 2);\ B = -2$

SOLUTION 3 - 40
Ans: C

$$\frac{x+2}{x^2-7x+12} = \frac{x+2}{(x-4)(x-3)} = \frac{A}{x-4} + \frac{B}{x-3}$$

$x + 2 = A(x - 3) + B(x - 4)$

$x + 2 = (A + B)x + (-3A - 4B)$

Equating coefficients:

$A + B = 1;\ A = 1 - B \quad \rightarrow (1)$

$2 = -3A - 4B \quad \rightarrow (2)$

Substitute A in Eq. (1) to Eq. (2):

$2 = -3(1 - B) - 4B$

$2 = -3 + 3B - 4B$

$B = -5$ and $A = 6$

$$\text{Thus; } \frac{x+2}{x^2-7x+12} = \frac{6}{x-4} - \frac{5}{x-3}$$

SOLUTION 3 - 41
Ans: A

Let a be the sum of the first 80 numbers, then;
$a / 80$ = mean = 55; $a = 4{,}400$

If the two numbers 250 and 850 were removed, then the arithmetic mean of the remaining numbers is: (78 numbers were left)

$(a - 250 - 850)/78 = (4400 - 250 - 850)/78 = \mathbf{42.31}$

SOLUTION 3 - 42
Ans: B

Let a be the sum of the first 6 numbers, then a / 6 =17 or $a = 102$.
Let x and y be the two number for which $x - y = 4$, or $x = 4 + y$, then the arithmetic of the new set of number is:

$$\text{mean} = \frac{a + x + y}{8} = 19$$
$$19(8) = 102 + (4 + y) + y$$
$$y = 23$$
$$x = 27$$

Note: The answer for this problem can easily be seen from the choices. The only choice whose difference is 4 is letter b (23 and 27).

SOLUTION 3 - 43
Ans: D

$2x - 3y = x + y$
$x = 4y$
$x / y = 4/1$
$(x/y)^2 = 4^2 = 16/1$
$x^2 : y^2 = \mathbf{16 : 1}$

SOLUTION 3 - 44
Ans: A

$$\text{If } \frac{1}{a} : \frac{1}{b} : \frac{1}{c} = 2 : 3 : 4$$
$$\text{then } \frac{1}{a} = k(2),\ \frac{1}{b} = k(3)$$
$$\frac{1}{c} = k(4)$$

$a = 1/2k$, $b = 1/3k$, $c = 1/4k$, then
$(a + b + c) : (b + c) = (1/2 + 1/3 + 1/4) : (1/3 + 1/4)$
$= (13/12) : (7/12)$
$= \mathbf{13{:}7}$

SOLUTION 3 - 45
Ans: B

Let x be the mean proportional to 5 and 20, then 5 : x : 20,
$$\text{or } \frac{5}{x} = \frac{x}{20}$$
$x^2 = 100$; $x = 10$

SOLUTION 3 - 46
Ans: A

Let x be the fourth proportional, then $7 : 12 : 21 : x$
or $7/12 = 21/x$
$x = 21(12)/7 = 36$

SOLUTION 3 - 47
Ans: C

$$\frac{x + 3}{10} = \frac{3x - 2}{8}$$
$8x + 24 = 30x - 20$
$x = 2$
$2x - 1 = 2(2) - 1$
$\mathbf{2x - 1 = 3}$

SOLUTION 3 - 48
Ans: B

$-4 < 3x - 1 < 11$

Considering the left side:
$-4 < 3x - 1$
$-4 + 1 < 3x$
$-3 < 3x$, thus $-1 < x$

Considering the right side:
$3x - 1 < 11$
$3x < 12$, thus $x < 4$
Therefore; $-1 < x < 4$

Another Solution:

$-4 < 3x - 1 < 11$	Add 1 to all the terms
$-3 < 3x < 12$	Divide all terms by 3
$-1 < x < 4$	

SOLUTION 3 - 49
Ans: A

$x^2 + 4x - 12 > 0$
$(x + 6)(x - 2) > 0$
Thus, $\mathbf{-6 > x > 2}$

?

SOLUTION 3 - 50
Ans: A

$f(x) = x^4 + ax^3 + 5x^2 + bx + 6$
$f(2) = 2^4 + a(2)^3 + 5(2)^2 + b(2) + 6 = 16$
$8a + 2b = -26$
$4a + b = -13$ → (1)

$f(-1) = (-1)^4 + a(-1)^3 + 5(-1)^2 + b(-1) + 6 = 10$
$a + b = 2$
$b = 2 - a$ → (2)

$4a + (2 - a) = -13$
$3a + 2 = -13$
$3a = -15$
$a = -5$
$b = 2 - (-5) = 7$

Problems - Set 4
Logarithm, Binomial Theorem, Quadratic Equation

PROBLEM 4 - 1

If $\dfrac{\log_{10} x}{1-\log_{10} 2} = 2$, what is the value of x?

A. ¼
B. 25
C. 4
D. 5

PROBLEM 4 - 2
EE Oct. 1992

Solve for x: $\log 6 + x \log 4 = \log 4 + \log (32 + 4^x)$

A. 1
B. 2
C. 3
D. 4

PROBLEM 4 - 3

Which of the following cannot be used as a base of a system of logarithm?

A. e
B. 10
C. 2
D. 1

PROBLEM 4 - 4

If $\log_{52} 1000 = x$, what is the value of x?

A. 4.19
B. 5.23
C. 3.12
D. 4.69

PROBLEM 4 - 5

Find the value of a in the equation $\log_a 2187 = 7/2$.

A. 3
B. 6
C. 9
D. 12

PROBLEM 4 - 6

If $\log 2 = x$ and $\log 3 = y$, find $\log 1.2$.

A. $2x + y$
B. $2xy/10$
C. $2x + y - 1$
D. $xy - 1$

PROBLEM 4 - 7

$\dfrac{\log x^y}{\log y^x}$ is equal to:

A. x^y / y^x
B. $y \log x - x \log y$
C. $(y \log x) / (x \log y)$
D. 1

PROBLEM 4 - 8

If $10^{ax+b} = P$, what is the value of x?

A. $(1/a)(\log P - b)$
B. $(1/a)\log (P - b)$
C. $(1/a)\, P^{10-b}$
D. $(1/a)\log P^{10}$

PROBLEM 4 - 9

Find the value of $\log(a^a)^a$

A. $2a \log a$
B. $a^2 \log a$
C. $a \log a^2$
D. $(a \log a)^a$

PROBLEM 4 - 10

Solve for x: $x = \log_b a \times \log_c d \times \log_d c$

A. $\log_b a$
B. $\log_a c$
C. $\log_b c$
D. $\log_d a$

PROBLEM 4 - 11

Find the positive value of x if $\log_x 36 = 2$

A. 2
B. 4
C. 6
D. 8

PROBLEM 4 - 12

Find x if $\log_x 27 + \log_x 3 = 2$.

A. 9
B. 12
C. 8
D. 7

PROBLEM 4 - 13

Find a if $\log_2 (a + 2) + \log_2 (a - 2) = 5$.

A. 2
B. 4
C. 6
D. 8

PROBLEM 4 - 14

Solve for x if $\log_5 x = 3$.

A. 115
B. 125
C. 135
D. 145

PROBLEM 4 - 15

Find $\log P$ if $\ln P = 8$.

A. 2980.96
B. 2542.33
C. 3.47
D. 8.57

PROBLEM 4 - 16

If $\log_8 x = -n$, then x is equal to:

A. 8^n
B. $1 / 8^{-n}$
C. $1 / 8^n$
D. $81 / n$

PROBLEM 4 - 17

If $3 \log_{10} x - \log_{10} y = 0$, find y in terms of x.

A. $y = \sqrt[3]{x}$
B. $y = \sqrt{x^3}$
C. $y = x^3$
D. $y = x$

PROBLEM 4 - 18

Which of the following is correct?

A. $-2 \log 7 = 1/49$
B. $\log_7 (-2) = 1/49$
C. $\log_7 (1/49) = -2$
D. $\log_7 (1/49) = 2$

PROBLEM 4 - 19
ME April 1996

Log of the nth root of x equals log of x to the $1/n$ power and also equal to:

A. $\dfrac{\log(x)}{n}$
B. $n\log(x)$
C. $\dfrac{\log(x)^{1/n}}{n}$
D. $(n\text{-}1)\log(x)$

PROBLEM 4 - 20
ME April 1996

What is the natural logarithm of e to the xy power?

A. $1/xy$
B. $2.718/xy$
C. xy
D. $2.718xy$

PROBLEM 4 - 21
ME April 1997

What expression is equivalent to $\log x - \log (y + z)$?

A. $\log x + \log y + \log z$
B. $\log [\, x / (y + z)\,]$
C. $\log x - \log y - \log z$
D. $\log y + \log (x + z)$

PROBLEM 4 - 22
ME April 1997

What is the value of log to the base 10 of $1000^{3.3}$?

A. 9.9
B. 99.9
C. 10.9
D. 9.5

PROBLEM 4 - 23

If $\log_x 2 + \log_2 x = 2$, then the value of x is:

A. 1
B. 2
C. 3
D. 4

PROBLEM 4 - 24
CE Nov. 1997

$\text{Log}_6 845 = ?$

A. 4.348
B. 6.348
C. 5.912
D. 3.761

PROBLEM 4 - 25
CE May 1998, Similar to Nov. 1998

The logarithms of the quotient and the product of two numbers are 0.352182518 and 1.556302501, respectively. Find the first number?

A. 9
B. 10
C. 11
D. 12

PROBLEM 4 - 26

The sum of the logarithms of two numbers is 1.748188 and the difference of their logarithms is -0.0579919. One of the numbers is:

A. 9
B. 6
C. 8
D. 5

PROBLEM 4 - 27
CE Nov. 1999

Solve for y: $y = \ln \dfrac{e^x}{e^{x-2}}$.

A. 2
B. x
C. -2
D. x-2

PROBLEM 4 - 28
ECE April 1998

What is the value of (log 5 to the base 2) + (log 5 to the base 3)?

A. 3.97
B. 7.39
C. 9.37
D. 3.79

PROBLEM 4 - 29
ME Oct. 1997

The logarithm of negative number is:

A. irrational number
B. real number
C. imaginary number
D. complex number

PROBLEM 4 - 30
ME April 1998

38.5 to the x power = 6.5 to the x - 2 power, solve for x using logarithms.

A. 2.70
B. 2.10
C. -2.10
D. -2.02

PROBLEM 4 - 31
CE Nov. 1996

Find the 6th term of the expansion of $(1/2a - 3)^{16}$

A. $-\dfrac{22113}{256a^{11}}$
B. $-\dfrac{66339}{128a^{11}}$
C. $-\dfrac{22113}{128a^{11}}$
D. $-\dfrac{66339}{256a^{11}}$

PROBLEM 4 - 32
ECE April 1998

In the expansion of $(x + 4y)^{12}$, the numerical coefficient of the 5th term is

A. 253,440
B. 126,720
C. 63,360
D. 506,880

PROBLEM 4 - 33

The middle term in the expansion of $(x^2 - 3)^8$ is:

A. $-70\,x^8$
B. $70\,x^8$
C. $-5670\,x^8$
D. $5670\,x^8$

PROBLEM 4 - 34
CE Board

The term involving x^9 in the expansion of $(x^2 + 2/x)^{12}$ is:

A. $25434\,x^9$
B. $52344\,x^9$
C. $25344\,x^9$
D. $23544\,x^9$

PROBLEM 4 - 35

The constant term in the expansion of $\left(x + \dfrac{1}{x^{3/2}}\right)^{15}$ is:

A. 3003
B. 5005
C. 6435
D. 7365

PROBLEM 4 - 36

Find the sum of the coefficients in the expansion of $(x + 2y - z)^8$.

A. 256
B. 1024
C. 1
D. 6

PROBLEM 4 - 37
The sum of the coefficients in the expansion of $(x + 2y + z)^4 (x + 3y)^5$ is:
A. 524,288 C. 131,072
B. 65,536 D. 262,144

PROBLEM 4 - 38
ECE April 1995
The sum of the coefficients in the expansion of $(x + y - z)^8$ is:
A. less than 2 C. from 2 to 5
B. above 10 D. from 5 to 10

PROBLEM 4 - 39
ECE Nov. 1995
What is the sum of the coefficients of the expansion of $(2x - 1)^{20}$?
A. 1 C. 215
B. 0 D. 225

PROBLEM 4 - 40
In the quadratic equation $Ax^2 + Bx + C = 0$, the product of the roots is:
A. C/A C. $-C/A$
B. $-B/A$ D. B/A

PROBLEM 4 - 41
If 1/4 and -7/2 are the roots of the quadratic equation $Ax^2 + Bx + C = 0$, what is the value of B?
A. -28 C. -7
B. 4 D. 26

PROBLEM 4 - 42
In the equation $3x^2 + 4x + (2h - 5) = 0$, find h if the product of the roots is 4.
A. -7 / 2 C. 17 / 2
B. -10 / 2 D. 7 / 2

PROBLEM 4 - 43
If the roots of $ax^2 + bx + c = 0$ are u and v, then the roots of $cx^2 + bx + a = 0$ are:
A. u and v C. $1/u$ and $1/v$
B. $-u$ and v D. $-1/u$ and $-1/v$

PROBLEM 4 - 44
If the roots of the quadratic equation $ax^2 + bx + c = 0$ are 3 and 2 and a, b, and c are all whole numbers, find $a + b + c$.
A. 12 C. 2
B. -2 D. 6

PROBLEM 4 - 45
ECE March 1996
The equation whose roots are the reciprocals of the roots of $2x^2 - 3x - 5 = 0$ is:
A. $5x^2 + 3x - 2 = 0$ C. $5x^2 - 2x - 3 = 0$
B. $3x^2 - 5x - 2 = 0$ D. $2x^2 - 5x - 3 = 0$

PROBLEM 4 - 46
ECE Nov. 1997
The roots of a quadratic equation are 1/3 and ¼. What is the equation?
A. $12x^2 + 7x + 1 = 0$ C. $12x^2 - 7x + 1 = 0$
B. $12x^2 + 7x - 1 = 0$ D. $12x^2 - 7x - 1 = 0$

PROBLEM 4 - 47
Find k so that the expression $kx^2 - 3kx + 9$ is a perfect square.
A. 3 C. 12
B. 4 D. 6

PROBLEM 4 - 48
EE Oct. 1990
Find k so that $4x^2 + kx + 1 = 0$ will only have one real solution.
A. 1 C. 3
B. 4 D. 2

PROBLEM 4 - 49
The only root of the equation $x^2 - 6x + k = 0$ is:
A. 3 C. 6
B. 2 D. 1

PROBLEM 4 - 50
Two engineering students are solving a problem leading to a quadratic equation. One student made a mistake in the coefficient of the first-degree term, got roots of 2 and -3. The other student made a mistake in the coefficient of the constant term got roots of -1 and 4. What is the correct equation?
A. $x^2 - 6x - 3 = 0$ C. $x^2 + 3x + 6 = 0$
B. $x^2 + 6x + 3 = 0$ D. $x^2 - 3x - 6 = 0$

$r_1 + r_2 = 7/2 =$

$r_1 \cdot r_2 =$ [illegible]

$b^2 - 4ac = 0$

$k^2 - 4(4)$

ANSWER SHEET

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Solutions to Set 4
Logarithm, Binomial Theorem, Quadratic Equation

SOLUTION 4 - 1
Ans: B

$$\frac{\log_{10} x}{1-\log_{10} 2} = 2$$

$$\log_{10} x = 2 - 2\log_{10} 2 = 2 - \log_{10} 4$$

$$x = 10^{2-\log 4} = \frac{10^2}{10^{\log 4}} \qquad \text{but } 10^{\log 4} = 4$$

$$x = 100/4 = \mathbf{25}$$

SOLUTION 4 - 2
Ans: C

$$\log 6 + x\log 4 = \log 4 + \log(32 + 4^x)$$

$$\log 6 + \log 4^x = \log 4 + \log(32 + 4^x)$$

$$\log(6 \times 4^x) = \log[4(32 + 4^x)] \qquad \text{(But if } \log a = \log b,\ a = b\text{)}$$

$$6 \times 4^x = 4(32 + 4^x)$$

$$6 \times 4^x = 128 + 4 \times 4^x$$

$$2 \times 4^x = 128\,;\ 4^x = 64;\ x = \mathbf{3}$$

SOLUTION 4 - 3
Ans: D

A logarithm cannot have a base of 1

SOLUTION 4 - 4
Ans: A

$$\text{If } \log_{5.2} 1000 = x \qquad \text{then } 5.2^x = 1000$$

$$\ln 5.2^x = \ln 1000$$

$$x\ln 5.2 = \ln 1000;\ x = 4.19$$

SOLUTION 4 - 5
Ans: C

$$\text{If } \log_a 2187 = 7/2 \text{ then } a^{7/2} = 2187$$

$$a = 2187^{2/7} = \mathbf{9}$$

SOLUTION 4 - 6
Ans: C

$$\log 1.2 = \log\frac{12}{10} = \log 12 - \log 10$$

$$= \log(2)(2)(3) - 1 = \log(2) + \log(2) + \log(3) - 1$$

$$\text{but: } \log 2 = x \text{ and } \log 3 = y$$

$$\log 1.2 = x + x + y - 1 = \mathbf{2x + y - 1}$$

SOLUTION 4 - 7
Ans: C

$$\frac{\log x^y}{\log y^x} = \frac{y\log x}{x\log y}$$

SOLUTION 4 - 8
Ans: A

$$10^{ax+b} = P, \text{ taking the logarithm of both sides}$$

$$\log 10^{ax+b} = \log P \qquad \text{but } \log 10^{ax+b} = ax + b$$

$$ax + b = \log P$$

$$ax = \log P - b; \quad x = \frac{1}{a}(\log P - b)$$

SOLUTION 4 - 9
Ans: B

$$\log\left(a^a\right)^a = \log a^{a^2} = a^2 \log a$$

SOLUTION 4 - 10
Ans: A

$$x = \frac{\log a}{\log b} \times \frac{\log d}{\log c} \times \frac{\log c}{\log d}$$

$$x = \frac{\log a}{\log b} = \log_b a$$

SOLUTION 4 - 11
Ans: C

If $\log_x 36 = 2$, then $x^2 = 36$ and $x = 6$

SOLUTION 4 - 12
Ans: A

$\log_x 27 + \log_x 3 = 2$
$\log_x (27 \times 3) = \log_x 81 = 2$
$x^2 = 81; x = 9$

SOLUTION 4 - 13
Ans: C

$\log_2 (a + 2) + \log_2 (a - 2) = 5$
$\log_2 (a + 2)(a - 2) = \log_2 (a^2 - 4) = 5$
$a^2 - 4 = 2^5 = 32$
$a^2 = 36; \ a = 6$

SOLUTION 4 - 14
Ans: B

$\log_5 x = 3$
$x = 5^3 = 125$

SOLUTION 4 - 15
Ans: C

$\ln P = \log_e P = 8$
$P = e^8 = 2980.96$
$\log P = \log 2980.96 = 3.47$

SOLUTION 4 - 16
Ans: C

$\log_8 x = -n$
$x = 8^{-n} = 1/8^n$

SOLUTION 4 - 17
Ans: C

$3 \log x = \log y$
$\log x^3 = \log y$ (But if $\log a = \log b$, $a = b$)
Thus $y = x^3$

SOLUTION 4 - 18
Ans: C

Among the choices, choice C is correct.
$\log_7 (1/49) = -2$
$7^{-2} = 1/49$
$1/49 = 1/49$ (OK)

SOLUTION 4 - 19
Ans: A

$$\log \sqrt[n]{x} = \log(x)^{1/n}$$

$$\log \sqrt[n]{x} = \frac{\log(x)}{n}$$

SOLUTION 4 - 20
Ans: C

$\ln(e^{xy}) = \log_e (e^{xy})$
From the basic logarithm property; $\log_a a^x = x$
$\ln(e^{xy}) = xy$

SOLUTION 4 - 21
Ans: B

By principle, $\log a - \log b = \log (a / b)$
Thus, $\log x - \log (y + z) = \log [x / (y + z)]$

SOLUTION 4 - 22
Ans: A

$\log_{10} 1000^{3.3} = \log_{10} [10^3]^{3.3}$
$= \log_{10} 10^{9.9} = 9.9 \log_{10} 10 = 9.9$
Or using calculator, $\log_{10} 1000^{3.3} = 9.9$

SOLUTION 4 - 23
Ans: B

$\log_x 2 + \log_2 x = 2$

By definition, $\log_a b = \dfrac{\log_c b}{\log_c a}$

and $\log_a a = 1$, also $\log_a b = \dfrac{1}{\log_b a}$

$$\frac{\log_2 2}{\log_2 x} + \log_2 x = 2$$

$$\frac{1}{\log_2 x} + \log_2 x = 2 \quad \text{multiply by } \log_2 x$$

$1 + (\log_2 x)^2 = 2 \log_2 x$
$(\log_2 x)^2 - 2 \log_2 x + 1 = 0; \quad (\log_2 x - 1)^2 = 0$
$\log_2 x - 1 = 0; \log_2 x = 1$
$x = 2^1 = 2$

Note: This can also be solved by inspection or by substituting the values of x from the choices.

SOLUTION 4 - 24
Ans: D

By change base rule, $\log_b a = \log a / \log b = \ln a / \ln b$
$\log_6 845 = \log 845 / \log 6$
$\log_6 845 = 3.761$

SOLUTION 4 - 25
Ans: A

Let x and y be the numbers, then;
$\log_{10} (x/y) = 0.352182518$
$x/y = 10^{0.352182518}; \quad x/y = 2.25$
$x = 2.25\,y \quad \rightarrow (1)$

$\log_{10} (x\,y) = 1.556302501; \quad x\,y = 10^{1.556302501}$
$x\,y = 36 \quad \rightarrow (2)$

Substitute x in Eq. (1) to Eq. (2):
$(2.25y)\,y = 36$
$y^2 = 16; \ y = 4$
and $x = 2.25(4) = 9$

SOLUTION 4 - 26
Ans: C

$\log x + \log y = \log (xy) = 1.748188$

$xy = 10^{1.748188} = 56 \quad \rightarrow (1)$

$\log x - \log y = \log (x/y) = -0.0579919$

$x/y = 10^{-0.0579919}$

$x/y = 0.875$

$x = 0.875\, y \quad \rightarrow (2)$

Substitute x in Eq. (2) to Eq. (1):

$(0.875y)\, y = 56$

$y^2 = 64$

$y = 8$ and $x = 7$

SOLUTION 4 - 27
Ans: A

$$y = \ln \frac{e^x}{e^{x-2}} = \ln e^x - \ln e^{x-2}$$

$y = x \ln e - (x - 2) \ln e$ But $\ln e = 1$

$y = x - (x - 2)$; $y = \mathbf{2}$

SOLUTION 4 - 28
Ans: D

$\log_2 5 + \log_3 5 = (\log 5)/(\log 2) + (\log 5)/(\log 3)$

$\log_2 5 + \log_3 5 = \mathbf{3.787}$

SOLUTION 4 - 29
Ans: D

The logarithm of a negative number can be evaluated as follows:

$\log (-a) = \log (a)(-1)$

knowing that $i^2 = -1$,

$\log (-a) = \log (a)(i^2)$

$\log (-a) = \log a + \log i^2$

$\log (-a) = \log a + 2 \log i$

The exponential form of the imaginary number $i = e^{i(\pi/2)}$

$\log (-a) = \log a + 2 \log e^{i(\pi/2)}$

$\log (-a) = \log a + 2\, i(\pi/2) \log e$

$\log (-a) = \log a + \pi\, i \log e$

Thus, the logarithm of a negative number is a complex number.

SOLUTION 4 - 30
Ans: C

$$38.5^x = 6.5^{x-2} = \frac{6.5^x}{6.5^2}$$

$$6.5^2 = \frac{6.5^x}{38.5^x} = \left(\frac{6.5}{38.5}\right)^x$$

Taking the logarithm of both sides:

$\ln (6.5)^2 = \ln(6.5/38.5)^x$

$\ln (6.5)^2 = x \ln (6.5/38.5)$

$$x = \frac{\ln(6.5)^2}{\ln(6.5/38.5)} = -2.1$$

Another Solution:

$\ln(38.5)^x = \ln(6.5)^{x-2}$

$x \ln 38.5 = (x - 2) \ln 6.5$

$1.95x = x - 2$

$0.95x = -2;\ \ x = \mathbf{-2.1}$

Or we can just substitute the choices to the given equation.

SOLUTION 4 - 31
Ans: B

$$r^{th}\text{ term} = \frac{n!}{(n-r+1)!(r-1)!}\, x^{n-r+1}\, y^{r-1}$$

$$x = \frac{1}{2a};\ y = -3;\ r = 6$$

$$6^{th}\text{ term} = \frac{16!}{(16-6+1)!(6-1)!}\left(\frac{1}{2a}\right)^{16-6+1}(-3)^{6-1}$$

$$6^{th}\text{ term} = 4368\ \frac{1}{2048a^{11}}(-243) = -\frac{66339}{128a^{11}}$$

SOLUTION 4 - 32
Ans: B

$$r^{th}\text{ term} = \frac{n!}{(n-r+1)!(r-1)!}\, a^{n-r+1}\, b^{r-1}$$

The 5th term of $(x + 4y)^{12}$ is:

$$\frac{12!}{(12-5+1)!(5-1)!}(x)^{(12-5+1)}\,(4y)^{(5-1)} = 126720\, x^8 y^4$$

The numerical coefficient is **126,720**

SOLUTION 4 - 33
Ans: D

$$r^{th}\text{ term} = \frac{n!}{(n-r+1)!(r-1)!}\, a^{n-r+1}\, b^{r-1}$$

For the middle term, $r = \dfrac{n}{2} + 1$

(for even values of n only)

$r = 8/2 + 1 = 5$ (5th term)

$a = x^2;\ \ b = -3$

$$\text{Middle term (5}^{th}\text{ term)} = \frac{8!}{(8-5+1)!(5-1)!}\left(x^2\right)^{8-5+1}(-3)^{5-1}$$

$= 70\, x^8\, (-3)^4$

$= \mathbf{5670\, x^8}$

SOLUTION 4 - 34
Ans: C

$$r^{th}\text{ term} = \frac{n!}{(n-r+1)!(r-1)!}\ a^{n-r+1}\ b^{r-1}$$

The variable part is found in $a^{n-r+1}\ b^{r-1}$. We can find the term (r) by equating the variable of this expression to x^9.

$$a^{n-r+1}\ b^{r-1} = \left(x^2\right)^{12-r+1}\ \left(\frac{2}{x}\right)^{r-1}$$

$$= x^{26-2r}\ \frac{2^{r-1}}{x^{r-1}} = 2^{r-1}\ x^{26-2r-(r-1)}$$

$$= 2^{r-1}\ x^{27-3r}$$

Equating: $x^{27-3r} = x^9$ (But if $x^a = x^b$, $a = b$)
$27 - 3r = 9;\ r = 6$

$$6^{th}\text{ term} = \frac{12!}{(12-6+1)!(6-1)!}\left(x^2\right)^{12-6+1}\ \left(\frac{2}{x}\right)^{6-1}$$

$$6^{th}\text{ term} = 792\ x^{14}\frac{32}{x^5} = \mathbf{25344\ x^9}$$

SOLUTION 4 - 35
Ans: B

The constant term in the expansion of $\left(x + \frac{1}{x^{3/2}}\right)^{15}$ is the term that contains x^0.

The r^{th} term of $(a+b)^n$ is $\dfrac{n!}{(n-r+1)!(r-1)!}a^{n-r+1}b^{r-1}$, where $a = x$, $b = 1/x^{3/2}$ and $n = 15$.

$$a^{n-r+1}b^{r-1} = x^{15-r+1}\left(\frac{1}{x^{3/2}}\right)^{r-1} = x^0$$

$$\frac{x^{16-r}}{x^{3(r-1)/2}} = x^0$$

$$x^{(16-r)-3(r-1)/2} = x^0$$ (But if $x^a = x^b$, then $a = b$)

$16 - r - 3r/2 + 3/2 = 0$
$r = 7$ (7th term)

$$7^{th}\text{ term} = \frac{15!}{(15-7+1)!(7-1)!}x^{15-7+1}\left(\frac{1}{x^{3/2}}\right)^{7-1}$$

$$7^{th}\text{ term} = 5005\ x^9/x^9 = \mathbf{5005}$$

SOLUTION 4 - 36
Ans: A

To get the sum of the coefficients in the expansion, replace all the variables with 1.

Sum = $[1 + 2(1) - 1]^8$ = **256**

SOLUTION 4 - 37
Ans: D

Set all the variables to 1:
Sum of coefficients = $(1 + 2 + 1)^4(1 + 3)^5$ = **262,144**

SOLUTION 4 - 38
Ans: A

To get the sum of the coefficients (and constants) of a binomial expansion, set all variables to 1.

Sum of coefficients = $(1 + 1 - 1)^8 = 1^8 = 1$ (less than 2)

SOLUTION 4 - 39
Ans: B

Sum = $[2(1) - 1]^{20} = 1^{20} = 1$

Note: the last term of the expansion of this binomial is $(-1)^{20} = 1$, which is constant (not a coefficient). Therefore the sum of the coefficients is:

Sum of coefficients = 1 - 1 = **0**

SOLUTION 4 - 40
Ans: A

The product of the roots of the quadratic equation $Ax^2 + Bx + C = 0$ is C/A

SOLUTION 4 - 41
Ans: D

If 1/4 and -7/2 are roots of a quadratic equation, then;
$(x - 1/4)(x + 7/2) = 0$
$x^2 + 7x/2 - x/4 - 7/8 = 0$ multiply both side by 8
$8x^2 + 28x - 2x - 7 = 0$
$8x^2 + 26x - 7 = 0$
Therefore, $B = 26$

SOLUTION 4 - 42
Ans: C

For the quadratic equation $3x^2 + 4x + (2h - 5) = 0$
$A = 3$
$B = 4$
$C = 2h - 5$
The product of the roots is C/A
$C/A = (2h - 5)/3 = 4$
$2h - 5 = 12;\ h = \mathbf{17/2}$

SOLUTION 4 - 43
Ans: C

For the quadratic equation $ax^2 + bx + c = 0$ whose roots are u and v, the sum of the roots is $-b/a = u + v$ and the product of the roots is $c/a = uv$.

Dividing both sides of the equation $cx^2 + bx + a = 0$ by a:
$(c/a)x + (b/a)x + 1 = 0$
but $c/a = uv$ and $b/a = -(u + v)$

$(uv)\ x^2 - (u + v)\ x + 1 = 0$
$(ux - 1)(vx - 1) = 0$
$x = 1/u$ and $1/v$

SOLUTION 4 - 44
Ans: C

If the roots of $ax^2 + bx + c = 0$ are 3 and 2, then;
$(x - 3)(x - 2) = 0$
$x^2 - 5x + 6 = 0$

Thus; $a = 1$, $b = -5$, and $c = 6$
$a + b + c = 1 + (-5) + 6 = \mathbf{2}$

SOLUTION 4 - 45
Ans: A

For the given equation $2x^2 - 3x - 5 = 0$ or $(2x - 5)(x + 1) = 0$
$2x - 5 = 0; x = 5/2$
$x + 1 = 0; x = -1$

For the required equation, the roots are: $x = 2/5$ and -1 then;
$(x - 2/5)(x + 1) = 0$
$x^2 + x - 2x/5 - 2/5 = 0$
$x^2 + 3x/5 - 2/5 = 0$ multiply both side by 5
$\mathbf{5x^2 + 3x - 2 = 0}$

SOLUTION 4 - 46
Ans: C

If the roots of the equation are 1/3 and ¼, then the equation may
be written as:
$(x - 1/3)(x - ¼) = 0$
$x^2 - (7/12)x + 1/12 = 0$

Multiply both sides by 12:
$\mathbf{12x^2 - 7x + 1 = 0}$

SOLUTION 4 - 47
Ans: B

The expression $Ax^2 + Bx + C$ is a perfect square if $B^2 = 4AC$.
In the expression $kx^2 - 3kx + 9$, $A = k$, $B = -3k$, and $C = 9$.
$(-3k)^2 = 4(k)(9)$
$9k^2 = 36k$
$9k = 36$
$\mathbf{k = 4}$

SOLUTION 4 - 48
Ans: B

For the equation $Ax^2 + Bx + C = 0$ to have only one real solution, $B^2 - 4AC = 0$.

For $4x^2 + kx + 1 = 0$, $A = 4$, $B = k$, and $C = 1$.
$k^2 - 4(4)(1) = 0$
$\mathbf{k = \pm 4}$

SOLUTION 4 - 49
Ans: A

"The only root" means that the given quadratic equation has only one root, thus it is a perfect square.

For the equation $Ax^2 + Bx + C = 0$ to be a perfect square,
$B^2 - 4AC = 0$.
$B^2 - 4AC = (-6)^2 - 4(1)k = 0; k = 9$
The equation is therefore $x^2 - 6x + 9 = 0$ or $(x - 3)^2 = 0$.
Thus, the only root is **3**.

SOLUTION 4 - 50
Ans: D

Let $Ax^2 + Bx + C = 0$ be the correct equation.

For the first student who got roots of 2 and -3:
$(x - 2)(x + 3) = 0$
$x^2 + x - 6 = 0$ → Equation made by the first student

Since he made a mistake on the coefficient of the constant term (B) only, then $A = 1$ and $C = \mathbf{-6}$ are correct.

For the other student who got roots of -1 and 4:
$(x + 1)(x - 4) = 0$
$x^2 - 3x - 4 = 0$ → Equation made by the other student

Since he made a mistake on the constant term (C) only, then $A = \mathbf{1}$ and $B = \mathbf{-3}$ are correct.

Thus, the correct equation is $x^2 - 3x - 6 = 0$

Problems - Set 5
Age, Mixture, Work, Clock, Number Problems

PROBLEM 5 - 1

Two times the father's age is 8 more than six times his sons age. Ten years ago, the sum of their ages was 44. The age of the son is:

A. 49 C. 20
B. 15 D. 18

PROBLEM 5 - 2

Peter's age 13 years ago was 1/3 of his age 7 years hence. How old is Peter?

A. 15 C. 23
B. 21 D. 27

PROBLEM 5 - 3

A man is 41 years old and in seven years he will be four times as old as his son is at that time. How old is his son now?

A. 9 C. 5
B. 4 D. 8

PROBLEM 5 - 4

A father is three times as old as his son. Four years ago, he was four times as old as his son was at that time. How old is his son?

A. 36 years C. 32 years
B. 24 years D. 12 years

PROBLEM 5 - 5

The ages of the mother and her daughter are 45 and 5 years, respectively. How many years will the mother be three times as old as her daughter?

A. 5 C. 15
B. 10 D. 20

PROBLEM 5 - 6
ECE Nov. 1995

Mary is 24 years old. Mary is twice as old as Ana was when Mary was as old as Ana is now. How old is Ana?

A. 16 C. 19
B. 18 D. 20

PROBLEM 5 - 7

The sum of the parent's ages is twice the sum of their children's ages. Five years ago, the sum of the parent's ages is four times the sum of their children's ages. In fifteen years, the sum of the parent's ages will be equal to the sum of their children's ages. How many children were in the family?

A. 2 C. 4
B. 3 D. 5

PROBLEM 5 - 8

Two thousand (2000) kg of steel containing 8% nickel is to be made by mixing a steel containing 14% nickel with another steel containing 6% nickel. How much of the steel containing 14% nickel is needed?

A. 1500 kg C. 750 kg
B. 800 kg D. 500 kg

PROBLEM 5 - 9

A 40-gram alloy containing 35% gold is to be melted with a 20-gram alloy containing 50% gold. How much percentage of gold is the resulting alloy?

A. 40% C. 45%
B. 30% D. 35%

PROBLEM 5 - 10

In what ratio must a peanut costing P 240.00 per kg be mixed with a peanut costing P 340.00 per kg so that a profit of 20% is made by selling the mixture at P 360.00 per kg.?

A. 1:2 C. 2:3
B. 3:2 D. 3:5

PROBLEM 5 - 11

A 100-kilogram salt solution originally 4% by weight. Salt in water is boiled to reduce water content until the concentration is 5% by weight salt. How much water is evaporated?

A. 10 C. 20
B. 15 D. 25

PROBLEM 5 - 12

A pound of alloy of lead and nickel weighs 14.4 ounces in water, where lead losses 1/11 of its weight and nickel losses 1/9 of its weight. How much of each metal is in the alloy?

A. Lead = 7.2 ounces; Nickel = 8.8 ounces
B. Lead = 8.8 ounces; Nickel = 7.2 ounces
C. Lead = 6.5 ounces; Nickel = 5.4 ounces
D. Lead = 7.8 ounces; Nickel = 4.2 ounces

PROBLEM 5 - 13

An alloy of silver and gold weighs 15 oz. in air and 14 oz. in water. Assuming that silver losses 1/10 of its weight in water and gold losses 1/18 of its weight, how many oz at each metal are in the alloy?

A. silver = 4.5 oz; gold = 10.5 oz.
B. silver = 3.75 oz.; gold 11.25 oz.
C. silver = 5 oz; gold = 10 oz.
D. silver = 2.75 oz; gold = 12.25 oz.

PROBLEM 5 - 14
ME April 1998

A pump can pump out a tank in 11 hours. Another pump can pump out the same tank in 20 hours. How long will it take both pumps together to pump out the tank?

A. 1/2 hours
B. 1/2 hours
C. 6 hours
D. 7 hours

PROBLEM 5 - 15

Mr. Brown can wash his car in 15 minutes, while his son John takes twice as long to do the same job. If they work together, how many minutes can they do the washing?

A. 6
B. 8
C. 10
D. 12

PROBLEM 5 - 16

One pipe can fill a tank in 5 hours and another pipe can fill the same tank in 4 hours. A drainpipe can empty the full content of the tank in 20 hours. With all the three pipes open, how long will it take to fill the tank?

A. 2 hours
B. 2.5 hours
C. 1.92 hours
D. 1.8 hours

PROBLEM 5 - 17

A swimming pool is filled through its inlet pipe and then emptied through its outlet pipe in a total of 8 hours. If water enters through its inlet and simultaneously allowed to leave through its outlet, the pool is filled in 7 ½ hours. Find how long will it take to fill the pool with the outlet closed

A. 6
B. 2
C. 3
D. 5

PROBLEM 5 - 18

Three persons can do a piece of work alone in 3 hours, 4 hours, and 6 hours, respectively. What fraction of the job can they finish in one hour working together?

A. ¾
B. 4/3
C. ½
D. 2/3

PROBLEM 5 - 19

A father and his son can dig a well if the father works 6 hours and his son works 12 hours or they can do it if the father works 9 hours and the son works 8 hours. How long will it take for the son to dig the well alone?

A. 5 hours
B. 10 hours
C. 15 hours
D. 20 hours

PROBLEM 5 - 20

Peter and Paul can do a certain job in 3 hours. On a given day, they worked together for 1 hour then Paul left and Peter finishes the rest of the work in 8 more hours. How long will it take for Peter to do the job alone?

A. 10 hours
B. 11 hours
C. 12 hours
D. 13 hours

PROBLEM 5 - 21
ECE Nov. 1995

Pedro can paint a fence 50% faster than Juan and 20% faster than Pilar and together they can paint a given fence in 4 hours. How long will it take Pedro to paint the same fence if he had to work alone?

A. 10 hrs.
B. 11 hrs.
C. 13 hrs.
D. 15 hrs.

PROBLEM 5 - 22

Nonoy can finish a certain job in 10 days if Imelda will help for 6 days. The same work can be done by Imelda in 12 days if Nonoy helps for 6 days. If they work together, how long will it take for them to do the job?

A. 8.9
B. 8.4
C. 9.2
D. 8

PROBLEM 5 - 23

A pipe can fill up a tank with the drain open in three hours. If the pipe runs with the drain open for one hour and then the drain is closed, it will take 45 more minutes for the pipe to fill the tank. If the drain will be closed right at the start of filling, how long will it take for the pipe to fill the tank?

A. 1.15 hrs
B. 1.125 hrs
C. 1.325 hrs
D. 1.525 hrs

PROBLEM 5 - 24

Delia can finish a job in 8 hours. Daisy can do it in 5 hours. If Delia worked for 3 hours and then Daisy was asked to help her finish it, how long will Daisy have to work with Delia to finish the job?

A. 2/5 hour
B. 25/14 hours
C. 28 hours
D. 1.923 hours

PROBLEM 5 - 25
CE Nov. 1998

A job could be done by eleven workers in 15 days. Five workers started the job. They were reinforced with four more workers at the beginning of the 6th day. Find the total number of days it took them to finish the job.

A. 22.36
B. 21.42
C. 23.22
D. 20.56

PROBLEM 5 - 26

On one job, two power shovels excavate 20,000 m^3 of earth, the larger shovel working for 40 hours and the smaller for 35 hours. Another job, they removed 40,0000 m^3 with the larger shovel working 70 hours and the smaller working 90 hours. How much earth can the larger shovel move in one hour?

A. 173.91 C. 368.12
B. 347.83 D. 162.22

PROBLEM 5 - 27
EE April 1996

A and *B* can do a piece of work in 42 days, *B* and *C* in 31 days, and *A* and *C* in 20 days. Working together, how many days can all of them finish the work?

A. 18.9 C. 17.8
B. 19.4 D. 20.9

PROBLEM 5 - 28

Eight men can dig 150 ft of trench in 7 hrs. Three men can backfill 100 ft of the trench in 4 hrs. The time that it will take 10 men to dig and fill 200 ft of trench is:

A. 9.867 hrs C. 8.967 hrs
B. 9.687 hrs D. 8.687 hrs

PROBLEM 5 - 29

In two hours, the minute hand of the clock rotates through an angle of:

A. 45° C. 360°
B. 90° D. 720°

PROBLEM 5 - 30

In one day (24 hours), how many times will the hour-hand and minute-hand of a continuously driven clock be together?

A. 21 C. 23
B. 22 D. 24

PROBLEM 5 - 31

How many minutes after 3:00 PM will the minute hand of the clock overtakes the hour hand?

A. 14/12 minutes C. 16-4/11 minutes
B. 16-11/12 minutes D. 14/11 minutes

PROBLEM 5 - 32

How many minutes after 10:00 o'clock will the hands of the clock be opposite each other for the first time?

A. 21.41 C. 21.81
B. 22.31 D. 22.61

PROBLEM 5 - 33

What time between the hours of 12:00 noon and 1:00 pm would the hour-hand and the minute-hand of a continuously driven clock be in straight line?

A. 12:33 pm C. 12:37 pm
B. 12:30 pm D. 12:287 pm

PROBLEM 5 - 34
GE Feb. 1997

At what time after 12:00 noon will the hour hand and the minute hand of a clock first form an angle of 120°?

A. 21.818 C. 21.181
B. 12:21.181 D. 12:21.818

PROBLEM 5 - 35
GE Feb. 1994

From the time 6:15 PM to the time 7:45 PM of the same day, the minute hand of a standard clock describe an arc of:

A. 360° C. 540°
B. 120° D. 720°

PROBLEM 5 - 36

It is now between 3 and 4 o'clock and in twenty minutes the minute-hand will be as much as the hour-han as it is now behind it. What is the time now?

A. 3:06.36 C. 3:09.36
B. 3:07.36 D. 3:08.36

PROBLEM 5 - 37
EE October 1990

A man left his home at past 3:00 o'clock PM as indicated in his wall clock. Between two to three hours after, he returned home and noticed that the hands of the clock interchanged. At what time did he left his home?

A. 3:27.27 C. 3:22.22
B. 3:31.47 D. 3:44.44

PROBLEM 5 - 38

The sum of the reciprocals of two numbers is 11. Three times the reciprocal of one of the numbers is three more than twice the reciprocal of the other number. Find the numbers.

A. 5 and 6 C. 1/5 and 1/6
B. 7 and 4 D. 1/7 and 1/4

PROBLEM 5 - 39

If a two-digit number has x for its unit's digit and y for its ten's digit, represent the number.

A. yx C. $10x + y$
B. $10y + x$ D. $x + y$

PROBLEM 5 - 40

One number if five less than the other number. If their sum is 135, what are the numbers?

A. 70 & 75 C. 65 & 70
B. 60 & 65 D. 75 & 80

PROBLEM 5 - 41

In a two-digit number, the unit's digit is 3 greater than the ten's digit. Find the number if it is 4 times as large as the sum of its digits.

A. 47 C. 63
B. 58 D. 25

PROBLEM 5 - 42 Find two consecutive even integers such that the square of the larger is 44 greater than the square of the smaller integer.
A. 10 & 12 C. 8 & 10
B. 12 & 14 D. 14 & 16

PROBLEM 5 - 43 Twice the middle digit of a three-digit number is the sum of the other two. If the number is divided by the sum of its digit, the answer is 56 and the remainder is 12. If the digits are reversed, the number becomes smaller by 594. Find the number.
A. 258 C. 852
B. 567 D. 741

PROBLEM 5 - 44 The product of three consecutive integers is 9240. Find the third integer.
A. 20 C. 22
B. 21 D. 23

PROBLEM 5 - 45 The product of two numbers is 1400. If three (3) is subtracted from each number, their product becomes 1175. Find the bigger number.
A. 28 C. 32
B. 50 D. 40

PROBLEM 5 - 46 The sum of the digits of a three-digit number is 14. The hundreds digit being 4 times the units digit. If 594 is subtracted from the number, the order of the digits will be reversed. Find the number.
A. 743 C. 653
B. 563 D. 842

PROBLEM 5 - 47
ECE March 1996
The sum of two numbers is 21, and one number is twice the other. Find the numbers.
A. 7 and 14 C. 8 and 13
B. 6 and 15 D. 9 and 12

PROBLEM 5 - 48
ECE March 1996
Ten less than four times a certain number is 14. Determine the number.
A. 4 C. 6
B. 5 D. 7

PROBLEM 5 - 49
ECE Nov. 1997
The denominator of a certain fraction is three more than twice the numerator. If 7 is added to both terms of the

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fraction, the resulting fraction is 3/5. Find the original fraction.
A. 8/5 C. 13/5
B. 5/13 D. 3/5

PROBLEM 5 - 50 Three times the first of three consecutive odd integers is three more than twice the third. Find the third integer.
A. 9 C. 13
B. 11 D. 15

ANSWER SHEET

1. A B C D E
2. A B C D E
3. A B C D E
4. A B C D E
5. A B C D E
6. A B C D E
7. A B C D E
8. A B C D E
9. A B C D E
10. A B C D E
11. A B C D E
12. A B C D E
13. A B C D E
14. A B C D E
15. A B C D E
16. A B C D E
17. A B C D E
18. A B C D E
19. A B C D E
20. A B C D E
21. A B C D E
22. A B C D E
23. A B C D E
24. A B C D E
25. A B C D E
26. A B C D E
27. A B C D E
28. A B C D E
29. A B C D E
30. A B C D E
31. A B C D E
32. A B C D E
33. A B C D E
34. A B C D E
35. A B C D E
36. A B C D E
37. A B C D E
38. A B C D E
39. A B C D E
40. A B C D E
41. A B C D E
42. A B C D E
43. A B C D E
44. A B C D E
45. A B C D E
46. A B C D E
47. A B C D E
48. A B C D E
49. A B C D E
50. A B C D E
51. A B C D E
52. A B C D E
53. A B C D E
54. A B C D E
55. A B C D E
56. A B C D E
57. A B C D E
58. A B C D E
59. A B C D E
60. A B C D E

Solutions to Set 5
Age, Mixture, Work, Clock, Number Problems

SOLUTION 5 - 1
Ans: B

Let x be the age of the son and y be the age of the father.

$$2y = 6x + 8$$
$$y = 3x + 4 \qquad \rightarrow (1)$$

Ten years ago:

$$(x - 10) + (y - 10) = 44$$
$$x + y = 64 \qquad \rightarrow (2)$$

Substitute y in Eq. (1) to Eq. (2)

$$x + (3x + 4) = 64$$
$$4x = 60;\ x = \mathbf{15}$$

SOLUTION 5 - 2
Ans: C

Let x be the age of Peter now. His age 13 years ago was $x - 13$ and his age 7 years hence is $x + 7$.

$$x - 13 = (1/3)(x + 7)$$
$$3x - 39 = x + 7$$
$$2x = 46$$
$$x = \mathbf{23}$$

SOLUTION 5 - 3
Ans: C

Let x be the age of the son, then

$$41 + 7 = 4(x + 7)$$
$$48 = 4x + 28;\ x = \mathbf{5}$$

SOLUTION 5 - 4
Ans: D

Let x be the present age of the father and y be the present age of the son.

$$x = 3y \qquad \rightarrow (1)$$

Four years ago:

$$x - 4 = 4(y - 4)$$
$$x - 4y + 12 = 0 \qquad \rightarrow (2)$$

Substitute x in Eq. (1) to Eq. (2)

$$3y - 4y + 12 = 0$$
$$y = \mathbf{12\ years}$$

SOLUTION 5 - 5
Ans: C

Let x be the required number of years, then;

$$45 + x = 3\,(5 + x) = 15 + 3x$$
$$x = 15 \text{ years}$$

SOLUTION 5 - 6
Ans: B

Let A = present age Ana

"when Mary was as old as Ana is now" means that the past age of Mary is equal to the present age of Ana.

"Mary is twice as old as Ana was" means that the present age of Mary is twice the past age of Ana, or the past age of Ana is half the present age of Mary, i.e. 24/2 = 12.

The following table shows the relationship of their past and present ages:

	Past	Present
Mary	A	24
Ana	24/2	A

Note: the difference (or gap) between the ages of two persons is constant.
then;

$$A - 12 = 24 - A$$
$$2A = 36;\ A = \mathbf{18\ years} \text{ (present age of Ana)}$$

Check:
Their gap is 24 - 18 = 6. So when Mary was 18 years old, Ana was 12, and 24 (Mary's age now) is twice of 12 (Ana's past age).

SOLUTION 5 - 7
Ans: D

Let x = number of children in the family
y = sum of parent's ages
z = sum of children's ages

$$y = 2z \qquad \rightarrow (1)$$

Five years ago:

$$y - 5(2) = 4\,(z - 5x)$$
$$20x + y - 4z = 10 \qquad \text{But } y = 2z$$
$$20x - 2z = 10 \qquad \rightarrow (2)$$

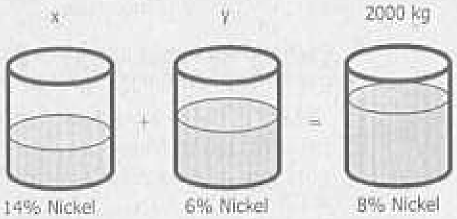
In fifteen years:

$$y + 15(2) = z + 15x$$
$$15x - y + z = 30 \quad \text{But } y = 2z$$
$$15\,x - z = 30 \qquad \rightarrow (3)$$

Eliminating from Equations (2) and (3) and solving for x:

$$\begin{array}{l} 20x - 2z = 10 \\ \underline{30x - 2z = 60} \\ -10x \quad = -50 \\ x = 5 \end{array}$$

SOLUTION 5 - 8
Ans: D



From the diagram shown:

$x + y = 2000$

$y = 2000 - x \qquad \rightarrow (1)$

$14x + 6y = 8(2000)$

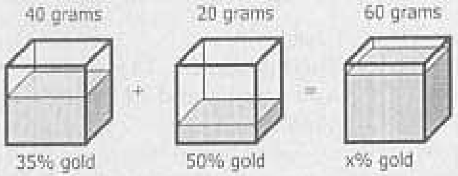
$7x + 3y = 8000 \qquad \rightarrow (2)$

Substitute y in Eq. (1) to Eq. (2):

$7x + 3(2000 - x) = 8000$

$\mathbf{x = 500\ kg}$

SOLUTION 5 - 9
Ans: A



$35\%(40) + 50\%(20) = x(60)$

$\mathbf{x = 40\%}$

SOLUTION 5 - 10
Ans: C

Let x be the quantity of P 240.00/kg peanut and y be the quantity of P340.00/kg peanut.

Capital = $240x + 340y$

Total sales = $360\,(x + y) = 360x + 360y$

Profit = Total sales - Capital = $360x + 360y - (240x + 340y)$

Profit = $120x + 20y$

But Profit is 20% of Capital

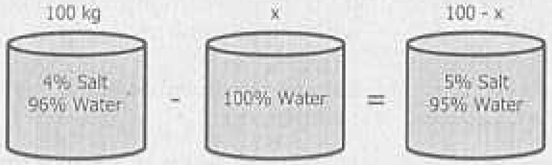
$120x + 20y = 0.20(240x + 340y)$

$120x + 20y = 48x + 68y$

$72x = 48y$

$$\text{Ratio} = \frac{x}{y} = \frac{48}{72} = \frac{2}{3}$$

SOLUTION 5 - 11
Ans: C



In Salt:

$4\%(100) - 0 = 5\%(100 - x)$

$400 = 500 - 5x;\ \mathbf{x = 20\ kg}$

In Water (check):

$96\%(100) - 100\%(x) = 95\%(100 - x)$

$1920 - 20x = 1900 - 19x;\ \mathbf{x = 20\ kg}$

SOLUTION 5 - 12
Ans: B

Let x be the weight of lead and y be the weight of nickel in the alloy.

$x + y = 1$ pound = 16 oz.

$y = 16 - x \qquad \rightarrow (1)$

In water:

$(x - x/11) + (y - y/9) = 14.4$

$(10/11)x + (8/9)y = 14.4 \qquad$ but $y = 16 - x$

$(10/11)x + (8/9)(16 - x) = 14.4$

$\mathbf{x = 8.8}$ **oz. (lead)**

$\mathbf{y = 7.2}$ **oz. (nickel)**

SOLUTION 5 - 13
Ans: B

Let x be the weight of silver in air and y be the weight of gold in air.

In air:

$x + y = 15$

$y = 15 - x \qquad \rightarrow (1)$

In water:

$(x - x/10) + (y - y/18) = 14$

$9x/10 + 17y/18 = 14 \qquad \rightarrow (2)$

Solving for x and y:

$\mathbf{x = 3.75}$ **oz (silver)**

$\mathbf{y = 11.25}$ **oz (gold)**

SOLUTION 5 - 14
Ans: D

Let t be required number of hours:

$$\frac{1}{11}t + \frac{1}{20}t = 1$$

$$\frac{31}{220}t = 1$$

$t = 220/31 = \mathbf{7.096}$ **hours**

SOLUTION 5 - 15
Ans: C

If Mr. brown can finish the job in 15 minutes, his son will take 30 minutes.

Let t be the time for both of them to finish the washing together, then,

$$\frac{1}{15}t + \frac{1}{30}t = 1;\quad t = \mathbf{10\ minutes}$$

SOLUTION 5 - 16
Ans: B

Let t be the time required, then with all pipes open:

$$\frac{1}{5}t + \frac{1}{4}t - \frac{1}{20}t = 1;\quad t = \mathbf{2.5\ hours}$$

SOLUTION 5 - 17
Ans: C

Let x be the time required for the inlet pipe to fill the pool and y be thetime required for the outlet pipe to empty the full content of the pool.

$$x + y = 8$$

$$y = 8 - x \qquad \rightarrow (1)$$

$$\frac{1}{x}(7\tfrac{1}{2}) - \frac{1}{y}(7\tfrac{1}{2}) = 1$$

$$\frac{1}{x} - \frac{1}{y} = \frac{2}{15} \qquad \rightarrow (2)$$

Substitute y in Eq. (1) to Eq. (20):

$$\frac{1}{x} - \frac{1}{8-x} = \frac{8-x-x}{x(8-x)} = \frac{2}{15} \qquad \text{Simplify}$$

$$120 - 30x = 16x - 2x^2$$

$$2x^2 - 46x + 120 = 0$$

$$x^2 - 23x + 60 = 0 \text{ or } (x - 20)(x - 3) = 0$$

$$x = 20 > 8 \text{ (absurd) and } x = \mathbf{3\ hours}$$

SOLUTION 5 - 18
Ans: A

In one (1) hour, the work done by the three persons is:

$$\frac{1}{3}(1) + \frac{1}{4}(1) + \frac{1}{6}(1) = \frac{3}{4}$$

Thus, they have done ¾ of the job in one hour.

SOLUTION 5 - 19
Ans: D

Let x and y be the number of hours that the father and his son, respectively, can dig the well working alone.

With the father working 6 hours and the son 12 hours:

$$\frac{1}{x}(6) + \frac{1}{y}(12) = 1 \qquad \text{Divide both sides by 6 and}$$

simplify

$$\frac{1}{x} = \frac{1}{6} - \frac{2}{y} \qquad \rightarrow (1)$$

With the father working 9 hours and the son 8 hours:

$$\frac{1}{x}(9) + \frac{1}{y}(8) = 1 \qquad \rightarrow (2)$$

Substitute $1/x$ in Eq. (1) to Eq. (2):

$$\left(\frac{1}{6} - \frac{2}{y}\right)(9) + \frac{1}{y}(8) = 1$$

$$\frac{9}{6} - 1 = \frac{10}{y};\quad y = \mathbf{20\ hours}$$

SOLUTION 5 - 20
Ans: C

Let x and y be the number of hours Peter and Paul, respectively, can finish the job working alone, then;

Working together for three (3) hours:

$$\frac{1}{x}(3) + \frac{1}{y}(3) = 1 \qquad \text{Divide both sides by 3}$$

$$\frac{1}{y} = \frac{1}{3} - \frac{1}{x} \qquad \rightarrow (1)$$

With Peter and Paul working for together for one (1) hour and Peter finishing the rest of the work in 8 hours without Paul:

$$\frac{1}{x}(1) + \frac{1}{y}(1) + \frac{1}{x}(8) = 1 \qquad \rightarrow (2)$$

Substitute $1/y$ in Eq. (1) to Eq. (2):

$$\frac{9}{x} + \frac{1}{3} - \frac{1}{x} = 1$$

$$\frac{8}{x} = 1 - \frac{1}{3};\quad x = \mathbf{12\ hours}$$

SOLUTION 5 - 21
Ans: A

Let x = no. of hours for Pedro alone to paint the fence
y = no. of hours for Juan alone to paint the fence
z = no. of hours for Pilar alone to paint the fence

With the three of them finishing the work together in 4 hours:

$$\frac{1}{x}(4) + \frac{1}{y}(4) + \frac{1}{z}(4) = 1$$

$$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{1}{4} \quad \rightarrow (1)$$

With Pedro being 50% faster than Juan

$$\frac{1}{x} = \frac{1}{y} + 50\% \frac{1}{y} = \frac{3}{2y}$$

$$\frac{1}{y} = \frac{2}{3x} \quad \rightarrow (2)$$

With Pedro being 20% faster than Pilar

$$\frac{1}{x} = \frac{1}{z} + 20\% \frac{1}{z} = \frac{6}{5z}$$

$$\frac{1}{z} = \frac{5}{6x} \quad \rightarrow (3)$$

Substitute $1/y$ in Eq. (2) and $1/z$ in Eq. (3) to Eq. (1):

$$\frac{1}{x} + \frac{2}{3x} + \frac{5}{6x} = \frac{1}{4}; \quad \frac{5}{2x} = \frac{1}{4}; \; x = \mathbf{10\ hours}$$

SOLUTION 5 - 22
Ans: B

Let x = Number of days for Nonoy alone to do the job,
y = Number of days for Imelda alone to do the job,
t = time for both of them to do the job together

With Nonoy working for 10 days and Imelda for 6 days:

$$\frac{1}{x}(10) + \frac{1}{y}(6) = 1 \quad \rightarrow (1)$$

With Imelda working for 12 days and Nonoy for 6 days:

$$\frac{1}{x}(6) + \frac{1}{y}(12) = 1$$

Divide both sides by 2 and simplify

$$\frac{1}{y}(6) = \frac{1}{2} - \frac{1}{x}(3) \quad \rightarrow (2)$$

Substitute $(1/y)(6)$ in Eq. (2) to Eq. (1)

$$\frac{1}{x}(10) + \frac{1}{2} - \frac{1}{x}(3) = 1; \quad \frac{1}{x}(7) = \frac{1}{2}$$

x = 14 days and y = 21 days

With Nonoy and Imelda working together for the entire period:

$$\frac{1}{14}t + \frac{1}{21}t = 1$$

t = 8.4 days

SOLUTION 5 - 23
Ans: B

Let x = time required for the inlet pipe to fill the empty tank with the drain pipe closed
y = time for the drain pipe to empty the full content of the tank with the inlet pipe closed in hours.

With the inlet pipe (and the drain open) filling the tank for 3 hours:

$$\frac{1}{x}(3) - \frac{1}{y}(3) = 1 \quad \rightarrow (1)$$

With the inlet and the drain open for 1 hour and the inlet filling the tank with the drain closed in 45 minutes (3/4 hr):

$$\frac{1}{x}(1) - \frac{1}{y}(1) + \frac{1}{x}\frac{3}{4} = 1$$

$$\frac{7}{x} - \frac{4}{y} = 4 \quad \rightarrow (2)$$

Subtract: Eq. (1) × 4 - Eq. (2) × 3 to eliminate y:

$$\begin{array}{rl} & \frac{12}{x} - \frac{12}{y} = 4 \\ - & \frac{21}{x} - \frac{12}{y} = 12 \\ \hline & -\frac{9}{x} \qquad = -8; \; x = \mathbf{1.125\ hrs.} \end{array}$$

SOLUTION 5 - 24
Ans: D

Let t = number of hours required.

With Delia working for 3 hours then Daisy helped her and both of them (working together) finished the job for "t" hours more:

$$\tfrac{1}{8}(3) + \tfrac{1}{8}t + \tfrac{1}{5}t = 1$$

$$\frac{13}{40}t = 1 - \frac{3}{8} = 0.625; \quad t = \mathbf{1.923\ hours}$$

SOLUTION 5 - 25
Ans: D

Let x be number of days one (1) worker can finish the job alone:

Then $\quad 11\frac{1}{x}(15) = 1; \; x = 165$ days

With five (5) workers starting the work for 5 days and were reinforced with four more workers to finish the job together (9 workers) in t days more:

$$5\frac{1}{165}(5) + 9\frac{1}{165}t = 1$$
$$t = 15.56$$

Thus, they finish the job in 5 + 15.56 = **20.56 days**

SOLUTION 5 - 26
Ans: A

Let x and y be the respective rate of the larger and smaller shovels in m³ per hour, then;

$$40x + 35y = 20000 \quad \rightarrow (1)$$
$$70x + 90y = 40000 \quad \rightarrow (2)$$

Solving for x and y:

$$x = 347.83 \text{ m}^3/\text{hr}$$
$$\mathbf{y = 173.91\ m^3/hr.}$$

SOLUTION 5 - 27
Ans: A

Let x, y, and z be the number of days A, B, and C, respectively can finish the job alone. Then;

$$\frac{1}{x} + \frac{1}{y} = \frac{1}{42} \quad \rightarrow (1)$$
$$\frac{1}{y} + \frac{1}{z} = \frac{1}{31} \quad \rightarrow (2)$$
$$\frac{1}{x} + \frac{1}{z} = \frac{1}{20} \quad \rightarrow (3)$$

Let t be the required number of days, then

$$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{1}{t} \quad \rightarrow (4)$$

Add: Eq. (1) + Eq. (2) + Eq. (3)

$$\left(\frac{1}{x} + \frac{1}{y}\right) + \left(\frac{1}{y} + \frac{1}{z}\right) + \left(\frac{1}{x} + \frac{1}{z}\right) = \frac{1}{42} + \frac{1}{31} + \frac{1}{20}$$
$$\frac{2}{x} + \frac{2}{y} + \frac{2}{z} = 0.10607 \text{ or } \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 0.05303 \quad \rightarrow (5)$$

Substitute Eq. (5) to Eq. (4)

$$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{1}{t} = 0.05303; \ \mathbf{t = 18.86\ days}$$

SOLUTION 5 - 28
Ans: A

Rate to dig = 8(7)/150 = 56/150 man-hr per ft.
Rate to backfill = 3(4)/100 = 12/100 man-hr per ft.
Required no. of man-hours to dig and fill 200 ft:
200(56/150) + 200(12/100) = 98.67 man-hours
Required time = 98.67 man-hours / 10 man
Required time = **9.867 hrs.**

SOLUTION 5 - 29
Ans: D

In one hour, the minute-hand rotates 360°.
In two hours it will rotate **720°**.

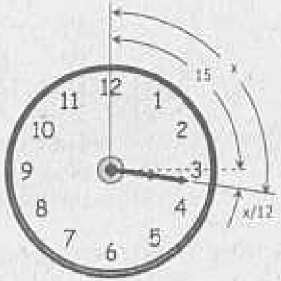
SOLUTION 5 - 30
Ans: B

In 12 hours, the minute-hand and hour-hand are together for eleven times.

Therefore in one day they will be together for **22 times.**

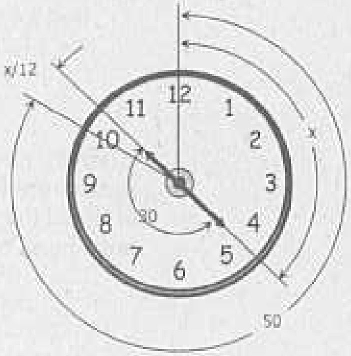
SOLUTION 5 - 31
Ans: C

$$15 + \frac{x}{12} = x$$
$$x = 16\frac{4}{11} \text{ min.}$$



SOLUTION 5 - 32
Ans: C

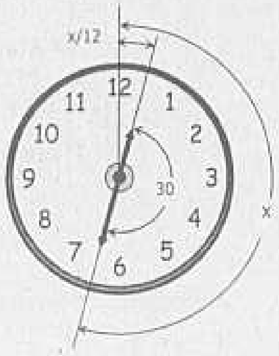
$$x + 30 = 50 + x/12$$
$$\frac{11}{12}x = 20$$
$$\mathbf{x = 21.81\ min.}$$



SOLUTION 5 - 33
Ans: A

$$x/12 + 30 = x$$
$$x = 32.73 \text{ minutes}$$

Therefore, the time is **12:32.73**



SOLUTION 5 - 34
Ans: D

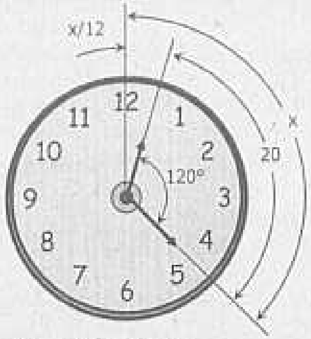
(Note: 5 minutes = 30°)
120° = 20 minutes

From the figure:

$$x = \frac{x}{12} + 20$$

$$\frac{11}{12}x = 20$$

$$x = 21.818 \text{ minutes}$$



Time: **12:21.818**

SOLUTION 5 - 35
Ans: C

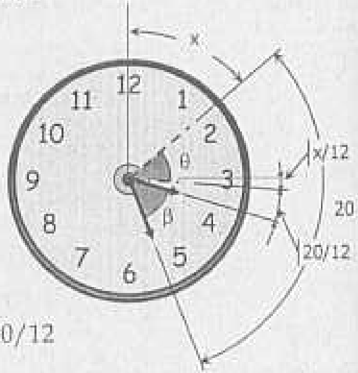
Total time elapsed from 6:15 to 7:45 is 1.5 hours. Since in one hour the minute-hand will travel 360°, then the total angle is 1.5(360) = **540°**

SOLUTION 5 - 36
Ans: A

At present (now), the minute-hand is behind the hour-hand by θ:

$$\theta = 15 + x/12 - x$$

$$\theta = 15 - 11x/12$$



After 20 minutes, the minute-hand is ahead the hour-hand by β:

$$\beta = x + 20 - 15 - x/12 - 20/12$$

$$\beta = 11x/12 + 10/3$$

β = θ (according to the condition in the problem)

$$11x/12 + 10/3 = 15 - 11x/12$$

$$x = 6.36 \text{ minutes}$$

Therefore, the time now is **3:06.36**

SOLUTION 5 - 37
Ans: B

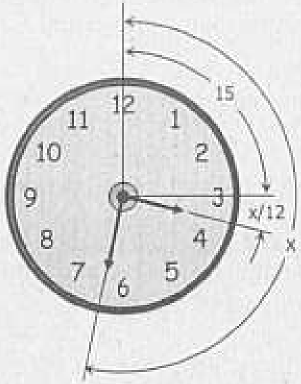


Fig. (a): When he left home past 3:00 o'clock

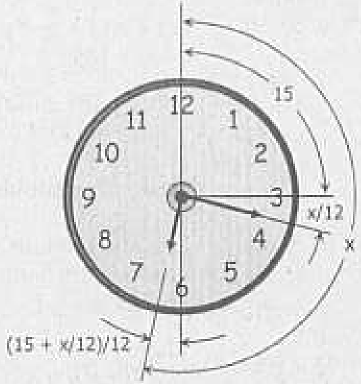


Fig. (b): When he returned home after 2 to 3 hours

In Figure (*b*)

$$x = 30 + \frac{15 + \frac{x}{12}}{12}$$

$$12x = 360 + 15 + \frac{x}{12}$$

$$\frac{143}{12}x = 375$$

$$x = 31.47$$

Thus, the time when he left home is **3:31.47**

SOLUTION 5 - 38
Ans: C

Let x and y be the numbers, then:

$$1/x + 1/y = 11$$

$$1/y = 11 - 1/x \quad \rightarrow (1)$$

$$3(1/x) = 2(1/y) + 3 \quad \rightarrow (2)$$

Substitute $1/y$ in Eq. (1) to Eq. (2):

$$3(1/x) = 2(11 - 1/x) + 3$$

$$3(1/x) = 22 - 2(1/x) + 3$$

$$5/x = 25$$

x = **1/5** and y = **1/6**

Note: This problem can be solved by inspection.

SOLUTION 5 - 39
Ans: B

If the unit's digit digit is x and the ten's digit is y, the number is $10y + x$.

SOLUTION 5 - 40
Ans: C

Let x and y be the numbers, then $x = y - 5$ and $x + y = 135$.

Solving the simultaneous equation:

x = **65** and y = **70**.

Note: This problem can be solved by inspection.

SOLUTION 5 - 41
Ans: C

Let u be the unit's digit and t be the ten's digit, then $u = t + 3$, the sum of the digits is $u + t$, and the number is $10t + u$

Then;

$$10t + u = 4(u + t), \text{but } u = t + 3$$
$$10t + t + 3 = 4(t + 3 + t)$$
$$11t + 3 = 8t + 12$$
$$t = 3 \text{ and } u = 6$$

The number is **63**

Note: This problem can be solved by inspection.

SOLUTION 5 - 42
Ans: A

Let x be the smaller integer, then the larger integer is $x + 2$.

$$(x + 2)^2 - x^2 = 44$$
$$x^2 + 4x + 4 - x^2 = 44$$
$$x = \mathbf{10} \text{ and } x + 2 = \mathbf{12}$$

Note: This problem can be solved by inspection.

SOLUTION 5 - 43
Ans: C

Let h, t, and u be the hundred's, ten's, and unit's digits, respectively.

The number is $100h + 10t + u$

Reversed number = $100u + 10t + h$

$$2t = h + u \qquad \rightarrow (1)$$

$$\frac{100h + 10t + u}{h + t + u} = 56 + \frac{12}{h + t + u}$$
$$\frac{(100h + 10t + u) - 12}{h + t + u} = 56$$
$$100h + 10t + u - 12 = 56h + 56t + 56u$$
$$44h - 46t - 55u = 12 \qquad \rightarrow (2)$$

$$100u + 10t + h = 100h + 10t + u - 594$$
$$99u - 99h = -594$$
$$h = 6 + u \qquad \rightarrow (3)$$

Substitute h in Eq. (3) to Eq. (1)

$$2t = (6 + u) + u = 6 + 2u; \quad t = 3 + u \rightarrow (4)$$

Substitute h in Eq. (3) to Eq. (2)

$$44(6 + u) - 46(3 + u) - 55u = 12$$
$$u = 2$$
$$h = 8 \text{ and } t = 5$$

The number is 852

Note: This problem can be solved by inspection.

SOLUTION 5 - 44
Ans: C

Let x be the third integer, then the second integer is $(x - 1)$ and the first integer is $(x - 2)$.

Product: $x(x - 1)(x - 2) = 9420$

By trial and error using the choices: x = **22**

SOLUTION 5 - 45
Ans: B

Let x and y be the numbers, then;

$$x\,y = 1400 \text{ or } y = 1400 / x \qquad \rightarrow (1)$$
$$(x - 3)(y - 3) = 1175 \qquad \rightarrow (2)$$

Substitute y in Eq. (1) to Eq. (2):

$(x - 3)(1400/x - 3) = 1175$ (results to quadratic equation)

By trial and error using the choices: x = **50**

SOLUTION 5 - 46
Ans: D

Let h be the hundreds' digit, t be the tens' digit, and u be the unit's digit.

$$\text{Number} = 100h + 10t + u$$
$$h + t + u = 14 \qquad \rightarrow (1)$$
$$h = 4u \qquad \rightarrow (2)$$

$$100h + 10t + u - 594 = 100u + 10t + h$$
$$99h - 99u = 594 \qquad \rightarrow (3)$$

Substitute h in Eq. (2) to Eq. (1):

$$99(4u) - 99u = 594; \; u = 2$$
$$h = 4(2) = 8, \; t = 14 - 8 - 2 = 4$$

Therefore, the number is **842.**

Note: This problem can be solved by inspection.

SOLUTION 5 - 47
Ans: A

Let x and y be the numbers

$$x + y = 21$$
$$x = 2y$$

Solving for x and y:

$$y = 7$$
$$x = 2(7) = 14$$

Note: This problem can be solved by inspection.

SOLUTION 5 - 48
Ans: C

Let x be the number

$$4x - 10 = 14$$
$$4x = 24$$
$$x = 6$$

SOLUTION 5 - 49
Ans: B

If we let x be the numerator, then the denominator is $2x + 3$ (three more than twice the numerator)

The original fraction is $\frac{x}{2x+3}$

If 7 is added to both terms:

$$\frac{(x)+7}{(2x+3)+7} = \frac{3}{5} = \frac{x+7}{2x+10}$$
$$6x + 30 = 5x + 35$$
$$x = 5$$

$$\text{Original fraction} = \frac{5}{2(5)+3} = 5/13$$

This can clearly be seen from the choices. Considering that the denominator is 3 more than twice the numerator, only 5/13 is satisfied.

SOLUTION 5 - 50
Ans: D

Let x, $x + 2$, and $x + 4$ be the three consecutive odd integers.

Then

$$3x = 2(x + 4) + 3$$
$$x = 11$$

The third integer is $11 + 4 = 15$

Problems - Set 6
Motion, Variation, Percent, Miscellaneous Problems

PROBLEM 6 - 1

Nonoy left Pikit to drive to Davao at 6:15 PM and arrived at 11:45 PM. If he averaged 30 mph and stopped 1 hour for dinner, how far is Davao from Pikit.

A. 128   C. 160
B. 135   D. 256

PROBLEM 6 - 2

A man fires a target 420 m away and hears the bullet strike 2 seconds after he pulled the trigger. An observer 525 m away from the target and 455 m from the man heard the bullet strike the target one second after he heard the report of the rifle. Find the velocity of the bullet.

A. 525 m/s   C. 350 m/s
B. 360 m/s   D. 336 m/s

PROBLEM 6 - 3

A man travels in a motorized banca at the rate of 12 kph from his barrio to the poblacion and come back to his barrio at the rate of 10 kph. If his total time of travel back and forth is 3 hours and 10 minutes, the distance from the barrio to the poblacion is:

A. 17.27 km   C. 12.77 km
B. 17.72 km   D. 17.32 km

PROBLEM 6 - 4

It takes Michael 60 seconds to run around a 440-yard track. How long does it take Jordan to run around the track if they meet in 32 seconds after they start together in a race around the track in opposite directions?

A. 58.76 seconds   C. 65.87 seconds
B. 68.57 seconds   D. 86.57 seconds

PROBLEM 6 - 5

Juan can walk from his home to his office at the rate of 5 mph and back at the rate of 2 mph. What is his average speed in mph?

A. 2.86   C. 4.12
B. 3.56   D. 5.89

PROBLEM 6 - 6

Kim and Ken traveled at the same time at the rate of 20 m/min, from the same point on a circular track of radius 600 m. If Kim walks along the circumference and Ken towards the center, find their distance after 10 minutes.

A. 193 m   C. 241 m
B. 202 m   D. 258 m

PROBLEM 6 - 7

Two ferryboats ply back and forth across a river with constant but different speeds, turning at the riverbanks without loss of time. They leave the opposite shores at the same instant, meet for the first time 900 meters from one shore, and meet for the second time 500 meters from the opposite shore. What is the width of the river?

A. 1500 m   C. 2000 m
B. 1700 m   D. 2200 m

PROBLEM 6 - 8
CE May 1998

A boat takes 2/3 as much time to travel downstream from C to D, as to return, If the rate of the river's current is 8 kph, what is the speed of the boat in still water?

A. 38   C. 40
B. 39   D. 41

PROBLEM 6 - 9
ECE Nov. 1998

A man rows downstream at the rate of 5 mph and upstream at the rate of 2 mph. How far downstream should he go if he is to return in 7/4 hours after leaving?

A. 2 mi   C. 3 mi
B. 3.5 mi   D. 2.5 mi

PROBLEM 6 - 10
EE April 1997

A jogger starts a course at a steady rate of 8 kph. Five minutes later, a second jogger the same course at 10 kph. How long will it take for the second jogger to catch the first?

A. 20 min   C. 30 min
B. 25 min   D. 15 min

PROBLEM 6 - 11
CE May 1999

At 2:00 pm, an airplane takes off at 340 mph on an aircraft carrier. the aircraft carrier moves due south at 25 kph in the same direction as the plane. At 4:05 pm, the communication between the plane and the aircraft carrier was lost. Determine the communication range in miles between the plane and the carrier.

A. 656 miles   C. 557 miles
B. 785 miles   D. 412 miles

PROBLEM 6 - 12

A boat going across a lake 8 km wide proceed 2 km at a certain speed and then completes the trip at a speed 1/2 kph faster. By doing this, the boat arrives 10 minutes earlier than if the original speed had been maintained. Find the original speed of the boat.

A. 2 kph   C. 9 kph
B. 4 kph   D. 5 kph

PROBLEM 6 - 13
CE May 1993

Given that w varies directly as the product of x and y and inversely as the square of z and that $w = 4$ when $x = 2$, $y = 6$, and $z = 3$. Find w when $x = 1$, $y = 4$, and $z = 2$.

A. 4   C. 1
B. 2   D. 3

PROBLEM 6 - 14
ECE Nov. 1993

If x varies directly as y and inversely as z, and $x = 14$ when $y = 7$ and $z = 2$, find x when $z = 4$ and $y = 16$.

A. 14   C. 16
B. 4   D. 8

PROBLEM 6 - 15

The electrical resistance of a cable varies directly as its length and inversely as the square of its diameter. If a cable 600 meters long and 25 mm in diameter has a resistance of 0.1 ohm, find the length of the cable 75 mm in diameter with resistance of 1/6 ohm.

A. 6000 m   C. 8000 m
B. 7000 m   D. 9000 m

PROBLEM 6 - 16

The electrical resistance offered by an electric wire varies directly as the length and inversely as the square of the diameter of the wire. Compare the electrical resistance offered by two pieces of wire of the same material, one being 100 m long and 5 mm in diameter, and the other is 50 m long and 3 mm in diameter.

A. $R_1 = 0.57\ R_2$ C. $R_1 = 0.84\ R_2$
B. $R_1 = 0.72\ R_2$ D. $R_1 = 0.95\ R_2$

PROBLEM 6 - 17

The time required for an elevator to lift a weight varies directly with the weight and the distance through which it is to be lifted and inversely as the power of the motors. If it takes 20 seconds for a 5-hp motor to lift 50 lbs. through 40 feet, what weight can an 80-hp motor lift through a distance of 40 feet within 30 seconds?

A. 1000 lbs.   C. 1175 lbs.
B. 1150 lbs.   D. 1200 lbs.

PROBLEM 6 - 18
ECE Nov. 1995

The time required by an elevator to lift a weight, vary directly with the weight and the distance through which it is to be lifted and inversely as the power of the motor. If it takes 30 seconds for a 10-hp motor to lift 100 lbs. through 50 feet, what size of motor is required to lift 800 lbs. in 40 seconds through a distance of 40 feet?

A. 48 hp   C. 56 hp
B. 50 hp   D. 58 hp

PROBLEM 6 - 19

In a certain department store , the monthly salary of a saleslady is partly constant and varies as the value of her sales for the month. When the value of her sales for the month is P10,000.00, her salary for that month is P900.00. When her sales goes up to P12,000.00, her monthly salary goes up to P1,000.00. What must be the value of her sales for the month so that her salary for that month will be P2,000.00.

A. P25,000.00   C. P32,000.00
B. P28,000.00   D. P36,000.00

PROBLEM 6 - 20

A man sold 100 eggs. Eighty of them were sold at a profit of 30% while the rest were sold at a loss of 40%. What is the percentage gain or loss on the whole stock?

A. 14%   C. 16%
B. 15%   D. 17%

PROBLEM 6 - 21

The population of the country increases 5% each year. Find the percentage it will increase in three years.

A. 5%   C. 15.15%
B. 15%   D. 15.76%

PROBLEM 6 - 22

Pedro bought two cars, one for P 600,000.00 and the other for P 400,000.00. He sold the first at a gain of 10% and the second at a loss of 12%. What was his total percentage gain or loss?

A. 6% gain   C. 1.20% gain
B. 0% gain   D. 6% loss

PROBLEM 6 - 23

A grocery owner raises the prices of his goods by 10%. Then he starts his Christmas sale by offering the customers a 10% discount. How many percent of discount does the customers actually get?

A. nothing   C. 9% discount
B. 1% discount   D. they pay 1% more

PROBLEM 6 - 24

Kim sold a watch for P 3,500.00 at a loss of 30% on the cost price. Find the corresponding loss or gain if he sold it for P 5,050.00

A. 1% loss   C. 1% gain
B. 10% loss   D. 10% gain

PROBLEM 6 - 25

By selling balut at P 5.00 each, a vendor gains 20%. The cost price of egg rises by 12.5%. If he sells the balut at the same price as before, find his new gain in percent.

A. 7.5%   C. 8%
B. 5%   D. 6.25%

PROBLEM 6 - 26

The enrollment at college *A* and college *B* both grew up by 8% from 1980 to 1985. If the enrollment in college *A* grew up by 800 and the enrollment in college *B* grew up by 840, the enrollment at college *B* was how much greater than the enrollment in college *A* in 1985?

A. 650   C. 483
B. 504   D. 540

PROBLEM 6 - 27

A group consists of n boys and n girls. If two of the boys are replaced by two other girls, then 49% of the group members will be boys. Find the value of n.

A. 100   C. 50
B. 49   D. 51

PROBLEM 6 - 28

On his Christmas Sale, a merchant marked a pair of slipper P180.00, which is 20% off the normal retail price. If the retail price is 50% higher than the wholesale price, what is the wholesale price of the slipper?

A. P 18.00   C. P 15.00
B. P 17.00   D. P 22.50

PROBLEM 6 - 29

A certain Xerox copier produces 13 copies every 10 seconds. If the machine operates without interruption, how many copies will it produce in an hour?

A. 780   C. 1825
B. 46800   D. 4680

PROBLEM 6 - 30

At a certain printing plant, each of the machines prints 6 newspapers every second. If all machines work together but independently without interruption, how many minutes will it take to print the entire run of 18000 newspapers? (Hint: let x = number of machines)

A. $50x$ C. $50 / x$
B. $3000 / x$ D. $3000x$

PROBLEM 6 - 31
ME April 1996

A manufacturing firm maintains one product assembly line to produce signal generators. Weekly demand for the generators is 35 units. The line operates for 7 hours per day, 5 days per week. What is the maximum production time per unit in hours required for the line to meet the demand?

A. 1 hour C. 3 hours
B. 0.75 hour D. 2.25 hours

PROBLEM 6 - 32

Of the 316 people watching a movie, there are 78 more children than women and 56 more women the men. The number of men in the movie house is:

A. 176 C. 42
B. 98 D. 210

PROBLEM 6 - 33

A certain department store has an inventory of Q units of a certain product at time $t = 0$. The store sells the product at a steady rate of Q/A units per week, and exhausts the inventory in A weeks. The amount of product in inventory at any time t is:

A. $Q - (Q / A)\,t$ C. $Q\,t - Q / A$
B. $Q + (Q / A)\,t$ D. $Q\,t - (Q / A)\,t$

PROBLEM 6 - 34
ECE March 1996

A merchant has three items on sale: namely, a radio for P50, a clock for P30, and a flashlight for P1. At the end of the day, she has sold a total of 100 of the three items and has taken exactly P1000 on the total sales. How many radios did he sell?

A. 80 C. 16
B. 4 D. 20

PROBLEM 6 - 35

The price of 8 calculators ranges from P200 to P1000. If their average price is P950, what is the lowest possible price of any one of the calculators?

A. 500 C. 600
B. 550 D. 650

PROBLEM 6 - 36

A deck of 52 playing cards is cut into two piles. The first pile contains 7 times as many black cards as red cards. The second pile contains the number of red cards that is an exact multiple as the number of black cards. How many cards are there in the first pile.

A. 14 C. 16
B. 15 D. 17

PROBLEM 6 - 37
ECE Nov. 1997

The Population of the Philippines doubled in the last 30 years from 1967 to 1997. Assuming that the rate of population increase will remain the same in what year will the population triple?

A. 2030 C. 2021
B. 2027 D. 2025

PROBLEM 6 - 38

Determine the unit's digit in the expansion of 3^{855}.

A. 3 C. 7
B. 9 D. 1

PROBLEM 6 - 39
ECE April 1998

Find the 1987th digit in the decimal equivalent of 1785/9999 starting from the decimal point.

A. 1 C. 8
B. 7 D. 5

PROBLEM 6 - 40

Find the sum of all positive integral factors of 2048.

A. 4095 C. 4560
B. 3065 D. 1254

PROBLEM 6 - 41

In how many ways can two integers be selected from the numbers 1,2,3,...50 so that their difference is exactly 5?

A. 50 C. 45
B. 5 D. 41

PROBLEM 6 - 42

A box contains 8 white balls, 15 green balls, 6 black balls. a8 red balls, and 13 yellow balls. How many balls must be drawn to ensure that there will be three balls of the same color?

A. 8 C. 10
B. 9 D. 11

PROBLEM 6 - 43

A shoe store sells 10 different sizes of shoes, each in both high-cut and low-cut variety, each either rubber or leather, and each with white or black color. How many different kinds of shoes does he sell?

A. 64 C. 72
B. 80 D. 92

PROBLEM 6 - 44
ME Oct. 1999

An engineer was told that a survey had been made on a certain rectangular field but the dimensions had been lost. An assistant remembered that if the field had been 100 ft longer and 25 ft narrower, the area would have been increased by 2500 sq. ft, and that if it had been 100 ft shorter and 50 ft wider, the area would have been decreased 5000 sq. ft. What was the area of the field?

A. 25,000 ft² C. 20,000 ft²
B. 15,000 ft² D. 22,000 ft²

PROBLEM 6 - 45
EE April 1994

A 10-meter tape is 5 mm short. What is the correct length in meters.

A. 9.995 m C. 9.95 m
B. 10.05 m D. 10.005 m

PROBLEM 6 - 46
ME Oct. 1997

The distance between two points measured with a steel tape was recorded as 916.58 ft. Later, the tape was checked and found to be only 99.9 ft long. What is the true distance between the points?

A. 935.66 ft C. 955.66 ft
B. 966.15 ft D.. 915.66 ft

PROBLEM 6 - 47
ME April 1996

A certain steel tape is known to be 100.000 feet long at a temperature of 70 °F. When the tape is at a temperature of 10 °F, what tape reading corresponds to a distance of 90 ft? Coefficient of linear expansion of the tape is 5.833 × 10-6 per °F.

A. 85.931 C. 90.031
B. 88.031 D. 93.031

PROBLEM 6 - 48
ME April 1996

A line was measured with a steel tape when the temperature was 30 °C. The measured length of the line was found to be 1,256.271 feet. The tape was afterwards tested when the temperature was 10 °C and it was found to be 100.042 feet long. What was the true length of the line if the coefficient of expansion of the tape was 0.000011 per °C?

A. 1,275.075 feet C. 1,256.547 feet
B. 1,375.575 feet D. 1,249.385 feet

PROBLEM 6 - 49
ME April 1997

The standard deviation of the numbers 1, 4, & 7 is:

A. 2.3567 C. 3.2256
B. 2.4495 D. 3.8876

PROBLEM 6 - 50

Three cities are connected by roads forming a triangle, all of different lengths. It is 30 km around the circuit. One of the roads is 10 km long and the longest is 10 km longer than the shortest. What is the length of the longest road?

A. 5 km C. 15 km
B. 10 km D. 20 km

ANSWER SHEET

1.	A	B	C	D	E	21.	A	B	C	D	E	41.	A	B	C	D	E
2.	A	B	C	D	E	22.	A	B	C	D	E	42.	A	B	C	D	E
3.	A	B	C	D	E	23.	A	B	C	D	E	43.	A	B	C	D	E
4.	A	B	C	D	E	24.	A	B	C	D	E	44.	A	B	C	D	E
5.	A	B	C	D	E	25.	A	B	C	D	E	45.	A	B	C	D	E
6.	A	B	C	D	E	26.	A	B	C	D	E	46.	A	B	C	D	E
7.	A	B	C	D	E	27.	A	B	C	D	E	47.	A	B	C	D	E
8.	A	B	C	D	E	28.	A	B	C	D	E	48.	A	B	C	D	E
9.	A	B	C	D	E	29.	A	B	C	D	E	49.	A	B	C	D	E
10.	A	B	C	D	E	30.	A	B	C	D	E	50.	A	B	C	D	E
11.	A	B	C	D	E	31.	A	B	C	D	E	51.	A	B	C	D	E
12.	A	B	C	D	E	32.	A	B	C	D	E	52.	A	B	C	D	E
13.	A	B	C	D	E	33.	A	B	C	D	E	53.	A	B	C	D	E
14.	A	B	C	D	E	34.	A	B	C	D	E	54.	A	B	C	D	E
15.	A	B	C	D	E	35.	A	B	C	D	E	55.	A	B	C	D	E
16.	A	B	C	D	E	36.	A	B	C	D	E	56.	A	B	C	D	E
17.	A	B	C	D	E	37.	A	B	C	D	E	57.	A	B	C	D	E
18.	A	B	C	D	E	38.	A	B	C	D	E	58.	A	B	C	D	E
19.	A	B	C	D	E	39.	A	B	C	D	E	59.	A	B	C	D	E
20.	A	B	C	D	E	40.	A	B	C	D	E	60.	A	B	C	D	E

Solutions to Set 6
Motion, Variation, Percent, Miscellaneous Problems

SOLUTION 6 - 1
Ans: B

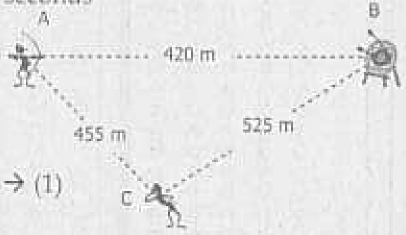
For uniform rate (speed), distance = rate × time

His total travel time = 11.75 - 6.25 = 5.5 hours. Since he stopped for one (1) hour, his total running time is 4.5 hours.

Distance = 30 × 4.5 = **135 miles**

SOLUTION 6 - 2
Ans: A

Let x = speed of the bullet in m/s
Let y = speed of sound in m/s
$t_{AB(\text{bullet})} + t_{BA(\text{sound})} = 2$ seconds



$$\frac{420}{x} + \frac{420}{y} = 2 \quad \rightarrow (1)$$

$$t_{AC(\text{sound of bullet})} + 1 = t_{AD(\text{bullet})} + t_{BC(\text{sound of target})}$$

$$\frac{455}{y} + 1 = \frac{420}{y} + \frac{525}{y}$$

$$\frac{420}{x} = 1 - \frac{70}{y} \quad \rightarrow (2)$$

Substitute $420/x$ from Eq. (2) to Eq. (1)

$$1 - \frac{70}{y} + \frac{420}{y} = 2$$

$$\frac{350}{y} = 1$$

$y = 350$ m/s
$x =$ **525 m/s**

SOLUTION 6 - 3
Ans: A

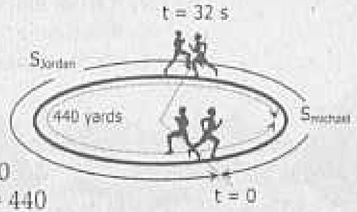
Let x be the distance from the barrio to the poblacion.
$t_{BP} + t_{PB} = 3 + 10/60 = 19/6$

$$\frac{x}{12} + \frac{x}{10} = \frac{19}{6}$$

$x =$ **17.27 km**

SOLUTION 6 - 4
Ans: B

Let x be the rate of Jordan in yards/sec.
Rate of Michael = 440/60 = 22/3 yards/sec.



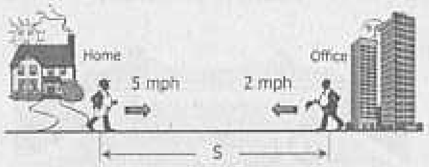
Running in opposite direction from the same point and meet in 32 seconds.

$S_{\text{Jordan}} + S_{\text{Michael}} = 440$
$x(32) + (22/3)(32) = 440$
$x = 77/12$ yards/sec.

Time for Jordan to run around the track
$t = 440 / (77/12) =$ **68.57 seconds**

SOLUTION 6 - 5
Ans: A

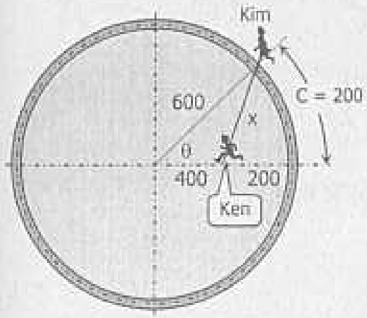
Let s be the distance from his home to his office:



$$v_{\text{ave}} = \frac{s_{\text{total}}}{t_{\text{total}}} = \frac{s+s}{\frac{s}{5}+\frac{s}{2}} \; ; \; v_{\text{ave}} = 2.86 \text{ mph}$$

SOLUTION 6 - 6
Ans: D

After 10 minutes, each of them had traveled 20(10) = 200 m.

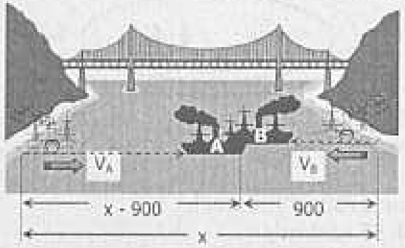


From the figure shown:
Solve for θ:
$C = R\theta_r$
$200 = 600\,\theta_r$
$\theta = 1/3$ radian × (180°/2π rad)
$\theta = 19.1°$

Solve for x by cosine law:
$x^2 = 400^2 + 600^2 - 2(400)(600)\cos(19.1°)$
$x =$ **258 m**

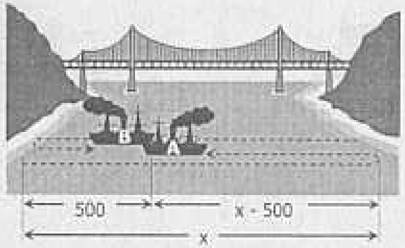
SOLUTION 6 - 7
Ans: D

Let x = width of the river in m
V_A & V_B = speed of ferryboats in m/s
t_1 = time elapsed when they first meet in seconds
t_2 = time elapsed when they meet for the second time
S_A & S_B = total distance traveled in m



Meeting for the first time

When they meet for the first time:
$S_A = V_A t_1 = x - 900$ → (1)
$S_B = V_B t_1 = 900$ → (2)



Meeting for the second time

When they meet for the second time:
$S_A' = V_A t_2 = x + (x - 500)$
$S_A' = V_A t_2 = 2x - 500$ → (3)
$S_B' = V_B t_2 = x + 500$ → (4)

Dividing Eq. (1) by Eq. (2):

$$\frac{V_A t_1}{V_B t_1} = \frac{x-900}{900} = \frac{V_A}{V_B} \quad \rightarrow (5)$$

Dividing Eq. (3) by Eq. (4):

$$\frac{V_A t_2}{V_B t_2} = \frac{2x-500}{x+500} = \frac{V_A}{V_B} \quad \rightarrow (6)$$

Compare (Equate) Eq. (5) & Eq. (6)

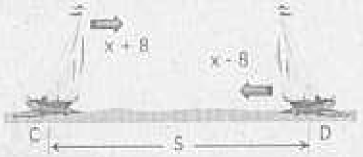
$$\frac{V_A}{V_B} = \frac{V_A}{V_B}$$
$$\frac{x-900}{900} = \frac{2x-500}{x+500}$$
$$x^2 - 400x - 450000 = 1800x - 450000$$
$$x^2 - 2200x = 0$$
$$x(x - 2200) = 0$$
$x = 0$ (absurd)
x = **2200 m**

SOLUTION 6 - 8
Ans: C

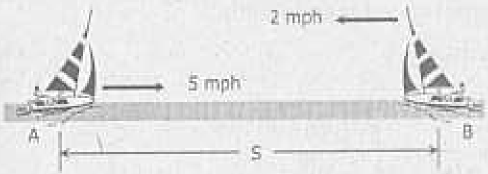
Let x be the speed of the boat (kph) in still water, then its rate downstream is $x + 8$ and its rate upstream is $x - 8$.

$[\,t_{CD} = (2/3)\,t_{DC}\,]$

$$\frac{S}{x+8} = \frac{2}{3} \times \frac{S}{x-8}$$
$$3x - 24 = 2x + 16$$
x = **40 kph**



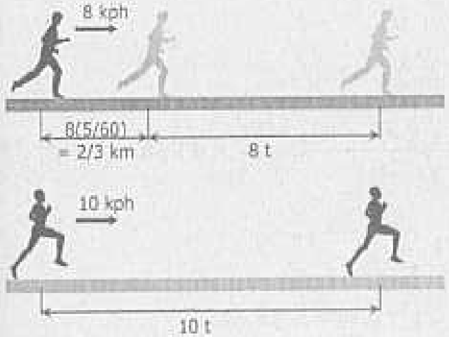
SOLUTION 6 - 9
Ans: D



$$t_{AB} + t_{BA} = 7/4$$
$$\frac{S}{5} + \frac{S}{2} = \frac{7}{4}$$
S = **2.5 miles**

SOLUTION 6 - 10
Ans: A

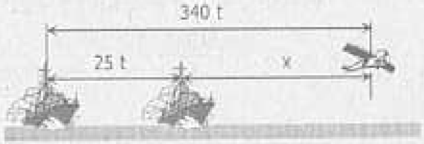
Let t be the time the second jogger will take to catch the first.



From the figure shown:
$2/3 + 8t = 10t$
$2t = 2/3$
$t = 1/3$ hour = **20 minutes**

SOLUTION 6 - 11
Ans: A

Let t be the time after 2:00 pm.
At 4:05 pm, t = 2 hrs & 5 min = $2 + 5/60$



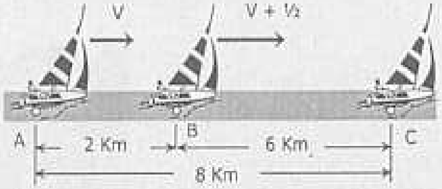
From the figure shown:
$x = 340\,t - 25\,t = 315\,t$
$x = 315(2 + 5/60)$ = **656.25 miles**

SOLUTION 6 - 12
Ans: B

Let V be the original speed in kph

$$t_{AC} = t_{AB} + t_{BC}$$

$$t_{AC} = \frac{2}{V} + \frac{6}{V+0.5} \text{ (hrs.)}$$



Had the original speed maintained:

$$t_{AC}' = \frac{8}{V} \text{ (hours)}$$

$$t_{AC} = t_{AC}' - 10 \text{ minutes}$$

$$\frac{2}{V} + \frac{6}{V+0.5} = \frac{8}{V} - \frac{10}{60}$$

$$\frac{6}{V} - \frac{6}{V+0.5} = \frac{1}{6}$$

$$\frac{V+0.5-V}{V(V+0.5)} = \frac{1}{36}$$

$$18 = V^2 + 0.5V$$

$$V^2 + 0.5V - 18 = 0$$

$$V = \frac{-0.5 \pm \sqrt{(0.5)^2 - 4(1)(-18)}}{2(1)} = 4 \text{ kph}$$

SOLUTION 6 - 13
Ans: D

$$w \propto \frac{xy}{z^2} \text{ or } w = k\frac{xy}{z^2}$$

$w = 4$ when $x = 2$, $y = 6$, and $z = 3$

$$4 = k\frac{2(6)}{3^2}; \quad k = 3$$

When $x = 1$, $y = 4$, and $z = 2$:

$$w = 3\frac{1(4)}{2^2} = 3$$

SOLUTION 6 - 14
Ans: C

$x \propto y/z$ or $x = k(y/z)$

$14 = k(7/2)$; $k = 4$

$x = 4(16/4) = 16$

SOLUTION 6 - 15
Ans: D

Resistance, $R \propto$ Length, L / diameter, D^2

$$R = kL / D^2$$

$0.1 = k(600)/(25)^2$, $k = 0.104167$
$1/6 = 0.104167L / (75)^2$
$L = 9000$ m

SOLUTION 6 - 16
Ans: B

Resistance, $R \propto L / D^2$

$$R = kL/D^2$$

$$\frac{R_1}{R_2} = \frac{k(100)/(5)^2}{k(50)/(3)^2} = 0.72$$

$R_1 = 0.72\,R_2$

SOLUTION 6 - 17
Ans: D

$$t \propto \frac{WS}{P} \text{ or } t = k\frac{WS}{P}$$

where: t = time, S = distance, W = weight,
k = constant of proportionality, P = power

Solve for k:

$$20 = k\frac{50(40)}{5}; \quad k = 0.05$$

$$30 = 0.05\frac{W(40)}{80}; \quad W = 1200 \text{ lbs.}$$

SOLUTION 6 - 18
Ans: A

$$\left[t \propto \frac{Wd}{P}\right] \quad t = k\frac{Wd}{P} \text{ or } P = k\frac{Wd}{t}$$

$$30 = k\frac{100(50)}{10}; \quad k = 0.06$$

$$\text{Power, } P = 0.06\frac{800(40)}{40}$$

Power = 48 hp

SOLUTION 6 - 19
Ans: C

Let x be the fixed salary and c be a constant factor.

Her salary can be expressed as:
Salary = x + sales × c

Then;
$900 = x + 10000c$
$x = 900 - 10000c$ → (1)
$1000 = x + 12000c$ → (2)

Substitute x in Eq. (1) to Eq. (2)
$1000 = (900 - 10000c) + 12000c$
$c = 0.05$ and $x =$ P400

Salary = 400 + 0.05 × sales
2,000 = 400 + 0.05 × sales

sales = **P32,000.00**

SOLUTION 6 - 20
Ans: C

Let x be the buying price of each egg.
His total capital is $100x$
Income for 80 eggs $= 80(x + 0.3x) = 104x$
Income for 20 eggs $= 20(x - 0.4x) = 12x$
Total sales $= 104x + 12x = 116x$
Profit = total sales - capital $= 116x - 100x = 16x$
Percent gain $= (16x/100x) \times 100\%$
Percent gain= **16%**

SOLUTION 6 - 21
Ans: D

Let x be the present population of the Philippines.
Population on the first year $= x + 0.05x = 1.05x$
Population on the second year $= 1.05x + 0.05(1.05x)$
$= 1.1025x$
Population on the third year $= 1.1025x + 0.05(1.1025x)$
$= \underline{1.157625x}$

Therefore, the population on the third year is increased by 0.1576 or **15.76%**

Another solution: (Geometric progression)
From $a_n = a_1 r^{n-1}$ $(r = 1.05)$
$P_3 = x\,(1.05)^3$
$P_3 = \underline{1.157625\,x}$

SOLUTION 6 - 22
Ans: C

Gain in the first car = 10%(600,000) = P60,000
Loss in the second car = 12%(400,000) = P48,000
Total gain = P60,000 - P48,000 = P12,000
Percent gain = P12,000 / (600,000 + 400,000) × 100%
Percent gain = **1.2%**

SOLUTION 6 - 23
Ans: B

Let x be the original price of his goods.
Raised price $= x + 10\%x = 1.1x$
Christmas sale discounted price $= 1.1x - 10\%(1.1x)$
Christmas sale discounted $= 0.99x = 99\%x$

Therefore, the customers actually got **1% discount.**

SOLUTION 6 - 24
Ans: C

Let x be the price of the watch.

Then, $x - 30\%x = 3{,}500$
$x =$ P5,000

By selling it at P5,050, Kim gains P50.00
Percent gain = (50 / 5,000) × 100% = **1%**

SOLUTION 6 - 25
Ans: D

Let x be the original price of an egg.
By selling the balut at P5.00 he gains 20% (of x), then
$0.2x = 5 - x$
$x =$ P4.1667 (original price of egg)
Increased price of egg $= x + 12.5\%x = 4.6875$
By selling it at the same price of P5.00, his gain
is 5 - 4.6875 = P0.3125

Percent gain = (0.3125/5) × 100% = **6.25%**

SOLUTION 6 - 26
Ans: D

Let x = enrollment in college A in 1980
y = enrollment in college B in 1980

Growth in enrollment in college A in 1985 $= 0.08x = 800$
$x = 10000$

Total enrollment in college A in 1985 = 10000 + 800
Total enrollment in college A in 1985 = 10800

Growth in enrollment in college B in 1985 $= 0.08y = 840$
$y = 10500$

Total enrollment in college A in 1985 = 10500 + 840
Total enrollment in college A in = 11340

Therefore, college B is greater than A by
11340 - 10800 = **540**

SOLUTION 6 - 27
Ans: A

Number of boys = n Total Number = $2n$
Number of girls = n

If two boys were replaced with two girls, then the number of boys is $n - 2$

Percent boys:
$(n - 2) / 2n = 49\% = 0.49$
$n - 2 = 0.98\,n$
$n =$ **100**

SOLUTION 6 - 28
Ans: C

Sale price = P 180.00
Sale price = Retail price - 20% of Retail price
108.00 = 0.80 × Retail Price
Retail Price = P 225

Retail price = Wholesale price + 50% × Wholesale price
225 = 1.5 × Wholesale price

Wholesale price = **P 150.00**

SOLUTION 6 - 29
Ans: D

No. of copies per hour = 13 copies/sec × (3600 sec/hr)
No. of copies per hour = **46800**

SOLUTION 6 - 30
Ans: C

Let x = number of machines
Time for one machine to run 18000 newspapers:

$$\text{Time} = 18000 \text{ newspapers} \times \frac{1 \text{second}}{6 \text{newspaper}} \times \frac{1 \text{ minute}}{60 \text{seconds}}$$

Time = 50 minutes

For x machines, the number of minutes is **$50/x$**

SOLUTION 6 - 31
Ans: A

Demand = 35 units/week
No. of working hours per week = 7(5) = 35 hours/week

$$\text{Required production time per unit} = \frac{35 \text{ hours/week}}{35 \text{ units/week}}$$

Required production time per unit = **1 hour/unit**

SOLUTION 6 - 32
Ans: C

Let x, y, and z be the number of men, women, and children, respectively.

$x + y + z = 316$ → (1)
$z = y + 78$ → (2)
$y = x + 56$ → (3)

Substitute y is Eq. (3) to Eq. (2):
$z = x + 56 + 78 = 134 + x$

Substitute y and z to Eq. (1)
$x + (x + 56) + (134 + x) = 316$; $x = \mathbf{42}$

SOLUTION 6 - 33
Ans: A

Initial inventory = Q
Inventory at any time t: $I = Q - (Q/A)t + C$
(where C = constant)

When t = A, $I = 0$
$0 = Q - (Q/A)A + C$
$C = 0$

Therefore; $I = Q - \mathbf{(Q/A)}t$

SOLUTION 6 - 34
Ans: C

Let x = number of radios sold
y = number of clocks sold
z = number of flashlights sold

Then;
$x + y + z = 100$; $z = 100 - x - y$ → (1)
$50x + 30y + 1z = 1000$ → (2)

Substitute z in Eq. (1) to Eq. (2):
$50x + 30y + 1(100 - x - y) = 1000$
$49x + 29y = 900$

Solve for x and y be series of trials using the four choices

x	80	4	16	20
y	-104.1	24.7	4	-2.75

From the table above, the only possible answer is x = **16**, which yields a value of y of 4 which is a whole number.

SOLUTION 6 - 35
Ans: C

The lowest price of any one of the calculators occurs when the rest (7) costs P1000.

Let x be the lowest priced calculator, then;

$$\text{Ave} = \frac{1000(7) + x}{8} = 950; \ x = \mathbf{P600.00}$$

SOLUTION 6 - 36
Ans: C

In a deck of 52 playing cards, 26 are black and 26 are red.
Let x = number of black cards in the first pile
y = number of red cards in the first pile
$x = 7y$

For the second pile:
Black cards = $26 - x = 26 - 7y$
Red cards = $26 - y$

Since the number of red cards is an exact multiple of the number of black, the ratio of Red to Black must be a whole number,

$$\frac{26 - y}{26 - 7y} = \text{whole number}$$

By trial and error, $y = 2$ (only this value will give a whole number)
$x = 7(2) = 14$

Therefore, there are 14 + 2 = **16** cards in the first pile.

SOLUTION 6 - 37
Ans: B

The general equation is $P = P_0 + r\,t$, where P is the population at any time t, P_0 is the initial population and r is the rate of increase.

For 30 years (1967 to 1997), $P = 2P_0$

$$P = P_0 + r(30) = 2P_0$$
$$P_0 = 30\,r$$
$$r = P_0/30$$

Then,

$$P = P_0 + (P_0/30)t = 3\,P_0$$
$$P_0\,t/30 = 3P_0 - P_0$$
$$t = 60 \text{ years}$$

Thus, the population is triple in the year 1997 + 60 = **2027**

SOLUTION 6 - 38
Ans: C

$3^1 = 3$
$3^2 = 9$
$3^3 = 27$
$3^4 = 81$
$3^5 = 243$
$3^6 = 729$
$3^7 = 2187$
$3^8 = 6561$

Remainder	Unit's Digit
1	3
2	9
3	7
none	1

Note: the unit's digit is either 3, 9, 7, or 1 and is repeated in every exponent interval of 4.

$$\frac{855}{4} = 213.75 \text{ or } 213 \text{ remainder } 3$$

Therefore; the units digit is 7.

SOLUTION 6 - 39
Ans: C

1785/9999 = 0.178517851785…

Notice that the digits (1, 7, 8, an 5) are repeated every interval of four (4). To get the n^{th} digit, we divide n by 4 and get the remainder. If the remainder is 1, the answer is 1, is 2 the answer is 7, if 3 the answer is 8 and if there is no remainder the answer is 5.

1987/4 = 496 remainder 3.
The 1987th digit is therefore 8

SOLUTION 6 - 40
Ans: A

The factors are: 1×2048; 2×1024; 4×512; 8×256; 16×128; 32×64

Sum = 1 + 2048 + 2 + 1024 + 4 + 512 + 8 + 256 + 16 + 128 + 32 + 64
Sum = **4095**

SOLUTION 6 - 41
Ans: C

The choices are as follows:

6 - 1 = 5
7 - 2 = 5
8 - 3 = 5
:
:
50 - 45 = 5

Therefore; there are **45 ways**

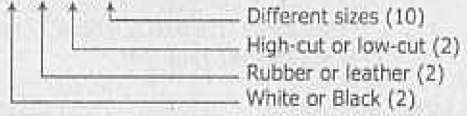
SOLUTION 6 - 42
Ans: D

There are 5 different colors in the box. The worst case to happen is when in each draw, different colors will be picked. So in 5 draws there will be one of each color, in 10 draws there will be two of each color. Therefore; the 11th draw will ensure that there will be three balls of the same color (either white, green, black, red, or yellow).

SOLUTION 6 - 43
Ans: B

The total number of kinds of shoes sold is:

$N = (2)(2)(2)(10) = 80$



SOLUTION 6 - 44
Ans: C

Let the original length and width be x and y, respectively.

The area of the field is xy.

$(x + 100)(y - 25) = xy + 2500$
$xy - 25x + 100y - 2500 = xy + 2500$
$x = 4y - 200 \quad \rightarrow (1)$

$(x - 100)(y + 50) = xy - 5000$
$xy + 50x - 100y - 5000 = xy - 5000$
$x = 2y \quad \rightarrow (2)$

Compare x in Eq. (1) with Eq. (2):

$4y - 200 = 2y$
$y = 100$ and $x = 200$

Area of the field, xy = **20,000 ft²**

SOLUTION 6 - 45
Ans: A

The correct length is 10 - 5/1000 = **9.995 m**

SOLUTION 6 - 46
Ans: D

$TD = MD + \text{Error} = MD + eN$ (where $N = MD / L_T$)

Length of tape used, $L_T = 100$ feet
Error per tape length, $e = -0.1$ (too short)
Measured distance, $MD = 916.58$ ft.

$$TD = 916.58 + (-0.1)\frac{916.58}{100} = \mathbf{915.66\ ft.}$$

SOLUTION 6 - 47
Ans: C

$e = KL_o(T - T_o) = 5.833 \times 10^{-6}\,(100)(10 - 70)$
$e = -0.035$ ft. or 0.035 ft. too short

$TD = MD + \text{Error} = MD + e\,N$ (where $N = MD / L_T$)

$$90 = MD + (-0.035)\frac{MD}{100}$$

$MD = \mathbf{90.031\ ft.}$

SOLUTION 6 - 48
Ans: C

$e = K\,L_o(T - T_o) = 0.000011(100.042)(30 - 10)$
$e = 0.022$ ft. (too long)
$TD = MD + eN = MD + e(MD/L_T)$
$TD = 1256.271 + (0.022)(1256.271/100)$
$TD = \mathbf{1256.547\ feet}$

SOLUTION 6 - 49
Ans: B

$$\text{Standard Deviation} = \sqrt{\frac{1}{n-1}\sum (v - v_{mean})^2}$$

$$v_{mean} = \frac{1+4+7}{3} = 4$$

$$\text{Standard Deviation} = \sqrt{\frac{1}{3-1}\left[(1-4)^2 + (4-4)^2 + (7-4)^2\right]}$$

Standard Deviation = **3**

SOLUTION 6 - 50
Ans: C

Let x be the shortest road, then the longest road is $x + 10$. Then,

$x + (x + 10) + 10 = 30$
$2x = 10$
$x = 5$ km

The longest road is $x + 10 = 15$ km

Problems - Set 7
Progression, Matrix, Determinant, Venn Diagram

PROBLEM 7 - 1
ECE Nov. 1996

How many terms of the sequence -9, -6, -3, ... must be taken so that the sum is 66?

A. 13
B. 12
C. 4
D. 11

PROBLEM 7 - 2
CE Nov. 1997

The sum of the progression 5, 8, 11, 14 ... is 1025. How many terms are there?

A. 22
B. 23
C. 24
D. 25

PROBLEM 7 - 3
CE May 1998

There are seven arithmetic means between 3 and 35. Find the sum of all the terms.

A. 169
B. 171
C. 167
D. 173

PROBLEM 7 - 4
CE May 1999

There are nine (9) arithmetic means between 11 and 51. The sum of the progression is:

A. 279
B. 341
C. 376
D. 254

PROBLEM 7 - 5

The sum of all even numbers from 0 to 420 is:

A. 43410
B. 44300
C. 44310
D. 44130

PROBLEM 7 - 6
CE May 1997

Which of the following numbers should be changed to make all the numbers form an arithmetic progression when properly arranged?

A. 27/14
B. 33/28
C. 45/28
D. 20/14

PROBLEM 7 - 7

The first term of an arithmetic progression (A.P.) is 6 and the 10th term is 3 times the second term. What is the common difference?

A. 1
B. 2
C. 3
D. 4

PROBLEM 7 - 8

The sum of five arithmetic means between 34 and 42 is:

A. 150
B. 160
C. 190
D. 210

PROBLEM 7 - 9

The positive value of a so that $4x$, $5x + 4$, $3x^2 - 1$ will be in arithmetic progression is:
A. 2
C. 4
B. 3
D. 5

PROBLEM 7 - 10

Solve for x if $x + 3x + 5x + 7x + \dots + 49x = 625$.
A. ¼
C. 1
B. ½
D. 1¼

PROBLEM 7 - 11

The 10th term of the series a, $a - b$, $a - 2b$, ... is:
A. $a - 6b$
C. $2a - b$
B. $a - 9b$
D. $a + 9b$

PROBLEM 7 - 12

If the sum of the first 13 terms of two arithmetic progressions are in the ratio 7:3, find the ratio of their corresponding 7th term.
A. 3:7
C. 7:3
B. 1:3
D. 6:7

PROBLEM 7 - 13

If $1/x$, $1/y$, $1/z$ are in arithmetic progression, then y is equal to:
A. $x - z$
C. $(x + z) / 2xz$
B. ½ $(x + 2z)$
D. $2xz / (x + z)$

PROBLEM 7 - 14
ECE Nov. 1997

Find the 30th term of the A.P. 4, 7, 10 ...
A. 88
C. 75
B. 91
D. 90

PROBLEM 7 - 15
ECE Nov. 1997

Find the 100th term of the sequence 1.01, 1.00, 0.99...
A. 0.05
C. 0.03
B. 0.04
D. 0.02

PROBLEM 7 - 16

The sum of all numbers between 0 and 10,000 which is exactly divisible by 77 is:
A. 546,546
C. 645,645
B. 645,568
D. 645,722

PROBLEM 7 - 17
ME April 1998

What is the sum of the following finite sequence of terms? 18, 25, 32, 39, ... , 67.
A. 234
C. 213
B. 181
D. 340

PROBLEM 7 - 18

Find x in the series: 1, 1/3, 0.2, x.
A. 1/6
C. 1/7
B. 1/8
D. 1/9

PROBLEM 7 - 19
ECE Nov. 1995

Find the fourth term of the progression 1/2, 0.2, 0.125,...
A. 0.102
C. 1/11
B. 1/10
D. 0.099

PROBLEM 7 - 20

The 10th term of the progression 6/5, 4/3, 3/2, ... is:
A. 12
C. 12/3
B. 10/3
D. 13/3

PROBLEM 7 - 21
ME Oct. 1997

The geometric mean of 4 and 64 is:
A. 48
C. 34
B. 16
D. 24

PROBLEM 7 - 22
ME Oct. 1997

The geometric mean of a and b is:
A. $\sqrt{ab}$
C. $1/b$
B. $(a + b)/2$
D. $ab/2$

PROBLEM 7 - 23
CE May 1998

Determine the sum of the infinite geometric series of 1, -1/5, +1/25, ...?
A. 4/5
C. 4/6
B. 5/7
D. 5/6

PROBLEM 7 - 24

There are 6 geometric means between 4 and 8748. Find the sum of all the terms.
A. 13120
C. 10250
B. 15480
D. 9840

PROBLEM 7 - 25
ECE April 1998

Find the sum of the infinite geometric progression 6, -2, 2/3 . . .
A. 5/2
C. 7/2
B. 9/2
D. 11/2

PROBLEM 7 - 26
ECE April 1998

Find the sum of the first 10 terms of the Geometric Progression 2, 4, 8, 16, ...
A. 1023
C. 1596
B. 2046
D. 225

PROBLEM 7 - 27

The 1st, 4th, and 8th terms of an A.P. are themselves geometric progression (G.P.). What is the common ratio of the G.P.
A. 4/3
C. 2
B. 5/3
D. 7/3

PROBLEM 7 - 28

Determine x so that x, $2x + 7$, $10x - 7$ will form a geometric progression.

A. -7   C. 7
B. 6   D. -6

PROBLEM 7 - 29

The fourth term of a geometric progression is 189 and the sixth term is 1701, the 8th term is:

A. 5103   C. 45927
B. 1240029   D. 15309

PROBLEM 7 - 30

The sum of three numbers in arithmetical progression is 45. If 2 is added to the first number, 3 to the second, and 7 to the third, the new numbers will be in geometrical progression. Find the common difference in A.P..

A. -5   C. 6
B. 10   D. 5

PROBLEM 7 - 31

The geometric mean and the harmonic mean of two numbers are 12 and 36/5 respectively. What are the numbers?

A. 36 & 4   C. 36 & 8
B. 72 & 8   D. 72 & 4

PROBLEM 7 - 32

If x, $4x + 8$, $30x + 24$ are in geometric progression, find the common ratio.

A. 2   C. 6
B. 4   D. 8

PROBLEM 7 - 33
ECE April 1995

A besiege fortress is held by 5700 men who have provisions for 66 days. If the garrison loses 20 men each day, for how many days can the provision hold out?

A. 60   C. 76
B. 72   D. 82

PROBLEM 7 - 34
ECE April 1999

If one third of the air in the tank is removed by each stroke of an air pump, what fractional part of the total air is removed in 6 strokes?

A. 0.9122   C. 0.8211
B. 0.0877   D. 0.7145

PROBLEM 7 - 35

A rubber ball is dropped from a height of 15 m. On each rebound, it rises 2/3 of the height from which it last fell. Find the distance traveled by the ball before it comes to rest.

A. 75 m   C. 100 m
B. 96 m   D. 85 m

PROBLEM 7 - 36
CE May 1991

In the recent Bosnia conflict, the NATO forces captured 6400 soldiers. The provisions on hand will last for 216 meals while feeding 3 meals a day. The provisions lasted 9 more days because of daily deaths. At an average, how many died per day?

A. 15.2   C. 18.3
B. 17.8   D. 19.4

PROBLEM 7 - 37

To build a dam, 60 men must work 72 days. If all 60 men are employed at the start but the number is decreased by 5 men at the end of each 12-day period, how long will it take to complete the dam?

A. 108 days   C. 94 days
B. 9 days   D. 60 days

PROBLEM 7 - 38
CE Nov. 1994

In a benefit show, a number of wealthy men agreed that the first one to arrive would pay 10 centavos to enter and each later arrival would pay twice as much as the preceding man. The total amount collected from all of them was P104,857.50. How many wealthy men had paid?

A. 18   C. 20
B. 19   D. 21

PROBLEM 7 - 39

Evaluate the following determinant: $\begin{vmatrix} 7 & 8 \\ 9 & 4 \end{vmatrix}$

A. 64   C. 54
B. 44   D. -44

PROBLEM 7 - 40

The following equation involves two determinants:

$$\begin{vmatrix} 3 & x \\ 2 & 2 \end{vmatrix} = \begin{vmatrix} 2 & -1 \\ x & -3 \end{vmatrix}$$

The value of x is:

A. 1   C. 4
B. 3   D. 3

PROBLEM 7 - 41
CE Nov. 1997

Evaluate the following determinant:

$$\begin{vmatrix} 1 & 5 & -2 \\ 2 & 1 & -3 \\ 3 & -2 & 1 \end{vmatrix}$$

A. -24   C. -46
B. 24   D. 46

PROBLEM 7 - 42
CE Nov. 1996

Compute the value of x from the following:

$$x = \begin{vmatrix} 4 & -1 & 2 & 3 \\ 2 & 0 & 2 & 1 \\ 10 & 3 & 0 & 1 \\ 14 & 2 & 4 & 5 \end{vmatrix}$$

A. 27 C. 26
B. -28 D. -29

PROBLEM 7 - 43

Evaluate the following determinant:

$$D = \begin{vmatrix} 1 & 4 & 2 & -1 \\ 2 & -1 & 0 & -3 \\ -2 & 3 & 1 & 2 \\ 0 & 2 & 1 & 4 \end{vmatrix}$$

A. 5 C. 4
B. -4 D. -5

PROBLEM 7 - 44

Given: Matrix $A = \begin{bmatrix} 1 & 3 \\ -2 & 1 \end{bmatrix}$

Matrix $B = \begin{bmatrix} -1 & -2 \\ -1 & 1 \end{bmatrix}$

Find $A + 2B$.

A. $\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$ C. $\begin{bmatrix} -1 & 3 \\ 0 & 1 \end{bmatrix}$

B. $\begin{bmatrix} 1 & 0 \\ 1 & 3 \end{bmatrix}$ D. $\begin{bmatrix} -1 & -1 \\ 0 & 3 \end{bmatrix}$

PROBLEM 7 - 45
CE May 1996

Elements of matrix $B = \begin{bmatrix} 1 & 2 \\ 0 & -5 \end{bmatrix}$

Elements of matrix $C = \begin{bmatrix} 3 & 6 \\ 4 & 1 \end{bmatrix}$

Find the elements of the product of the two matrices, matrix BC.

A. $\begin{bmatrix} 11 & 8 \\ -20 & -5 \end{bmatrix}$ C. $\begin{bmatrix} 12 & 10 \\ 20 & -4 \end{bmatrix}$

B. $\begin{bmatrix} 15 & 9 \\ -22 & 4 \end{bmatrix}$ D. $\begin{bmatrix} 15 & 15 \\ -17 & -6 \end{bmatrix}$

PROBLEM 7 - 46
CE Board

Solve for x and y from the given relationship:

$$\begin{bmatrix} 1 & 1 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

A. $x = -2; y = 6$ C. $x = -2; y = -6$
B. $x = 2; y = 6$ D. $x = 2; y = -6$

PROBLEM 7 - 47
EE Oct. 1993

In a class of 40 students, 27 students like Calculus and 25 like Geometry. How many students liked both Calculus and Geometry?

A. 10 C. 11
B. 14 D. 12

PROBLEM 7 - 48

A class of 40 took examination in Algebra and Trigonometry. If 30 passed Algebra, 36 passed Trigonometry, and 2 failed in both subjects, the number of students who passed the two subjects is:

A. 2 C. 28
B. 8 D. 25

PROBLEM 7 - 49
ECE Nov. 1992

The probability for the ECE board examinees from a certain school to pass the Mathematics subject is 3/7 and that for the Communications subject is 5/7. If none of the examinees failed in both subjects and there are 4 examinees who pass both subjects, how many examinees from the school took the examination?

A. 28 C. 26
B. 27 D. 32

PROBLEM 7 - 50
EE March 1998

In a commercial survey involving 1000 persons on brand preferences, 120 were found to prefer brand x only, 200 persons prefer brand y only, 150 persons prefer brand z only, 370 prefer either brand x or y but not z, 450 prefer brand y or z but not x, and 370 prefer either brand z or x but not y, and none prefer all the three brands at a time.. How many persons have no brand preference with any of the three brands?

A. 120 C. 70
B. 280 D. 320

ANSWER SHEET

1. A B C D E
2. A B C D E
3. A B C D E
4. A B C D E
5. A B C D E
6. A B C D E
7. A B C D E
8. A B C D E
9. A B C D E
10. A B C D E
11. A B C D E
12. A B C D E
13. A B C D E
14. A B C D E
15. A B C D E
16. A B C D E
17. A B C D E
18. A B C D E
19. A B C D E
20. A B C D E
21. A B C D E
22. A B C D E
23. A B C D E
24. A B C D E
25. A B C D E
26. A B C D E
27. A B C D E
28. A B C D E
29. A B C D E
30. A B C D E
31. A B C D E
32. A B C D E
33. A B C D E
34. A B C D E
35. A B C D E
36. A B C D E
37. A B C D E
38. A B C D E
39. A B C D E
40. A B C D E
41. A B C D E
42. A B C D E
43. A B C D E
44. A B C D E
45. A B C D E
46. A B C D E
47. A B C D E
48. A B C D E
49. A B C D E
50. A B C D E
51. A B C D E
52. A B C D E
53. A B C D E
54. A B C D E
55. A B C D E
56. A B C D E
57. A B C D E
58. A B C D E
59. A B C D E
60. A B C D E

Solutions to Set 7 Progression, Matrix, Determinant, Venn Diagram

SOLUTION 7 - 1
Ans: D

The sequence -9, -6, -3, ... forms an arithmetic progression with a common difference of -6 - (-9) = 3.

From the formula: $S = \frac{n}{2}[\,2a_1 + (n-1)d\,]$

$$66 = \frac{n}{2}[\,2(-9) + (n-1)(3)]$$
$$132 = -18n + 3n^2 - 3n$$
$$0 = 3n^2 - 21n - 132$$
$$0 = (3n + 12)(n - 11);\quad \mathbf{n = 11}$$

SOLUTION 7 - 2
Ans: D

The given sequence is an arithmetic progression with the following elements:

$a_1 = 5;\quad d = 8 - 5 = 3;\qquad S = 1025$

Using the formula: $S = \frac{n}{2}[\,2\,a_1 + (n-1)d\,]$

$$1025 = \frac{n}{2}[\,2(5) + (n-1)(3)\,]$$
$$2050 = n\,(10 + 3n - 3)$$
$$0 = 3n^2 + 7n - 2050;\quad \mathbf{n = 25}$$

SOLUTION 7 - 3
Ans: B

The sequence is 3, _, _, _, _, _, _, _, 35. Thus, there are 9 terms with the first term 3 and the ninth term 35.

$$\text{Sum} = \frac{n}{2}(a_1 + a_n) = \frac{9}{2}(3 + 35) = \mathbf{171}$$

SOLUTION 7 - 4
Ans: B

$$\text{Sum of A.P.} = \frac{n}{2}(a_1 + a_n)$$
$$n = 11;\quad a_1 = 11;\quad a_n = a_{11} = 51$$
$$\text{Sum} = \tfrac{11}{2}(11 + 51) = \mathbf{341}$$

SOLUTION 7 - 5
Ans: C

The even numbers from 0 to 420 are 2, 4, 6, ... 420. (A.P.)
There are 420/2 = 210 even numbers.
The sum of all even numbers is:

$$S = \frac{n}{2}(a_1 + a_n) = \tfrac{210}{2}(2 + 420) = \mathbf{44{,}310}$$

SOLUTION 7 - 6
Ans: C

The terms $\frac{27}{14}$, $\frac{33}{28}$, $\frac{45}{28}$, and $\frac{20}{14}$ may also be written as $\frac{54}{28}$, $\frac{33}{28}$, $\frac{45}{28}$, $\frac{40}{28}$, and in ascending order is $\frac{33}{28}$, $\frac{40}{28}$, $\frac{45}{28}$, $\frac{54}{28}$

From the sequence shown, the most possible common difference is 7/28. With this, the sequence is $\frac{33}{28}$, $\frac{40}{28}$, $\frac{47}{28}$, $\frac{54}{28}$. Therefore choice C must be changed.

SOLUTION 7 - 7
Ans: B

Let d = common difference
First term = $a_1 = 6$ and $a_{10} = 3a_2$
From $a_n = a_1 + (n - 1)d$
$a_{10} = 6 + (10 - 1)d = 6 + 9d$
$a_2 = 6 + (2 - 1)\mathrm{d} = 6 + d$
$a_{10} = 3\,a_2$
$6 + 9d = 3(6 + d) = 18 + 3d;\ d = \mathbf{2}$

SOLUTION 7 - 8
Ans: C

$$\underset{1^{st}}{34}\ \ \underset{2^{nd}}{\_\_}\ \ \underset{3^{rd}}{\_\_}\ \ \underset{4^{th}}{\_\_}\ \ \underset{5^{th}}{\_\_}\ \ \underset{6^{th}}{\_\_}\ \ \underset{7^{th}}{42} \qquad a_1 = 34,\ a_7 = 42$$

Sum of all terms = $\frac{7}{2}$ (34 + 42) = 266

Sum of the five arithmetic means = 266 – (34 + 42)
Sum of the five arithmetic means = **190**

SOLUTION 7 - 9
Ans: B

If $4x$, $5x + 4$, and $3x^2 - 1$ are in A.P., then
$(5x + 4) - 4x = (3x^2 - 1) - (5x + 4)$
$3x^2 - 6x - 9 = 0 = (x + 1)(x - 3)$
$x = 3$ and $x = -1$

SOLUTION 7 - 10
Ans: C

The given sequence x, $3x$, $5x$, … is an A.P. with $d = 2x$
Solve for n from $a_n = a_1 + (n - 1)d$
$49x = x + (n - 1)(2x)$
$49 = 2n - 1;\ n = 25$

From $S = \frac{n}{2}(a_1 + a_n)$

$625 = \frac{25}{2}(x + 49x);\ x = \mathbf{1}$

SOLUTION 7 - 11
Ans: B

The series a, $a - b$, $a - 2b$, … is an A.P. with a common difference $d = -b$.
The 10th term is:
$a_{10} = a_1 + (10 - 1)d$
$a_{10} = a + 9(-b) = \mathbf{a - 9b}$

SOLUTION 7 - 12
Ans: C

Sum of A.P. = $\frac{n}{2}\,[2a_1 + (n - 1)d]$

Let S, A_1, D and s, a_1, d be respectively the sum, first term, and common difference of the arithmetic progression, then;

$$\frac{S}{s} = \frac{(13/2)[2A_1 + (13-1)D]}{(13/2)[2a_1 + (13-1)d]} = \frac{7}{3}$$
$$\frac{2A_1 + 12D}{2a_1 + 12d} = \frac{7}{3};\quad \frac{A_1 + 6D}{a_1 + 6d} = \frac{7}{3}$$

n^{th} term of A.P. = $a_1 + (n - 1)d$
$$\frac{A_7}{a_7} = \frac{A_1 + (7-1)D}{a_1 + (7-1)d} = \frac{A_1 + 6D}{a_1 + 6d}$$
$$\text{but } \frac{A_1 + 6D}{a_1 + 6d} = \frac{7}{3}$$
$$\text{Thus, } \frac{A_7}{a_7} = \frac{7}{3}$$

SOLUTION 7 - 13
Ans: D

If $1/x$, $1/y$, and $1/z$ are in A.P., then
$$\frac{1}{y} - \frac{1}{x} = \frac{1}{z} - \frac{1}{y};\qquad \frac{1}{y} = \frac{1}{x} + \frac{1}{z} = \frac{x+z}{xz}$$
$\mathbf{y = 2xz/(x + z)}$

SOLUTION 7 - 14
Ans: B

The given arithmetic progression (4, 7, 10, …) has a common difference of 7 - 4 = 10 - 7 = 3, with a first term a_1 of 4.
$a_n = a_1 + (n - 1)d$
$a_{30} = 4 + (30 - 1)(3) = \mathbf{91}$

SOLUTION 7 - 15
Ans: D

The sequence 1.01, 1.00, 0.99, … is an A.P. with a common difference of -0.01
The 100th term is:
$a_{100} = a_1 + (100 - 1)\,d = 1.01 + 99(-0.01) = \mathbf{0.02}$

SOLUTION 7 - 16
Ans: C

The number of integers between 0 to 10,000 which is divisible by 77 is n = 10000/77 = 129.9 or 129
The sum is, $S = 77 + 154 + 231 + \ldots + 9933$ (sum of A.P.)
$S = \frac{n}{2}(a_1 + a_n)$
$S = \frac{129}{2}(77 + 9933) = \mathbf{645{,}645}$

SOLUTION 7 - 17
Ans: D

The sequence 18, 25, 32, 39, ... , 67 is an arithmetic progression with first term $a_1 = 18$, common difference $d = 7$, and last term $a_n = 67$.

Solving for n from $a_n = a_1 + (n-1)d$

$$67 = 18 + (n-1)7; \quad n = 8$$

$$\text{Sum} = \frac{n}{2}(a_1 + a_n) = \frac{8}{2}(18 + 67) = 340$$

SOLUTION 7 - 18
Ans: C

The given sequence 1, 1/3, 0.2, x can be written as 1/1, 1/3, 1/5, x which is a harmonic progression, the reciprocal of which has a common difference of 2. by inspection, $x = 1/7$.

SOLUTION 7 - 19
Ans: C

1/2, 0.2, 0.125, ... = 1/2, 1/5, 1/8, ...
The sequence is a harmonic progression, the reciprocal of which has a common difference of 3. By inspection, the fourth term is **1/11**.

SOLUTION 7 - 20
Ans: A

The sequence 6/5, 4/3, 3/2, ... is a harmonic progression.
In A.P. the sequence is: 5/6, ¾, 2/3, ... with $a_1 = 5/6$, and $d = 3/4 - 5/6 = -1/12$

The 10th term is:

$$a_n = a_1 + (n-1)d$$
$$a_{10} = 5/6 + (10-1)(-1/12) = 1/12$$

In H.P., the 10th term is **12**

SOLUTION 7 - 21
Ans: B

The geometric mean x of 4 and 64 is such that $4 : x : 64$.

$$\frac{4}{x} = \frac{x}{64}; \quad x^2 = 256; \quad x = 16$$

SOLUTION 7 - 22
Ans: A

The geometric mean "x" of a and b is such that $a : x : b$ or

$$\frac{a}{x} = \frac{x}{b}$$
$$x^2 = ab; \quad x = \sqrt{ab}$$

SOLUTION 7 - 23
Ans: D

The sum of infinite G.P. is given by the formula

$$S = \frac{a_1}{1-r}$$

where r = common ratio = $a_2 / a_1 = (-1/5) / 1$
$r = -1/5$

$$S = \frac{1}{1-(-1/5)} = 5/6$$

SOLUTION 7 - 24
Ans: A

First term, $a_1 = 4$
8th term, $a_8 = 8748$
$a_n = a_1 r^{n-1}$
$8748 = 4 r^{8-1}$
$r = 3$

$$\text{Sum, } S = \frac{a_1(r^n - 1)}{r-1} = \frac{4(3^8 - 1)}{3-1} = 13{,}120$$

SOLUTION 7 - 25
Ans: B

$$\text{Sum of infinite G.P.} = \frac{a_1}{1-r}$$

$a_1 = 6; \quad r = -2/6 = -1/3.$

$$\text{Sum} = \frac{6}{1-(-1/3)} = 9/2$$

SOLUTION 7 - 26
Ans: B

$$\text{The sum of G.P. is: } S = \frac{a_1(r^n - 1)}{r-1}$$

$a_1 = 2$, $r = 2$, and $n = 10$.

$$S = \frac{2(2^{10} - 1)}{2-1} = 2046$$

SOLUTION 7 - 27
Ans: A

In A.P.
Let the first term $a_1 = x$ and d = common difference
then $a_4 = x + 3d$, and $a_8 = x + 7d$.

Since a_1, a_4, and a_8 are in G.P. then $\frac{a_4}{a_1} = \frac{a_8}{a_4}$

$$\frac{x+3d}{x} = \frac{x+7d}{x+3d}$$
$$x^2 + 6xd + 9d^2 = x^2 + 7xd$$
$$9d^2 = xd$$
$$x = 9d$$

$$\text{Common ratio of G.P.,} \quad r = \frac{a_4}{a_1} = \frac{x+3d}{x}$$
$$r = \frac{9d+3d}{9d} = \frac{4}{3}$$

SOLUTION 7 - 28
Ans: C

If x, $2x + 7$, and $10x - 7$ are in G.P., then

$$\frac{2x+7}{x} = \frac{10x-7}{2x+7}$$
$$4x^2 + 28x + 49 = 10x^2 - 7x$$
$$6x^2 - 35x - 49 = 0$$
$$x = 7 \text{ and } -7/6$$

SOLUTION 7 - 29
Ans: D

Given: $a_4 = 189$ and $a_6 = 1701$.

$$a_n = a_m\, r^{n-m}$$
$$1701 = 189\, r^{(6-4)}$$
$$r = 3$$

The 8th term is, $a_8 = 189\,(3)^{8-4} = \mathbf{15309}$

SOLUTION 7 - 30
Ans: D

Let x be the first term in A.P. and d be the common difference.
In A.P.:

$$a_1 = x,\ a_2 = x + d,\ a_3 = x + 2d$$
$$\text{Sum} = \frac{n}{2}\left[2a_1 + (n-1)d\right];\quad 45 = \frac{3}{2}\left[2x + (3-1)d\right]$$
$$30 = 2x + 2d;\ x + d = 15;\quad x = 15 - d$$

In G.P.: $(x = 15 - d)$

$$a_1 = x + 2 = 15 - d + 2 = 17 - d$$
$$a_2 = x + d + 3 = (15 - d) + d + 3 = 18$$
$$a_3 = x + 2d + 7 = (15 - d) + 2d + 7 = 22 + d$$
$$\frac{a_2}{a_1} = \frac{a_3}{a_2};\ a_2^2 = a_1 a_3$$
$$18^2 = (17 - d)(22 + d)$$
$$324 = 374 - 5d - d^2$$
$$d^2 + 5d - 50 = 0$$
$$(d - 5)(d + 10) = 0;\ d = \mathbf{5} \text{ or } -10$$

SOLUTION 7 - 31
Ans: A

Let x and y be the numbers.
In G.P., the terms are x, 12, y, then

$$12/x = y/12;\ xy = 144 \qquad \rightarrow (1)$$

In H.P., the terms are x, $36/5$, y, then

$$\frac{1}{y} - \frac{5}{36} = \frac{5}{36} - \frac{1}{x}$$
$$\frac{1}{y} + \frac{1}{x} = \frac{5}{18} \text{ or } \frac{x+y}{xy} = \frac{5}{18} \qquad \rightarrow (2)$$

Substitute xy in Eq. (1) to Eq. (2)

$$\frac{x+y}{144} = \frac{5}{18}$$
$$x + y = 40,\quad y = 40 - x \qquad \rightarrow (3)$$

Substitute y in Eq. (3) to Eq. (1)

$$x(40 - x) = 144$$
$$x^2 - 40x + 144 = 0$$
$$x = 36 \text{ and } y = 4$$

SOLUTION 7 - 32
Ans: C

If x, $4x + 8$, and $30x + 24$ are in G.P., then

$$r = \frac{4x+8}{x} = \frac{30x+24}{4x+8}$$
$$16x^2 + 64x + 64 = 30x^2 + 24x$$
$$7x^2 - 20x - 32 = 0$$
$$(7x + 8)(x - 4) = 0$$
$$x = 4 \text{ and } x = -8/7$$

Using $x = 4$:

$$\text{Common ratio, } r = \frac{4(4)+8}{4} = 6$$

SOLUTION 7 - 33
Ans: C

Total number of days the provision can hold out, N

$$N = 5700(66) = 376200 \text{ man-days}$$

Day	Number of man-days
1	5700
2	5700 - 20 = 5680
3	5680 - 20 = 5660
.	.
.	.
n	?
	N

The provision will hold out until the total number of man-days becomes 376200.

$N = 5700 + 5680 + 5660 + \ldots$ (Sum of A.P. with $d = -20$)

From $S = \frac{n}{2}\left[2a_1 + (n-1)d\right]$

$$376200 = \frac{n}{2}\left[2(5700) + (n-1)(-20)\right]$$
$$752400 = 11400n - 20n^2 + 20n$$
$$20n^2 - 11420n + 752400 = 0$$
$$n = \frac{11420 \pm \sqrt{(11420)^2 - 4(20)(752400)}}{2(20)} = \mathbf{76 \text{ days}}$$

SOLUTION 7 - 34
Ans: A

Let V be the initial volume of air.
First stroke: $V_1 = V - V/3 = 2V/3$
Second stroke: $V_2 = V_1 - V_1/3 = 2V_1/3$

The volume in the tank after each stroke is 2/3 of the volume before the stroke. Hence, the series forms a geometric progression with $r = 2/3$ and $a_1 = 2V/3$. After the sixth stroke;

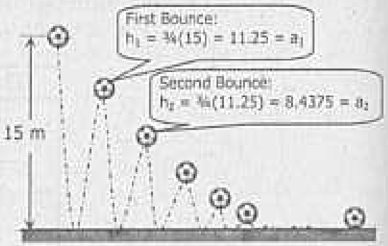
$$a_6 = a_1 r^{n-1} = (2V/3)(2/3)^{6-1} = 0.08779V$$

Thus, the volume removed is $V - 0.08779 = 0.9122V$ or **0.9122** of the total air.

SOLUTION 7 - 35
Ans: A

Total distance traveled, D
$D = 15 + 2 \times$ sum of height of each bounce.

Since the ball bounces infinite times and each succeeding heights form a G.P. with $r = 2/3$ and n = infinity.



We may use the sum of infinite G.P. with $a_1 = 10$ and

$$D = 15 + 2 \times \frac{a_1}{1-r} = 15 + 2 \times \frac{10}{1-2/3} = \mathbf{75\ m}$$

SOLUTION 7 - 36
Ans: B

Total provision on hand = 6400(216) = 1,382,400
Number of days the provision will hold out if no one dies:
= 216/3 = 72 days
Number of days to provision lasted because of deaths:
= 72 + 9 = 81 days

Let x be the number of soldiers that will die each day

Day	Consumption
1	6400(3) = 19200
2	$19200 - 3x$
3	$19200 - (3x + 3x)$

Thus, the series will form an A.P. with $a_1 = 19200$ and $d = -3x$. The provision will run out when the sum of the consumption equals 1,382,400.

$$S = \frac{n}{2}[\,2a_1 + (n-1)\,d\,]$$

$$1382400 = \tfrac{81}{2}[\,2(19200) + (81-1)(-3x)\,];\quad x = \mathbf{17.78}$$

SOLUTION 7 - 37
Ans: A

No. of man days required = 60(72) = 4320 man-days

Day	Man-days
First 12 days	60(12) = 720 man-days
Second 12 days	55(12) = 660 man-days
Third 12 days	50(12) = 600 man-days

The sequence is in A.P. with $a_1 = 720$ and $d = -60$

$$S = \frac{n}{2}[\,2a_1 + (n-1)\,d\,]$$

$$4320 = \frac{n}{2}[\,2(720) + (n-1)(-60)\,]$$

$$8640 = 1500n - 60n^2$$

$$n^2 - 25n + 144 = 0$$

$$n = \frac{25 \pm \sqrt{(-25)^2 - 4(1)(144)}}{2(1)} = 9$$

Number of days = 9 × 12 = **108 days**

SOLUTION 7 - 38
Ans: C

The first one to attend pays P0.10, the second one pays P0.20, the third one pays P0.40, and so on. The amount paid forms a geometric progression with $a_1 = 0.1$, and $r = 2$. The total amount collected represents the sum of the progression.

From $\quad S = \dfrac{a_1(r^n - 1)}{r-1}$

$$104{,}857.50 = \frac{0.1(2^n - 1)}{2-1}$$

$$2^n - 1 = 1{,}048{,}575 = 1{,}048{,}576$$

$$n \ln 2 = \ln 1{,}048{,}576;\quad n = \mathbf{20}$$

SOLUTION 7 - 39
Ans: D

$$\begin{vmatrix} 7 & 8 \\ 9 & 4 \end{vmatrix} = (7)(4) - (9)(8) = -44$$

SOLUTION 7 - 40
Ans: C

If $\begin{vmatrix} 3 & x \\ 2 & 2 \end{vmatrix} = \begin{vmatrix} 2 & -1 \\ x & -3 \end{vmatrix}$, then

$3(2) - 2(x) = 2(-3) - x(-1)$
$6 - 2x = -6 + x;\quad x = 4$

SOLUTION 7 - 41
Ans: C

$$D = \begin{vmatrix} 1 & 5 & -2 \\ 2 & 1 & -3 \\ 3 & -2 & 1 \end{vmatrix}\begin{matrix} 1 & 5 \\ 2 & 1 \\ 3 & -2 \end{matrix}\Bigg|$$

$D = (1 - 45 + 8) - (-6 + 6 + 10)$

$D = -46$

SOLUTION 7 - 42
Ans: B

By pivotal element method, select column 4 row 2 as the pivot:

$$x = \begin{vmatrix} 4 & -1 & 2 & 3 \\ 2 & 0 & 2 & 1 \\ 10 & 3 & 0 & 1 \\ 14 & 2 & 4 & 5 \end{vmatrix}$$

$$x = 1(-1)^{4+2}\begin{vmatrix} 4-3(2) & -1-3(0) & 2-3(2) \\ 10-1(2) & 3-1(0) & 0-1(2) \\ 14-5(2) & 2-5(0) & 4-5(2) \end{vmatrix}$$

$$x = \begin{vmatrix} -2 & -1 & -4 \\ 8 & 3 & -2 \\ 4 & 2 & -6 \end{vmatrix}\begin{matrix} -2 & -1 \\ 8 & 3 \\ 4 & 2 \end{matrix}\Bigg|$$

$x = (36 + 8 - 64) - (-48 + 8 + 48) = -28$

SOLUTION 7 - 43
Ans: D

$$D = \begin{vmatrix} 1 & 4 & 2 & -1 \\ 2 & -1 & 0 & -3 \\ -2 & 3 & 1 & 2 \\ 0 & 2 & 1 & 4 \end{vmatrix}$$

$$D = +(1)\begin{vmatrix} -1-(4)(2) & 0-(2)(2) & -3-(-1)(2) \\ 3-(4)(-2) & 1-(2)(-2) & 2-(-1)(-2) \\ 2-(4)(0) & 1-(2)(0) & 4-(-1)(0) \end{vmatrix}$$

$$D = \begin{vmatrix} -9 & -4 & -1 \\ 11 & 5 & 0 \\ 2 & 1 & 4 \end{vmatrix}\begin{matrix} -9 & -4 \\ 11 & 5 \\ 2 & 1 \end{matrix}\Bigg|$$

$D = (-180 + 0 - 11) - (-10 + 0 - 176) = -5$

SOLUTION 7 - 44
Ans: D

$$A + 2B = \begin{bmatrix} 1 & 3 \\ -2 & 1 \end{bmatrix} + 2\begin{bmatrix} -1 & -2 \\ -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 3 \\ -2 & 1 \end{bmatrix} + \begin{bmatrix} -2 & -4 \\ -2 & 2 \end{bmatrix} = \begin{bmatrix} 1-2 & 3-4 \\ -2-2 & 1+2 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & -1 \\ 0 & 3 \end{bmatrix}$$

SOLUTION 7 - 45
Ans: A

$$B = \begin{bmatrix} 1 & 2 \\ 0 & -5 \end{bmatrix}; \quad C = \begin{bmatrix} 3 & 6 \\ 4 & 1 \end{bmatrix}$$

$$BC = \begin{bmatrix} 1 & 2 \\ 0 & -5 \end{bmatrix}\begin{bmatrix} 3 & 6 \\ 4 & 1 \end{bmatrix} = \begin{bmatrix} 1(3)+2(4) & 1(6)+2(1) \\ 0(3)+(-5)(4) & 0(6)+(-5)(1) \end{bmatrix}$$

$$BC = \begin{bmatrix} 11 & 8 \\ -20 & -5 \end{bmatrix}$$

SOLUTION 7 - 46
Ans: A

The given $\begin{bmatrix} 1 & 1 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$ represents the following equations:

$1(x) + 1(y) = 2 \quad \rightarrow (1)$

$3(x) + 2(y) = 0 \quad \rightarrow (2)$

Solving for x and y:

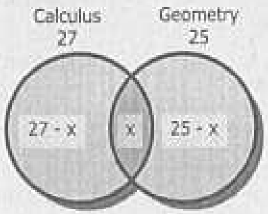
$x = -2$ and $y = 6$

SOLUTION 7 - 47
Ans: D

Let x be the number of students who liked both the subjects, then from the Venn Diagram shown:

$(27 - x) + x + (25 - x) = 40$

$x = 12$



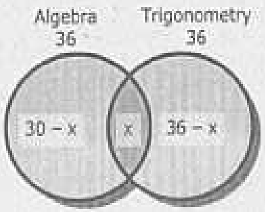
SOLUTION 7 - 48
Ans: C

Using Venn diagram:
Let x be the number of students who passed the two subjects

Since 2 students failed in two subject, then;

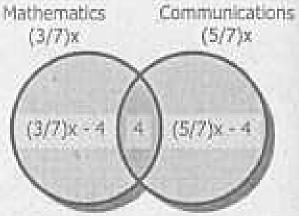
$(30 - x) + x + (36 - x) = 40 - 2$

$x = 28$



SOLUTION 7 - 49
Ans: A

Let x be the number of graduates who took the examinations.



From the Venn diagram shown:

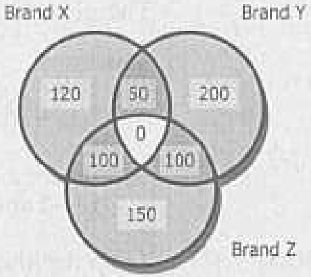
$[(3/7)x - 4] + 4 + [(5/7)x - 4] = x$

$x = 28$

SOLUTION 7 - 50
Ans: B

The total number of persons surveyed is 1000.

The Venn Diagram shows the number persons who have preferences. The total number of persons who have brand preference is the sum of all the numbers in the diagram.



Total = 120 + 200 + 150 + 50 + 100 + 100

Total = 720 persons

Therefore, the number of persons who have no brand preference is 1000 - 720 = 280

Problems - Set 8
Permutation, Combination, Probability

PROBLEM 8 - 1

How many permutations can be made out of the letters in the word ISLAND taking four letters at a time?

A. 360 C. 120
B. 720 D. 24

PROBLEM 8 - 2
CE Nov. 1996

How many 4 digit numbers can be formed without repeating any digit, from the following digits 1, 2, 3, 4 and 6.

A. 150 C. 140
B. 120 D. 130

PROBLEM 8 - 3

How many permutations can be made out of the letters of the word ENGINEERING?

A. 39,916,800 C. 55,440
B. 277,200 D. 3,326,400

PROBLEM 8 - 4

How many ways can 3 men and 4 women be seated on a bench if the women are to be together?

A. 720 C. 5040
B. 576 D. 1024

PROBLEM 8 - 5

In how many ways can 5 people line up to pay their electric bills?

A. 120 C. 72
B. 1 D. 24

PROBLEM 8 - 6

In how many ways can 5 people line up to pay their electric bills if two particular persons refuse to follow each other?

A. 120 C. 90
B. 72 D. 140

PROBLEM 8 - 7

How many ways can 7 people be seated at a round table?

A. 5040 C. 720
B. 120 D. 840

PROBLEM 8 - 8

In how many relative orders can we seat 7 people at a round table with a certain 3 people side by side.

A. 144 C. 720
B. 5040 D. 1008

PROBLEM 8 - 9

In how many ways can we seat 7 people in a round table with a certain 3 people not in consecutive order?

A. 576 C. 5320
B. 3960 D. 689

PROBLEM 8 - 10

The captain of a baseball team assigns himself to the 4th place in the batting order. In how many ways can he assign the remaining places to his eight teammates if just three men are eligible for the first position?

A. 2160 C. 5040
B. 40320 D. 15120

PROBLEM 8 - 11

In how many ways can PICE chapter with 15 directors choose a president, a vice-president, a secretary, a treasurer, and an auditor, if no member can hold more than one position?

A. 630630 C. 360360
B. 3300 D. 3003

PROBLEM 8 - 12

How many ways can a committee of five may be selected from an organization with 35 members?

A. 324632 C. 125487
B. 425632 D. 326597

PROBLEM 8 - 13

How many line segments can be formed by 13 distinct point?

A. 156 C. 98
B. 36 D. 78

PROBLEM 8 - 14

In how many ways can a hostess select six luncheon guests from 10 women if she is to avoid having a particular two of them together at the luncheon?

A. 210 C. 140
B. 84 D. 168

PROBLEM 8 - 15
ECE April 1998

A semiconductor company will hire 7 men and 4 women. In how many ways can the company choose from 9 men and 6 women who qualified for the position?

A. 680 C. 480
B. 840 D. 540

PROBLEM 8 - 16
ME Oct. 1997

How many ways can you invite one or more of five friends to a party?

A. 25 C. 31
B. 15 D. 62

PROBLEM 8 - 17

A bag contains 4 red balls, 3 green balls, and 5 blue balls. The probability of not getting a red ball in the first draw is:

A. 2 C. 1
B. 2/3 D. 1/3

PROBLEM 8 - 18

Which of the following cannot be a probability?

A. 1 C. 1/e
B. 0 D. 0.434343

PROBLEM 8 - 19
CE May 1996

A bag contains 3 white and 5 black balls. If two balls are drawn in succession without replacement, what is the probability that both balls are black?

A. 5/28 C. 5/32
B. 5/16 D. 5/14

PROBLEM 8 - 20

A bag contains 3 white and 5 red balls. If two balls are drawn at random, find the probability that both are white.

A. 3 / 28 C. 2 / 7
B. 3 / 8 D. 5 / 15

PROBLEM 8 - 21

In Problem 8 - 20, find the probability that one ball is white and the other is red.

A. 15 / 56 C. 1 / 4
B. 15 / 28 D. 225 / 784

PROBLEM 8 - 22

In Problem 8 - 20, find the probability that all are of the same color.

A. 13/30 C. 13/28
B. 14/29 D. 15/28

PROBLEM 8 - 23

The probability that both stages of a two-stage rocket to function correctly is 0.92. The reliability of the first stage is 0.97. The reliability of the second stage is:

A. 0.948 C. 0.968
B. 0.958 D. 0.8924

PROBLEM 8 - 24

Ricky and George each throw two dice. If Ricky gets a sum of 4, what is the probability that George will get less?

A. ½   C. 9/11
B. 5/6   D. 1/12

PROBLEM 8 - 25

Two fair dice are thrown. What is the probability that the sum shown on the dice is divisible by 5?

A. 7/36   C. 1/12
B. 1/9   D. ¼

PROBLEM 8 - 26
ME April 1996

An urn contains 4 black balls and 6 white balls. What is the probability of getting one black ball and one white ball in two consecutive draws from the urn?

A. 0.24   C. 0.53
B. 0.27   D. 0.04

PROBLEM 8 - 27

If three balls are drawn in succession from 5 white and 6 black balls in a bag, find the probability that all are of one color, if the first ball is replaced immediately while the second is not replaced before the third draw.

A. 10 / 121   C. 28 / 121
B. 18 / 121   D. 180 / 14641

PROBLEM 8 - 28

A First bag contains 5 white balls and 10 black balls and a second bag contains 20 white and 10 black balls. The experiment consists of selecting a bag and then drawing a ball from the selected bag. Find the probability of drawing a white ball.

A. 1 / 3   C. 1 / 2
B. 1 / 6   D. 1 18

PROBLEM 8 - 29

In Problem 8 - 28, find the probability of drawing a white ball from the first bag.

A. 5 / 6   C. 2 / 3
B. 1 / 6   D. 1 / 3

PROBLEM 8 - 30

If seven coins are tossed simultaneously, find the probability that they will just have three heads.

A. 33/128   C. 30/129
B. 35/128   D. 37/129

PROBLEM 8 - 31

If seven coins are tossed simultaneously, find the probability that there will be at least six tails.

A. 2 / 128   C. 1 / 6
B. 3 / 128   D. 2 / 16

PROBLEM 8 - 32
CE Nov. 1998

A face of a coin is either head or tail. If three coins are tossed, what is the probability of getting three tails?

A. 1/8   C. ¼
B. ½   D. 1/6

PROBLEM 8 - 33

The face of a coin is either head or tail. If three coins are tossed, what is the probability of getting three tails or three heads?

A. 1/8   C. ¼
B. ½   D. 1/6

PROBLEM 8 - 34

Five fair coins were tossed simultaneously. What is the probability of getting three heads and two tails?

A. 1/32   C. 1/8
B. 1/16   D. ¼

PROBLEM 8 - 35

Throw a fair coin five times. What is the probability of getting three heads and two tails?

A. 5/32   C. 1/32
B. 5/16   D. 7/16

PROBLEM 8 - 36
ECE March 1996

The probability of getting a credit in an examination is 1/3. If three students are selected at random, what is the probability that at least one of them got a credit?

A. 19/27   C. 2/3
B. 8/27   D. 1/3

PROBLEM 8 - 37

There are three short questions in mathematics test. For each question, one (1) mark will be awarded for a correct answer and no mark for a wrong answer. If the probability that Mary correctly answers a question in a test is 2/3, determine the probability that Mary gets two marks.

A. 4/27   C. 4/9
B. 8/27   D. 2/9

PROBLEM 8 - 38

A marksman hits 75% of all his targets. What is the probability that he will hit exactly 4 of his next 10 shots?

A. 0.01622   C. 0.004055
B. 0.4055   D. 0.001622

PROBLEM 8 - 39

A two-digit number is chosen randomly. What is the probability that it is divisible by 7?

A. 7/50   C. 1/7
B. 13/90   D. 7/45

PROBLEM 8 - 40

One box contains four cards numbered 1, 3, 5, and 6. Another box contains three cards numbered 2, 4, and 7. One card is drawn from each bag. Find the probability that the sum is even.

A. 5/12 C. 7/12
B. 3/7 D. 5/7

PROBLEM 8 - 41

Two people are chosen randomly from 4 married couples. What is the probability that they are husband and wife?

A. 1/28 C. 3/28
B. 1/14 D. 1/7

PROBLEM 8 - 42

One letter is taken from each of the words PARALLEL and LEVEL at random. What is the probability of getting the same letter?

A. 1/5 C. 3/20
B. 1/20 D. 3/4

PROBLEM 8 - 43

In a shooting game, the probabilities that Botoy and Toto will hit a target is 2/3 and ¾ respectively. What is the probability that the target is hit when both shoot at it once?

A. 13/5 C. 7/12
B. 5/13 D. 11/12

PROBLEM 8 - 44

A standard deck of 52 playing cards is well shuffled. The probability that the first four cards dealt from the deck will be the four aces is closest to:

A. 4×10^{-6} C. 3×10^{-6}
B. 2×10^{-6} D. 8×10^{-6}

PROBLEM 8 - 45

A card is chosen from a pack of playing cards. What is the probability that it is either red or a picture card?

A. 8/13 C. 19/26
B. 10/13 D. 8/15

PROBLEM 8 - 46

In a poker game consisting of 5 cards, what is the probability of holding 2 aces and 2 Queens?

A. 5!/52! C. 33/54145
B. 5/52 D. 1264/45685

PROBLEM 8 - 47

Dennis Rodman sinks 50% of all his attempts. What is the probability that he will make exactly 3 of his next 10 attempts?

A. 1/256 C. 30/128
B. 3/8 D. 15/128

PROBLEM 8 - 48

There are 10 defectives per 1000 items of a product in a long run. What is the probability that there is one and only one defective in a random lot of 100.

A. 0.3697 C. 0.3796
B. 0.3967 D. 0.3679

PROBLEM 8 - 49

The UN forces for Bosnia uses a type of missile that hits the target with a probability of 0.3. How many missiles should be fired so that there is at least an 80% probability of hitting the target?

A. 2 C. 5
B. 4 D. 3

PROBLEM 8 - 50
ME April 1997

In a dice game, one fair die is used. The player wins P20.00 if he rolls either 1 or 6. He losses P10.00 if he turns up any other face. What is the expected winning for one roll of the die?

A. P40.00 C. P20.00
B. P0.00 D. P10.00

ANSWER SHEET

1. Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
2. Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
3. Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
4. Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
5. Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
6. Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
7. Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
8. Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
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57. Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
58. Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
59. Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
60. Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ

Solutions to Set 8 Permutation, Combination, Probability

SOLUTION 8 - 1
Ans: A

The word "I S L A N D" has 6 letters with no letters alike.

Taking four letters at a time:

$$N = P(6, 4) = 360$$

SOLUTION 8 - 2
Ans: B

There are five (5) different things (numbers) given which will be arranged four (4) at a time. The number of arrangement is the permutation of five different things taken four at a time.

$$N = P(n, r) = \frac{n!}{(n-r)!}$$

$$N = P(5, 4) = \frac{5!}{(5-4)!} = 120$$

SOLUTION 8 - 3
Ans: B

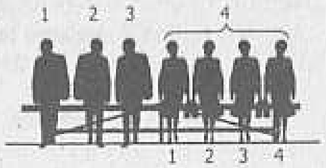
The permutation of n things (not all different) of which p are alike, q are alike, and so on is: $P = \frac{n!}{p!q!...}$.

For the word ENGINEERING, there are 11 objects (n = 11), with 3 E's, 3 N's, 2 G's, and 2 I's. Then the number of permutations is:

$$P = \frac{11!}{3!\,3!\,2!\,2!} = 277{,}200$$

SOLUTION 8 - 4
Ans: B

There are four groups involved. The three are the men and the fourth is the group of four women, but the women, being seated together, may also be arranged by within the group. Then



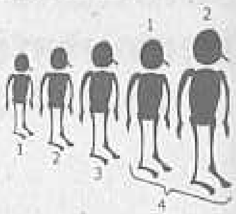
Number of ways = 4! × 4! = 576 ways

SOLUTION 8 - 5
Ans: A

$P = 5 \times 4 \times 3 \times 2 \times 1 =$ or $5!$
$P = 120$ ways

SOLUTION 8 - 6
Ans: B

No. of ways 5 people can line up in any order:
$N_1 = 5! = 120$



No. of ways 2 people can be together
$N_2 = (2! \times 4!) = 48$

Total no. of ways = 120 – 48
Total no. of ways = **72 ways**

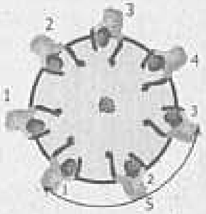
SOLUTION 8 - 7
Ans: C

The number of ways for n object be arranged in circular formation is:
$P = (n - 1)!$ "Cyclical permutation"

Thus, for 7 people:
$P = (7 - 1)! = 720$

SOLUTION 8 - 8
Ans: A

There will be 5 groups, 1 group consist of 3 people which should be seated together while the other 4 may be seated anywhere, then;



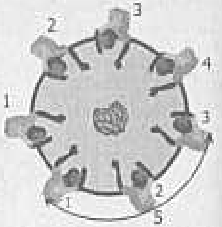
With the three seated side by side:
$N_1 = 3! = 6$

For 5 groups in circle, the number of arrangements is:
$N_2 = (5 - 1)! = 24$

Number of ways = 6(24) = **144 ways**

SOLUTION 8 - 9
Ans: A

No. of ways 7 people be seated in any order
$= (7 - 1)! = 720$



No. of ways 3 people be seated together
$= 3!\,(5 - 1)! = 144$

Total no. of ways = 720 - 144
Total no. of ways = **576**

SOLUTION 8 - 10
Ans: D

	1st Place	2nd Place	3rd Place	4th Place	5th Place	6th Place	7th Place	8th Place	9th Place
Players	3	7	6	1	5	4	3	2	1

Number of ways = 3(7)(6)(1)(5)(4)(3)(2)(1)
Number of = **15,120 ways**

SOLUTION 8 - 11
Ans: D

There are 15 members and 5 positions to be filled:

$$\text{Number of ways} = P(15, 5) = \frac{15!}{(15-5)!}$$

Number of ways = **360360 ways**

SOLUTION 8 - 12
Ans: A

$$\text{Number of ways} = C(n, r) = \frac{n!}{(n-r)!r!}$$

$$\text{Number of ways} = C(35, 5) = \frac{35!}{(35-5)!5!}$$

Number of ways = **324632 ways**

SOLUTION 8 - 13
Ans: D

Since a line segment is defined by any two distinct points, therefore the total number of line segments that can be drawn is the combination of 13 points taken 2 at a time.

$$\text{Number of line segments} = C(13, 2) = \frac{13!}{(13-2)!\,2!}$$

Number of line segments = **78**

SOLUTION 8 - 14
Ans: C

Let A and B be the two women who should not be together.
If B is excluded in the selection:

$$N_1 = C(9, 6) = \frac{9\,!}{(9-6)\,!\,6\,!} = 84$$

If A is excluded in the selection:

$$N_2 = C(9, 6) = \frac{9\,!}{(9-6)\,!\,6\,!} = 84$$

Since both cases includes the condition wherein the two of the women may not be selected, we have to subtract, from the total, the number of ways where the two may not be selected, which is equal to C(8, 6) = 28.

Thus, the number of ways is: 84 + 84 – 28 = **140**

SOLUTION 8 - 15
Ans: D

The number of ways of hiring 7 men and 4 women from 9 men and 6 women is:
$N = C(9, 7) \times C(6, 4)$
N = **540 ways**

SOLUTION 8 - 16
Ans: C

The combination of n things taken 1, 2, 3, ... n at a time is:
$C = 2^n - 1$
$C = 2^5 - 1 = 32 - 1 =$ **31 ways**

SOLUTION 8 - 17
Ans: B

$$P = \frac{\text{number of favorable ways}}{\text{total number of ways}}$$

There are 12 balls, 4 are red and 8 are not. The probability of NOT getting a red ball in the first draw is, $P = 8/12 = \mathbf{2/3}$.

SOLUTION 8 - 18
Ans: C

A probability P is $0 \le P \le 1$ and P must be a rational number.
Since e is not a rational number, $1/e$ cannot be a probability.

SOLUTION 8 - 19
Ans: D

There are 8 balls in the bag, five are black and three are white.
The probability that the first draw is black is:
$P_1 = 5/8$
The probability that the second draw is black is:
$P_2 = 4/7$
Then the probability that both are black is:
$P = P_1 \times P_2 = (5/8)(4/7) = \mathbf{5/14}$

SOLUTION 8 - 20
Ans: A

First draw white: $P_1 = 3/8$
Second draw white, $P_2 = 2/7$
The probability that both are white is: $3/8 \times 2/7 = \mathbf{3/28}$

SOLUTION 8 - 21
Ans: B

CASE I:
First draw white and second draw red:
$P_I = 3/8 \times 5/7 = 15/56$

CASE II:
First draw red and second draw white:
$P_{II} = 5/8 \times 3/7 = 15/56$

Thus, the probability is: $15/56 + 15/56 = \mathbf{15/28}$

SOLUTION 8 - 22
Ans: C

CASE I: Both balls are white:
$P_I = 3/8 \times 2/7 = 3/28$
Case II: Both balls are red:
$P_{II} = 5/8 \times 4/7 = 5/14$
Thus, the probability is: $3/28 + 5/14 = \mathbf{13/28}$

SOLUTION 8 - 23
Ans: A

Let P_1 & P_2 be the probability that the first and second rocket, respectively, to function correctly, then:
$P_t = P_1 \times P_2$
$0.92 = 0.97 \times P_2$
$P_2 = \mathbf{0.948}$

SOLUTION 8 - 24
Ans: D

Number of ways of getting a sum o less than 4: (1, 1) (2, 1)(1, 2) = 3 ways

$$P = \frac{3}{6(6)} = \frac{1}{12}$$

SOLUTION 8 - 25
Ans: A

The sum that is divisible by 5 are: 5 & 10
Number of ways of getting a sum of five is:
(4, 1) (1, 4) (2, 3) (3, 2) = 4 ways
Number of ways of getting a sum of ten is:
(5, 5) (6, 4) (4, 6) = 3 ways
Number of ways of getting a sum of 5 or 10 is: 4 + 3 = 7
Since two dice will fall in (6)(6) = 36 ways, the probability is:
$P = \mathbf{7/36}$

SOLUTION 8 - 26
Ans: C

Case I : First draw black and second draw white

$$P_1 = \frac{4}{10} \times \frac{6}{9} = \frac{4}{15}$$

Case II: First draw white and second draw black

$$P_2 = \frac{6}{10} \times \frac{4}{9} = \frac{4}{15}$$

then;

$$P = P_1 + P_2 = \frac{4}{15} + \frac{4}{15} = \frac{8}{15} = \mathbf{0.533}$$

SOLUTION 8 - 27
Ans: C

CASE I (all are white)
1st draw: $P_1 = 5/11$ 3rd draw: $P_3 = 4/10$
2nd draw: $P_2 = 5/11$
$P_I = 5/11 \times 5/11 \times 4/10 = 10/121$

CASE II (all are black)
1st draw: $P_1 = 6/11$ 3rd draw: $P_3 = 5/10$
2nd draw: $P_2 = 6/11$
$P_{II} = 6/11 \times 6/11 \times 5/10 = 18/121$

$P = P_I + P_{II} = 10/121 + 18/121 = \mathbf{28/121}$

SOLUTION 8 - 28
Ans: C

CASE I (First bag is selected)
Probability of selecting the first bag, $P_1 = 1/2$
Probability that a white ball is drawn, $P_2 = 5/15 = 1/3$
Probability that a white ball is drawn from the first bag
$P_I = P_1 \times P_2 = 1/2 \times 1/3 = 1/6$

CASE II (Second box is selected)
Probability of selecting the first bag, $P_1 = ½$
Probability that a white ball is drawn, $P_2 = 20/30 = 2/3$
Probability that a white ball is drawn from the second bag, $P_{II} = 1/2 \times 2/3 = 1/3$

Thus, the probability of drawing a white ball:
$$P = P_I + P_{II} = 1/6 + 1/3 = ½$$

SOLUTION 8 - 29
Ans: B

Probability of selecting the first bag = $P_1 = 1/2$
Probability of drawing a white ball = $P_2 = 5/15 = 1/3$
$P = P_1 \times P_2 = 1/2 \times 1/3 = \mathbf{1/6}$

SOLUTION 8 - 30
Ans: B

No. of ways each coin will fall = 2 (head or tail only)

No. of ways for 7 tosses can fall = $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$
No. of ways for 7 tosses can fall = $2^7 = 128$

No. of ways of getting 3 heads:
= (HHHTTTT), (HHTHTTT), (HTHHTTT), . . .
$$= C(7, 3) = \frac{7!}{(7-3)!\,3!} = 35$$

The probability is **35/128**

SOLUTION 8 - 31
Ans: C

Getting "at least six" tails means getting exactly 6 or exactly 7.

No. of ways that 7 coins will fall = $2^7 = 128$
No. of ways of at least getting 6 tails = $C(7, 6) + C(7, 7)$
No. of ways of at least getting 6 tails = 8

The probability is 8/128 = **1/6**

SOLUTION 8 - 32
Ans: A

$P = ½ \times ½ \times ½$
$P = \mathbf{1/8}$

SOLUTION 8 - 33
Ans: C

Probability of getting three heads:
$$P_1 = ½ \times ½ \times ½ = 1/8$$
Probability of getting three tails:
$$P_2 = ½ \times ½ \times ½ = 1/8$$
Probability of getting three tails or three heads:
$$P = P_1 + P_2 = 1/8 + 1/8 = ¼$$

SOLUTION 8 - 34
Ans: A

Each coin has equal chances of falling upright (head) or up-side-down (tail) which is one out of two (1/2). Then the probability of getting three heads and two tails is:
$$P = (1/2)(1/2)(1/2)\times(1/2)(1/2) = \mathbf{1/32}$$

SOLUTION 8 - 35
Ans: B

The possible outcomes are as follows:

TTHHH HTTHH HHTTH
THTHH HTHTH HHTHT
THHTH HTHHT HHHTT
THHHT

These arrangements is the permutation of 5 different thing where some are alike (3-H's and 2-T's), of which we can apply the formula:
$$P = \frac{n!}{p!\,q!} = \frac{5!}{3!\,2!} = 10$$
There are a total of 10 cases that the event can happen and the probability that each event can happen is:
$$P_1 = (1/2)(1/2)(1/2)(1/2)(1/2) = 1/32.$$

Therefore the total probability is: $P = (1/32)(10) = \mathbf{5/16}$

SOLUTION 8 - 36
Ans: A

Probability by repeated trials, $P = C(n,r)\,p^r\,q^{n-r}$, where $n = 3$; $p = 1/3$ (probability of getting a credit), $q = 2/3$ (probability of not getting a credit)
$$C(n,r) = \frac{n!}{(n-r)!\,r!}$$

The probability that one of them will get a credit (P_1), $r = 1$
$$P_1 = C(3,1)(1/3)^1(2/3)^{3-1} = \frac{3!}{(3-1)!\,1!} \times \frac{1}{3} \times \frac{4}{9}$$
$$P_1 = \frac{24}{54} = \frac{4}{9}$$

The probability that two of them will get a credit (P_2), $r = 2$
$$P_2 = C(3,2)(1/3)^2(2/3)^{3-2} = \frac{3!}{1!\,2!} \times \frac{1}{9} \times \frac{2}{3} = \frac{2}{9}$$

The probability that three of them will get a credit (P_3), $r = 3$
$$P_3 = C(3,3)(1/3)^3(2/3)^{3-3} = 1 \times \frac{1}{27} \times 1 = \frac{1}{27}$$

The probability that at least one of them will get a credit is:
$$P = P_1 + P_2 + P_3 = \frac{4}{9} + \frac{2}{9} + \frac{1}{27} = \frac{19}{27}$$

Another solution: (Using the "at least one" condition)

The probability that no one can get a credit is:

$Q = (2/3)(2/3)(2/3) = 8/27$

The probability that "at least one" got a credit:

$P = 1 - Q = 1 - 8/27 = \mathbf{19/27}$

SOLUTION 8 - 37
Ans: C

The probability of getting a correct answer is 2/3, then the probability of not getting a correct answer is 1/3.

The probability that Mary gets 2 marks (correct answers) in 3 questions is:

$P = C(n,r)p^r q^{n-r}$

where $p = 2/3$, $q = 1/3$, $n = 3$, and $r = 2$.

$P = C(3, 2)(2/3)^2 (1/3)^{3-2} = \mathbf{4/9}$

SOLUTION 8 - 38
Ans: A

Probability of hitting the target, $p = 75\% = 3/4$
Probability of not hitting the target, $q = 25\% = 1/4$

The probability that an event will occur exactly r times in n trials is given by the formula: (probability by repeated trials):

$P = C(n, r)p^r q^{n-r}$, $n = 10$; $r = 4$

$$P = \frac{10!}{(10-4)!\,4!}(3/4)^4 (1/4)^{10-4} = \mathbf{0.01622}$$

SOLUTION 8 - 39
Ans: B

The following are the two-digit numbers that are divisible by 7:

14, 21, 28, 35, 42, 49, 56, 63, 70, 77, 84, 91, 98 = 13 nos.

Since there are 90 two-digit numbers (10 ... 99), then the probability is:

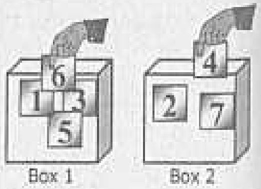
$P = \mathbf{13/90}$

SOLUTION 8 - 40
Ans: A

Note: odd + odd = even
even + even = even
odd + even = odd

Case 1: "odd + odd"
Box 1 (1, 3, 5), Box 2 (7)

$$P_1 = \frac{3}{4}\times\frac{1}{3} = \frac{1}{4}$$



Case 2: "even + even"
Box 1(6), Box 2 (2, 4)

$$P_2 = \frac{1}{4}\times\frac{2}{3} = \frac{1}{6}$$

The probability is: $P = P_1 + P_2 = \frac{1}{4} + \frac{1}{6}$

The probability is: $P = \frac{5}{12}$

SOLUTION 8 - 41
Ans: D

There are 4 married couples. The group therefore consists of 8 persons.

The probability that a couple will be chosen is:

$$P_1 = \frac{2}{8}\times\frac{1}{7} = \frac{1}{28}$$

Since there are four couples, then;

$$P = P_1 \times 4 = \frac{1}{28}\times 4 = \mathbf{1/7}$$

SOLUTION 8 - 42
Ans: A

From the words PARALLEL and LEVEL, only two letters are common (L & E).
PARALLEL has 3-L's and 1-E. LEVEL has 2-L's and 2-E's.

Case I: Both L: $P_I = \frac{3}{8}\times\frac{2}{5} = \frac{3}{20}$

Case II: Both E: $P_{II} = \frac{1}{8}\times\frac{2}{5} = \frac{1}{20}$

Then the probability is:

$$P = P_I + P_{II} = \frac{3}{20} + \frac{1}{20} = \frac{4}{20} = \frac{1}{5}$$

SOLUTION 8 - 43
Ans: D

Note: The condition of the problem is "the target is hit". It may be hit either only once or twice. Basically, if the target is NOT MISSED by both shooters, then it is therefore HIT. This idea is used to solve this problem knowing that the *probability of hitting* PLUS *the probability of missing* is one (1).

The probability that Botoy will miss the target:

$P_1 = 1 - 2/3 = 1/3$

SOLUTION 8 - 47
Ans: D

The probability that an event can happen exactly r times in n trials is:

$$P = C(n, r)\, p^r q^{n-r}$$

Where: p = probability of shooting the ball = ½
q = probability of missing = ½

then;

$$P = C(10, 3)(1/2)^3(1/2)^{10-3}$$

$$P = \frac{10!}{(10-3)!3!} \times \frac{1}{8} \times \frac{1}{128} = \frac{15}{128}$$

SOLUTION 8 - 48
Ans: A

Using the formula: (repeated trials)

$$P = C(n, r)\, p^r q^{n-r}$$

where: $n = 100$ $p = 10/1000 = 0.01$
$r = 1$ $q = 1 - 0.01 = 0.99$

then;

$$P = C(100, 1)(0.01)^1(0.99)^{100-1}$$

$$P = 0.3697$$

SOLUTION 8 - 49
Ans: C

The probability of hitting the target is 0.3
The probability of missing the target is 0.7
Using the "at least one" formula:

$$P = 1 - Q$$

where Q is the probability of missing the target
For n missiles fired, the probability of missing is $Q = 0.7^n$

$$P = 1 - 0.7^n = 80\%$$

$$0.7^n = 0.2$$

$$n \log 0.7 = \log 0.2$$

$$n = 4.51 \text{ say } 5$$

SOLUTION 8 - 50
Ans: B

A die has six (6) faces. The probability that a given number will fall in a single throw is 1/6.

The probability of winning (getting 1 or 6) is:

$$1/6 + 1/6 = 2/6 = 1/3$$

The probability of loosing is 2/3

The expected winning for one roll is $\frac{1}{3}(20) - \frac{2}{3}(10) = 0$

Problems - Set 9
Complex Numbers, Vectors, Elements

PROBLEM 9 - 1
CE May 1994

In the complex number $3 + 4i$, the absolute value is:
A. 10
B. 7.211
C. 5
D. 5.689

PROBLEM 9 - 2

In the complex number $8 - 2i$, the amplitude is:
A. 104.04°
B. 14.04°
C. 345.96°
D. 165.96°

PROBLEM 9 - 3

(6 cis 120°)(4 cis 30°) is equal to:
A. 10 cis 150°
B. 24 cis 150°
C. 10 cis 90°
D. 24 cis 90°

PROBLEM 9 - 4

$\frac{30 \text{ cis } 80^\circ}{10 \text{ cis } 50^\circ}$ is equal to:
A. 20 cis 30°
B. 3 cis 130°
C. 3 cis 30°
D. 20 cis130°

PROBLEM 9 - 5

The value of $x + y$ in the complex equation $3 + xi = y + 2i$ is:
A. 5
B. 1
C. 2
D. 3

PROBLEM 9 - 6

Multiply $(3 - 2i)(4 + 3i)$.
A. $12 + i$
B. $18 + i$
C. $6 + i$
D. $20 + i$

PROBLEM 9 - 7
EE Oct. 1997

Divide $\frac{4+3i}{2-i}$.
A. $\frac{11+10i}{5}$
B. $1 + 2i$
C. $\frac{5+2i}{5}$
D. $2 + 2i$

PROBLEM 9 - 8

Find the value of i^9.
A. i
B. $-i$
C. 1
D. -1

PROBLEM 9 - 9
ECE April 1999

Simplify $i^{1997} + i^{1999}$, where i is an imaginary number.
A. $1 + i$
B. i
C. $1 - i$
D. 0

PROBLEM 9 - 10

Expand $\left(2+\sqrt{-9}\right)^3$.
A. $46 + 9i$
B. $46 - 9i$
C. $-46 - 9i$
D. $-46 + 9i$

PROBLEM 9 - 11

Write $-4 + 3i$ in polar form.
A. $5 \angle 36.87^\circ$
B. $5 \angle 216.87^\circ$
C. $5 \angle 323.13^\circ$
D. $5 \angle 143.13^\circ$

PROBLEM 9 - 12

Simplify: $i^{30} - 2i^{25} + 3i^{17}$.
A. $i + 1$
B. $-1 - 2i$
C. $-1 + i$
D. $-1 + 5i$

PROBLEM 9 - 13
ME April 1997

Evaluate the value of $\sqrt{-10} \times \sqrt{-7}$
A. imaginary
B. $-\sqrt{70}$
C. $\sqrt{17}$
D. $\sqrt{70}$

PROBLEM 9 - 14
EE April 1994

Perform the indicated operation: $\sqrt{-9} \times \sqrt[3]{-343}$
A. 21
B. $21i$
C. $-21i$
D. -21

PROBLEM 9 - 15
ECE April 1999

What is the quotient when $4 + 8i$ is divided by i^3?
A. $8 + 4i$
B. $-8 + 4i$
C. $8 - 4i$
D. $-8 - 4i$

PROBLEM 9 - 16

What is the exponential form of the complex number $4 + 3i$?
A. $5e^{i53.13^\circ}$
B. $5e^{i36.87^\circ}$
C. $7e^{i53.13^\circ}$
D. $7e^{i36.87^\circ}$

PROBLEM 9 - 17

What is the algebraic form of the complex number $13e^{i67.38^\circ}$?
A. $12 + 5i$
B. $5 - 12i$
C. $12 - 5i$
D. $5 + 12i$

PROBLEM 9 - 18
ME April 1998

Solve for x that satisfy the equation $x^2 + 36 = 9 - 2x^2$.
A. $\pm 6i$
B. $\pm 3i$
C. $9i$
D. $-9i$

PROBLEM 9 - 19

Evaluate $\ln(5 + 12i)$.
A. $2.565 + 1.176i$
B. $2.365 - 0.256i$
C. $5.625 + 2.112i$
D. $3.214 - 1.254i$

PROBLEM 9 - 20
EE April 1994

Add the given vectors: (-4, 7) + (5, -9)
A. (1, 16) C. (9, 2)
B. (1, -2) D. (1, 2)

PROBLEM 9 - 21
EE April 1994

Find the length of the vector (2, 1, 4).
A. $\sqrt{17}$ C. $\sqrt{20}$
B. $\sqrt{21}$ D. $\sqrt{19}$

PROBLEM 9 - 22
ECE Nov. 1997

Find the length of the vector (2, 4, 4).
A. 8.75 C. 7.00
B. 6.00 D. 5.18

PROBLEM 9 - 23

What is the magnitude of the vector $F = 2i + 5j + 6k$?
A. 6.12 C. 6.18
B. 7.04 D. 8.06

PROBLEM 9 - 24

Find the value of x in the complex equation: $(x + yi)(1 - 2i) = 7 - 4i$.
A. 1 C. 4
B. 3 D. 2

PROBLEM 9 - 25

A statement the truth of which is admitted without proof is called:
A. an axiom C. a theorem
B. a postulate D. a corollary

PROBLEM 9 - 26

In a proportion of four quantities, the first and the fourth terms are referred to as the:
A. means C. extremes
B. denominators D. numerators

PROBLEM 9 - 27
ECE Nov. 1997

Convergent series is a sequence of decreasing numbers or when the succeeding term is ______ than the preceding term.
A. ten times more C. equal
B. greater D. lesser

PROBLEM 9 - 28
ECE Nov. 1997

It is the characteristic of a population which is measurable.
A. frequency C. sample
B. distribution D. parameter

PROBLEM 9 - 29
ECE Nov. 1997

The quartile deviation is a measure of:
A. division C. certainty
B. central tendency D. dispersion

PROBLEM 9 - 30
ECE Nov. 1995, 1997

In complex algebra, we use a diagram to represent a complex plane commonly called:
A. De Moivre's Diagram C. Argand Diagram
B. Funicular Diagram D. Venn Diagram

PROBLEM 9 - 31

A series of numbers which are perfect square numbers (i.e. 1, 4, 9, 16, ...) is caled:
A. Fourier series C. Euler's number
B. Fermat's number D. Fibonacci numbers

PROBLEM 9 - 32

A sequence of numbers where every term is obtained by adding all the preceding terms such as 1, 5, 14, 30, ... is called:
A. triangular number C. tetrahedral number
B. pyramidal number D. Euler's number

PROBLEM 9 - 33
ECE Nov. 1995

The graphical representation of the commulative frequency distribution in a set of statistical data is called:
A. Ogive C. Frequency polyhedron
B. Histogram D. mass diagram

PROBLEM 9 - 34
ECE March 1996

A sequence of numbers where the succeeding term is greater than the preceding term is called:
A. dissonant series C. isometric series
B. convergent series D. divergent series

PROBLEM 9 - 35
ECE March 1996

The number 0.123123123... is
A. irrational C. rational
B. surd D. transcendental

PROBLEM 9 - 36
ECE Nov. 1996

An array of $m \times n$ quantities which represent a single number system composed of elements in rows and column is known as:
A. transpose of a matrix C. co-factor of a matrix
B. determinant D. matrix

PROBLEM 9 - 37
ECE Nov. 1996

The characteristic is equal to the exponent of 10, when the number is written in:
A. exponential C. logarithmic
B. scientific notation D. irrational

PROBLEM 9 - 38
ECE Nov. 1996

Terms that differ only in numeric coefficients are known as:
A. unequal terms C. like terms
B. unlike terms D. equal terms

PROBLEM 9 - 39
ECE Nov. 1996

________ is a sequence of terms whose reciprocals form an arithmetic progression.
A. geometric progression C. algebraic progression
B. harmonic progression D. ratio and proportion

PROBLEM 9 - 40
ECE Nov. 1996

The logarithm of a number to the base e (2.718281828...) is called:
A. Naperian logarithm
B. Characteristic
C. Mantissa
D. Briggsian logarithm

PROBLEM 9 - 41
ECE Nov. 1996

The ratio or product of two expressions in direct or inverse relation with each other is called:
A. ratio and proportion
B. constant of variation
C. means
D. extremes

PROBLEM 9 - 42
ECE Nov. 1996

In any square matrix, when the elements of any two rows are exactly the same the determinant is:
A. zero
B. positive integer
C. negative integer
D. unity

PROBLEM 9 - 43
ECE Nov. 1996

Two or more equations are equal if and only if they have the same:
A. solution set
B. degree
C. order
D. variable set

PROBLEM 9 - 44

What is a possible outcome of an experiment called?
A. a sample space
B. a random point
C. an event
D. a finite set

PROBLEM 9 - 45

If the roots of an equation are zero, then they are classified as:
A. Trivial solutions
B. Extraneous roots
C. Conditional solutions
D. Hypergolic solutions

PROBLEM 9 - 46

A complex number associated with a phase-shifted sine wave in polar form whose magnitude is in RMS and angle is equal to the angle of the phase-shifted sine wave is known as:
A. Argand's number
B. Imaginary number
C. Phasor
D. Real number

PROBLEM 9 - 47

In raw data, the term, which occurs most frequently, is known as:
A. Mean
B. Median
C. Mode
D. Quartile

PROBLEM 9 - 48

Infinity minus infinity is:
A. Infinity
B. Zero
C. Indeterminate
D. None of these

PROBLEM 9 - 49

Any number divided by infinity is equal to:
A. 1
B. Infinity
C. Zero
D. Indeterminate

PROBLEM 9 - 50

The term in between any to terms of an arithmetic progression is called:
A. Arithmetic mean
B. Median
C. Middle terms
D. Mean

PROBLEM 9 - 51

Any equation which, because of some mathematical process, has acquired an extra root is sometimes called a:
A. redundant equation
B. literal equation
C. linear equation
D. defective equation

PROBLEM 9 - 52

A statement that one mathematical expression is greater than or less than another is called:
A. inequality
B. non-absolute condition
C. absolute condition
D. conditional expression

PROBLEM 9 - 53

A relation in which every ordered pair (x, y) has one and only one value of y that corresponds to the values of x, is called:
A. function
B. range
C. domain
D. coordinates

PROBLEM 9 - 54

An equation in which a variable appears under the radical sign is called:
A. literal equation
B. radical equation
C. irradical equation
D. irrational equation

PROBLEM 9 - 55

The number of favorable outcomes divided by the number of possible outcomes is:
A. permutations
B. probability
C. combination
D. chance

PROBLEM 9 - 56

Two factors are considered essentially the same if:
A. one is merely the negative of the other
B. one is exactly the same as the other
C. both of then are negative
D. both of them are positive

PROBLEM 9 - 57

An integer is said to be prime if:
A. it is factorable by any value
B. it is an odd integer
C. it has no other integer as a factor except itself or 1
D. it is an even integer

PROBLEM 9 - 58

Equations in which the members are equal for all permissible values of unknowns are called:
A. a conditional equation
B. an identity
C. a parametric equation
D. a quadratic equation

PROBLEM 9 - 59

Equations which satisfy only for some values of unknown are called:

A. a conditional equation
B. an identity
C. a parametric equation
D. a quadratic equation

PROBLEM 9 - 60
ME April 1996

The logarithm of 1 to any base is:

A. indeterminate
B. zero
C. infinity
D. one

ANSWER SHEET

1. A B C D E
2. A B C D E
3. A B C D E
4. A B C D E
5. A B C D E
6. A B C D E
7. A B C D E
8. A B C D E
9. A B C D E
10. A B C D E
11. A B C D E
12. A B C D E
13. A B C D E
14. A B C D E
15. A B C D E
16. A B C D E
17. A B C D E
18. A B C D E
19. A B C D E
20. A B C D E
21. A B C D E
22. A B C D E
23. A B C D E
24. A B C D E
25. A B C D E
26. A B C D E
27. A B C D E
28. A B C D E
29. A B C D E
30. A B C D E
31. A B C D E
32. A B C D E
33. A B C D E
34. A B C D E
35. A B C D E
36. A B C D E
37. A B C D E
38. A B C D E
39. A B C D E
40. A B C D E
41. A B C D E
42. A B C D E
43. A B C D E
44. A B C D E
45. A B C D E
46. A B C D E
47. A B C D E
48. A B C D E
49. A B C D E
50. A B C D E
51. A B C D E
52. A B C D E
53. A B C D E
54. A B C D E
55. A B C D E
56. A B C D E
57. A B C D E
58. A B C D E
59. A B C D E
60. A B C D E

Solutions to Set 9 Complex Numbers, Vectors, Elements

SOLUTION 9 - 1
Ans: C

The absolute value of the complex number $a + bi$ is

$$r = \sqrt{a^2 + b^2}$$

For the complex number $3 + 4i$,

$$r = \sqrt{3^2 + 4^2} = 5$$

SOLUTION 9 - 2
Ans: C

The amplitude of the complex number $a + bi$ is

$$\theta = \arctan b/a$$

For the complex number $8 - 2i$:

$$\theta = \arctan(-2/8)$$
$$\theta = -14.04 = 345.96°$$

SOLUTION 9 - 3
Ans: B

$$r_1 \text{ cis } \theta_1 \times r_2 \text{ cis } \theta_2 = r_1 r_2 \text{ cis}(\theta_1 + \theta_2)$$

$$(6 \text{ cis}120°)(4 \text{ cis}30°) = (6 \times 9) \text{ cis } (120° + 30°)$$
$$= \mathbf{24 \text{ cis } 150°}$$

SOLUTION 9 - 4
Ans: C

$$\frac{r_1 \text{ cis}\theta_1}{r_2 \text{ cis}\theta_2} = \frac{r_1}{r_2} \text{ cis } (\theta_1 - \theta_2)$$

$$\frac{30 \text{ cis } 80°}{10 \text{ cis } 50°} = \frac{30}{10} \text{ cis } (80° - 50°) = \mathbf{3 \text{ cis } 30°}$$

SOLUTION 9 - 5
Ans: A

If $a + bi = c + di$, the $a = c$ and $b = d$.

For $3 + xi = y + 2i$, $3 = y$ and $x = 2$

$$x + y = 5$$

SOLUTION 9 - 6
Ans: B

$$(3 - 2i)(4 + 3i) = 12 + 9i - 8i - 6i^2$$
$$= 12 + i - 6(-1)$$
$$= \mathbf{18 + i}$$

SOLUTION 9 - 7
Ans: B

$$\frac{4+3i}{2-i} \times \frac{2+i}{2+i} = \frac{8+10i+3i^2}{4-i^2}$$
$$= \frac{8+10i+3(-1)}{4-(-1)} = \frac{5+10i}{5}$$
$$= \mathbf{1 + 2i}$$

SOLUTION 9 - 8
Ans: A

Note: $i^2 = -1$

$i^9 = i^8\, i = (i^2)^4\, i = (-1)^4\, i$

$i^9 = i$

SOLUTION 9 - 9
Ans: D

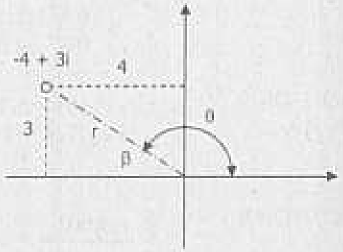
$$\begin{aligned} i^{1997} + i^{1999} &= i^{1996}\, i + i^{1998}\, i \\ &= (i^2)^{998}\, i + (i^2)^{999}\, i = (-1)^{998}\, i + (-1)^{999}\, i \\ &= i - i = 0 \end{aligned}$$

SOLUTION 9 - 10
Ans: D

$$\begin{aligned} \left(2+\sqrt{-9}\right)^3 &= (2 + 3i)^3 \\ &= 2^3 + 3(2^2)(3i) + 3(2)(3i)^2 + (3i)^3 \\ &= 8 + 36i + 54\, i^2 + 27\, i^3 \\ &= 8 + 36i + 54(-1) + 27i^2\, i \\ &= -46 + 36i + 27(-1)i \\ &= -46 + 9i \end{aligned}$$

SOLUTION 9 - 11
Ans: D

Rectangular form: $-4 + 3i$; $a = -4$ and $b = 3$



From the Argand chart shown:

$r = \sqrt{3^2 + 4^2} = 5$

$\tan \beta = 3/4$; $\beta = 36.87°$

$\theta = 180 - \beta = 143.13°$

Polar form: $r\angle\theta = 5\angle 143.13°$

SOLUTION 9 - 12
Ans: C

Note: $i^2 = -1$; $i^3 = i^2 \times i = -i$

$$\begin{aligned} i^{30} - 2i^{25} + 3i^{17} &= (i^2)^{15} - 2(i^{24} \times i) + 3(i^2)^8 \times i \\ &= (-1)^{15} - 2(i^2)^{12} \times i + 3(-1)^8 \times i \\ &= -1 - 2(-1)^{12}\, i + 3(1)i = -1 - 2i + 3i \\ i^{30} - 2i^{25} + 3i^{17} &= -1 + i \end{aligned}$$

SOLUTION 9 - 13
Ans: B

$$\begin{aligned} \sqrt{-10} \times \sqrt{-7} &= \sqrt{10}\, i \times \sqrt{7}\, i \\ &= \sqrt{10} \times \sqrt{7} \times i^2 \\ &= \sqrt{10 \times 7}\, i^2;\ \text{but } i^2 = -1 \\ &= -\sqrt{70} \end{aligned}$$

SOLUTION 9 - 14
Ans: C

$\sqrt{-9} \times \sqrt[3]{-343} = 3i \times -7 = -21i$

SOLUTION 9 - 15
Ans: B

$$\begin{aligned} \frac{4+8i}{i^3} &= \frac{4+8i}{i^2 i} = \frac{4+8i}{-i} \times \frac{i}{i} \\ &= \frac{4i+8i^2}{-i^2} = \frac{4i+(8)(-1)}{-(-1)} \\ &= -8 + 4i \end{aligned}$$

SOLUTION 9 - 16
Ans: B

The exponential form of the complex number $a + bi$ is $r\, e^{\,i \arctan(b/a)}$

$$r = \sqrt{a^2 + b^2}$$

then;

$$\begin{aligned} 4 + 3i &= \sqrt{4^2 + 3^2}\ e^{\,i \arctan(3/4)} \\ &= 5\, e^{\,i\, 36.87°} \end{aligned}$$

SOLUTION 9 - 17
Ans: D

A complex number in the form re^{ic} can be converted to algebraic form as $r\, e^{ic} = r \cos c + i(r \sin c)$

$$\begin{aligned} 13\, e^{i\, 67.38°} &= 13 \times \cos 67.38° + i\,(13 \times \sin 67.38°) \\ &= 5 + 12i \end{aligned}$$

SOLUTION 9 - 18
Ans: B

Given $x^2 + 36 = 9 - 2x^2$

Combine like terms:

$3x^2 = -27$

$x^2 = -9$; $x = \pm\sqrt{-9} = \pm 3i$

SOLUTION 9 - 19
Ans: A

Convert $5 + 12i$ to exponential form: $a + bi = r\, e^{\,i \arctan b/a}$

$5 + 12\, i = \sqrt{5^2 + 12^2}\ \ e^{i \arctan 12/5} = 13\ e^{i\, 67.38°}$

$$\begin{aligned} \ln(5 + 12\, i) &= \ln 13\, e^{i\, 67.38°} = \ln 13 + \ln e^{i\, 67.38°} \\ &= 2.565 + i(67.38°) \ln e \qquad \text{But } \ln e = 1 \\ &= 2.565 + 1.176\, i \end{aligned}$$

SOLUTION 9 - 20
Ans: B

$(-4, 7) + (5, -9) = [-4 + 5, 7 + (-9)] = (1, -2)$

SOLUTION 9 - 21
Ans: B

The length of the vector is $r = \sqrt{x^2 + y^2 + z^2}$

$r = \sqrt{(2)^2 + (1)^2 + (4)^2} = \sqrt{21}$

SOLUTION 9 - 22
Ans: B

The length of the vector (x, y, z) is $\sqrt{x^2 + y^2 + z^2}$

Length $= \sqrt{2^2 + 4^2 + 4^2} = 6$

SOLUTION 9 - 23
Ans: D

The magnitude of a vector $F = ai + bj + c\,k$ is

$$F=\sqrt{a^2+b^2+c^2}$$

Magnitude, $F=\sqrt{(2)^2+(5)^2+(6)^2}$

Magnitude, $F = 8.062$

SOLUTION 9 - 24
Ans: B

Note: Two complex numbers are equal if their corresponding real and imaginary parts are equal. i.e. $a + bi = c + di$ if $a = c$ and $b = d$

$$(x + yi)(1 - 2i) = 7 - 4i$$
$$x - 2xi + yi - 2yi^2 = 7 - 4i \qquad \text{Note: } i^2 = -1$$
$$x - 2xi + yi - 2y(-1) = 7 - 4i$$
$$(x + 2y) - (2x - y)i = 7 - 4i$$

Then; $x + 2y = 7$ → Eq. (1)
$2x - y = 4$ → Eq. (2)

Add (1) + (2)×2

$$\begin{array}{l} x + 2y = 7 \\ \underline{4x - 2y = 8} \\ 5x \quad = 15 \end{array}$$

$$x = 3$$

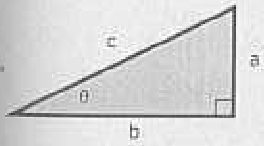
Answers to Problems 25 to 60

25. A
26. C
27. D
28. C
29. D
30. C
31. B
32. B
33. A
34. D
35. C
36. B
37. C
38. C
39. B
40. A
41. B
42. A
43 A
44. C
45. A
46. C
47. C
48. C
49. C
50. A
51. A
52. A
53. A
54. D
55. B
56. A
57. C
58. B
59. A
60. B

Part 2
PLANE & SPHERICAL TRIGONOMETRY

Plane Trigonometry

FUNCTIONS OF A RIGHT TRIANGLE



From the right triangle shown: (*soh-cah-toa*)

$$\sin\theta = \frac{\textit{opposite side}}{\textit{hypotenuse}} = \frac{a}{c} \qquad (soh)$$

$$\cos\theta = \frac{\textit{adjacent side}}{\textit{hypotenuse}} = \frac{b}{c} \qquad (cah)$$

$$\tan\theta = \frac{\textit{opposite side}}{\textit{adjacent side}} = \frac{a}{b} \qquad (toa)$$

$$\cot\theta = \frac{\textit{adjacent side}}{\textit{opposite side}} = \frac{b}{a} \qquad (tao)$$

$$\sec\theta = \frac{\textit{hypotenuse}}{\textit{adjacent side}} = \frac{c}{b} \qquad (cha)$$

$$\csc\theta = \frac{\textit{hypotenuse}}{\textit{opposite}} = \frac{c}{a} \qquad (sho)$$

Pythagorean Theorem

"In any right triangle, the square of the longest side (hypotenuse) equals the sum of the squares of the other two sides".

From the right triangle shown above:

$$c^2 = a^2 + b^2$$

SOLUTION 9 - 23
Ans: D

The magnitude of a vector $F = a\mathrm{i} + b\mathrm{j} + c\,\mathrm{k}$ is

$$F = \sqrt{a^2 + b^2 + c^2}$$

$$\text{Magnitude, } F = \sqrt{(2)^2 + (5)^2 + (6)^2}$$

$$\text{Magnitude, } F = 8.062$$

SOLUTION 9 - 24
Ans: B

Note: Two complex numbers are equal if their corresponding real and imaginary parts are equal. i.e. $a + bi = c + di$ if $a = c$ and $b = d$

$$(x + yi)\,(1 - 2i) = 7 - 4i$$
$$x - 2xi + yi - 2yi^2 = 7 - 4i \qquad \text{Note: } i^2 = -1$$
$$x - 2xi + yi - 2y(-1) = 7 - 4i$$
$$(x + 2y) - (2x - y)i = 7 - 4i$$

Then; $x + 2y = 7$ → Eq. (1)
$2x - y = 4$ → Eq. (2)

Add (1) + (2)×2

$$\begin{array}{l} x + 2y = 7 \\ \underline{4x - 2y = 8} \\ 5x \quad\;\; = 15 \end{array}$$

$$x = 3$$

Answers to Problems 25 to 60

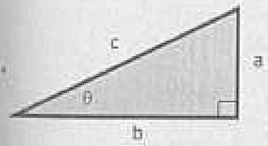
25. A
26. C
27. D
28. C
29. D
30. C
31. B
32. B
33. A
34. D
35. C
36. B
37. C
38. C
39. B
40. A
41. B
42. A
43 A
44. C
45. A
46. C
47. C
48. C
49. C
50. A
51. A
52. A
53. A
54. D
55. B
56. A
57. C
58. B
59. A
60. B

Part 2

PLANE & SPHERICAL TRIGONOMETRY

Plane Trigonometry

FUNCTIONS OF A RIGHT TRIANGLE



From the right triangle shown: (*soh-cah-toa*)

$$\sin\theta = \frac{\textit{opposite side}}{\textit{hypotenuse}} = \frac{a}{c} \qquad (\textit{soh})$$

$$\cos\theta = \frac{\textit{adjacent side}}{\textit{hypotenuse}} = \frac{b}{c} \qquad (\textit{cah})$$

$$\tan\theta = \frac{\textit{opposite side}}{\textit{adjacent side}} = \frac{a}{b} \qquad (\textit{toa})$$

$$\cot\theta = \frac{\textit{adjacent side}}{\textit{opposite side}} = \frac{b}{a} \qquad (\textit{tao})$$

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Pythagorean Theorem

"In any right triangle, the square of the longest side (hypotenuse) equals the sum of the squares of the other two sides".

From the right triangle shown above:

$$c^2 = a^2 + b^2$$

Applying similar procedure, the following formulas can be derived:

$$\cos\left(\frac{\theta}{2}\right) = \sqrt{\frac{1+\cos\theta}{2}}$$

$$\tan\left(\frac{\theta}{2}\right) = \frac{1-\cos\theta}{\sin\theta} = \frac{\sin\theta}{1+\cos\theta} = \sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$$

Powers of Functions

$$\sin^2 x = \frac{1-\cos 2x}{2}$$

$$\cos^2 x = \frac{1+\cos 2x}{2}$$

$$\tan^2 x = \frac{1-\cos 2x}{1+\cos 2x}$$

Product of Functions

$$\sin x \cos y = \tfrac{1}{2}[\sin(x+y) + \sin(x-y)]$$

$$\sin x \sin y = \tfrac{1}{2}[\cos(x-y) - \cos(x+y)]$$

$$\cos x \cos y = \tfrac{1}{2}[\cos(x+y) + \cos(x-y)]$$

Sum and Difference of Functions (Factoring Formulas)

$$\sin x + \sin y = 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

$$\sin x - \sin y = 2\cos\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$$

$$\cos x + \cos y = 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

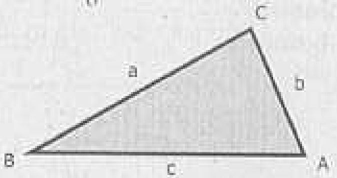
$$\cos x - \cos y = -2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$$

$$\tan x + \tan y = \frac{\sin(x+y)}{\cos x \cos y}$$

$$\tan x - \tan y = \frac{\sin(x-y)}{\cos x \cos y}$$

OBLIQUE TRIANGLES

An oblique triangle is any triangle that is not a right triangle. It could be an *acute triangle* (all three angles of the triangle are less than right angles) or it could be an *obtuse triangle* (one of the three angles is greater than a right angle). Actually, for the purposes of trigonometry, the class of *oblique triangles* might just as well include right triangles, too. Then the *study of oblique triangles is really the study of all triangles.*



Sine Law

In any triangle, the ratio of any one side to the sine of its opposite angle is constant. (This constant ratio is the diameter of the circle circumscribing the triangle.)

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Cosine Law

In any triangle, the square of any one side equals the sum of the squares of the other two sides, diminished by twice their product to the cosine of its included angle.

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$b^2 = a^2 + c^2 - 2ac\cos B$$

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Law of Tangents

$$\frac{a-b}{a+b} = \frac{\tan[(A-B)/2]}{\tan[(A+B)/2]}$$

$$\frac{b-c}{b+c} = \frac{\tan[(B-C)/2]}{\tan[(B+C)/2]}$$

$$\frac{c-a}{c+a} = \frac{\tan[(C-A)/2]}{\tan[(C+A)/2]}$$

Mollweide's Equations

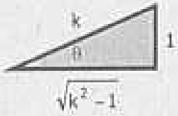
$$\frac{a-b}{c}=\frac{\sin[(A-B)/2]}{\cos(C/2)}$$

$$\frac{a+b}{c}=\frac{\cos[(A-B)/2]}{\sin(C/2)}$$

How to get the other trigonometric functions with one function known

Ex: If $\sin\theta = 1/k$, find the other functions.

From the right triangle shown:



$$\cos\theta=\frac{\sqrt{k^2-1}}{k}$$

$$\tan\theta=\frac{1}{\sqrt{k^2-1}}$$

$$\cot\theta=\frac{\sqrt{k^2-1}}{1}$$

$$\sec\theta=\frac{k}{\sqrt{k^2-1}}$$

$$\csc\theta=\frac{\sqrt{k^2-1}}{k}$$

ANGLES

The concept of angle is one of the most important concepts in geometry. The concepts of equality, sums, and differences of angles are important and used throughout geometry, but the subject of trigonometry is based on the measurement of angles.

Angle is the space between two rays that extend from a common point called the vertex.

An **acute angle** is an angle < 90°
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Complementary angles are angles whose sum is 90°.
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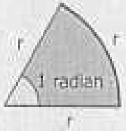
Units of Angle

90° = π/2 radians = 100 grades = 1600 mils

1 radian is the angle subtended by an arc of a circle whose length is one radius.

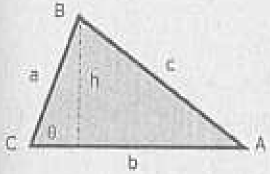


OTHER ELEMENTS AND PROPERTIES OF A TRIANGLE

Area of Triangle

Given base b and altitude h

$$\text{Area}=\frac{1}{2}bh$$



Given two sides a and b and included angle θ:

$$\text{Area}=\frac{1}{2}ab\sin\theta$$

Given three sides a, b, and c: (Hero's Formula)

$$\text{Area}=\sqrt{s(s-a)(s-b)(s-c)}$$

$$s=\frac{a+b+c}{2}$$

The area under this condition can also be solved by finding one angle using cosine law and apply the formula for two sides and included angle.

Given three angles A, B, and C and one side a:

$$\text{Area}=\frac{a^2\sin B\sin C}{2\sin A}$$

Applying similar procedure, the following formulas can be derived:

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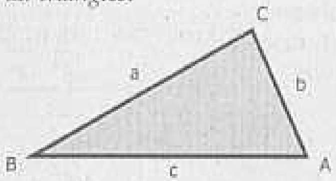

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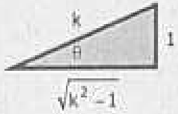
$$\csc\theta = \frac{\sqrt{k^2-1}}{k}$$



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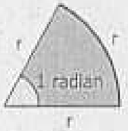

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Given base b and altitude h

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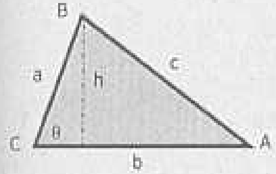


Given two sides a and b and included angle θ:

$$\text{Area} = \frac{1}{2}ab\sin\theta$$

Given three sides a, b, and c: (Hero's Formula)

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$s = \frac{a+b+c}{2}$$

The area under this condition can also be solved by finding one angle using cosine law and apply the formula for two sides and included angle.

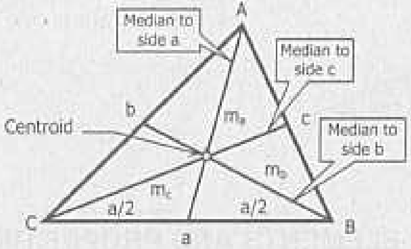
Given three angles A, B, and C and one side a:

$$\text{Area} = \frac{a^2\sin B\sin C}{2\sin A}$$

The area under this condition can also be solved by finding one side using sine law and apply the formula for two sides and included angle.

Median of a Triangle

The median of a triangle is the line drawn from one vertex to the midpoint of its opposite side. The medians of a triangle intersect at a common point called the **centroid** of the triangle.

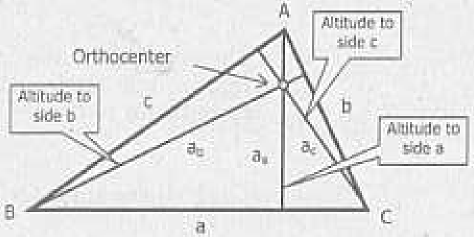


With all sides and angles already known, the median can be solved using cosine law or by the following formula:

$$4\,m_a^{\,2} = 2b^2 + 2c^2 - a^2$$
$$4\,m_b^{\,2} = 2a^2 + 2c^2 - b^2$$
$$4\,m_c^{\,2} = 2a^2 + 2b^2 - c^2$$

Altitudes of a Triangle

The altitude of a triangle is the line drawn from one vertex perpendicular to its opposite side. The altitudes of a triangle intersect at a point called the **orthocenter** of the triangle.

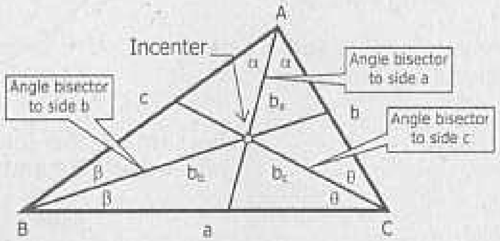


With all sides and angles already known, the altitudes of the triangle can be solved from the right triangles formed by these altitudes. If the area of the triangle A_T is known, the altitudes can be solved using the following formulas:

$$a_a = \frac{2A_T}{a}$$
$$a_b = \frac{2A_T}{b}$$
$$a_c = \frac{2A_T}{c}$$

Angle Bisectors of a Triangle

The angle bisector of a triangle is the line drawn from one vertex to the opposite side bisecting the angle included between the other two sides. The angle bisectors of a triangle intersect at a point called the **incenter** of the triangle.



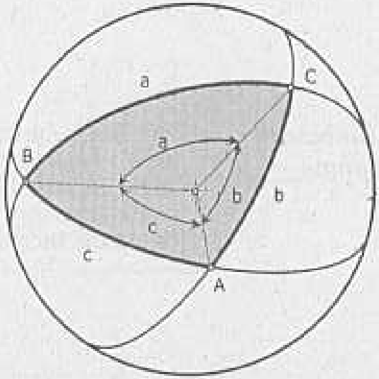
With all sides and angles already known, the angle bisectors of a triangle can be solved using sine law, or using the following formulas:

$$b_a = \frac{2}{b+c}\sqrt{bcs(s-a)}$$
$$b_b = \frac{2}{a+c}\sqrt{acs(s-b)}$$
$$b_c = \frac{2}{a+b}\sqrt{abs(s-c)}$$
$$s = \frac{a+b+c}{2} \text{ (semi-perimeter)}$$

Spherical Trigonometry

SPHERICAL TRIANGLE

A spherical triangle is the triangle enclosed by arcs of three great circles of a sphere.



The sum of the interior angles of a spherical is greater than 180° and less than 540°.

$$540° \geq (A + B + C) > 180°$$

Area of Spherical Triangle

The area of a spherical triangle of a sphere of radius R is:

$$A = \frac{\pi R^2 E}{180°}$$

Where E is the spherical excess in degrees and is given by the following equation:

$$E = A + B + C - 180°$$

or

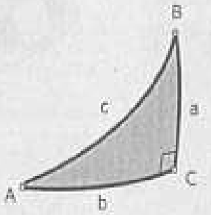
$$\tan\frac{E}{4} = \sqrt{\tan\frac{s}{2}\tan\frac{s-a}{2}\tan\frac{s-b}{2}\tan\frac{s-c}{2}}$$

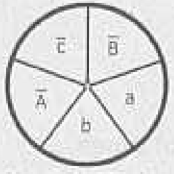
Where

$$s = \frac{a+b+c}{2}$$

For an arc of a great circle of the earth, the distance equivalent to 1 minute (0° 1') of the arc is one (1) nautical mile (6080 feet).

Right Spherical Triangle





Napier's Circle

The Napier's circle indicates the sides and angle of the triangle in consecutive order, not including the right angle. Where:

$$\bar{A} = 90 - A$$
$$\bar{B} = 90 - B$$
$$\bar{c} = 90 - c$$

Napier's Rules

1. In the Napier's circle, *the sine of any middle part is equal to product of the cosines of its opposite parts.* **SIN-COOP RULE.**

   If we take b as the middle part, its opposite parts are $\bar{c}$ and $\bar{B}$, then

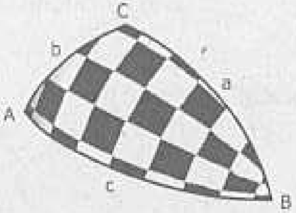
   $\sin b = \cos\bar{c} \times \cos\bar{B}$
   but $\cos\bar{c} = \cos(90 - c) = \sin c$,
   and $\cos\bar{B} = \sin B$,
   then $\mathbf{\sin b = \sin c \sin B}$

2. In the Napier's circle, *the sine of any middle part is equal to the product of the tangents of its adjacent parts.* **SIN-TAAD RULE.**

If we take $\overline{A}$ as the middle part, then is adjacent parts are $\bar{c}$ and b, then

$$\sin \overline{A} = \tan \bar{c} \times \tan b$$
$$\text{or } \cos A = \cot c \tan b$$

OBLIQUE SPHERICAL TRIANGLES



Law of Sines

$$\frac{\sin a}{\sin A} = \frac{\sin b}{\sin B} = \frac{\sin c}{\sin C}$$

Law of Cosines for Sides

$$\cos a = \cos b \cos c + \sin b \sin c \cos A$$
$$\cos b = \cos a \cos c + \sin a \sin c \cos B$$
$$\cos c = \cos a \cos b + \sin a \sin b \cos C$$

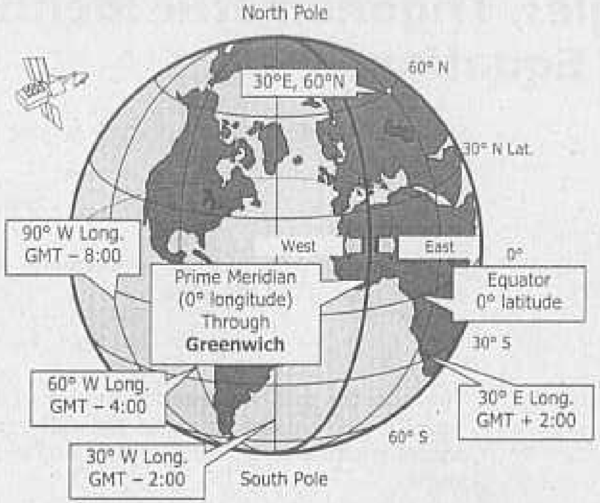
Law of Cosines for Angles

$$\cos A = -\cos B \cos C + \sin B \sin C \cos a$$
$$\cos B = -\cos A \cos C + \sin A \sin C \cos b$$
$$\cos C = -\cos A \cos B + \sin A \sin B \cos c$$

Napier's Analogies

$$\frac{\sin\frac{1}{2}(A-B)}{\sin\frac{1}{2}(A+B)} = \frac{\tan\frac{1}{2}(a-b)}{\tan\frac{1}{2}c}$$
$$\frac{\sin\frac{1}{2}(a-b)}{\sin\frac{1}{2}(a+b)} = \frac{\tan\frac{1}{2}(A-B)}{\cot\frac{1}{2}C}$$
$$\frac{\cos\frac{1}{2}(A-B)}{\cos\frac{1}{2}(A+B)} = \frac{\tan\frac{1}{2}(a+b)}{\tan\frac{1}{2}c}$$
$$\frac{\cos\frac{1}{2}(a-b)}{\cos\frac{1}{2}(a+b)} = \frac{\tan\frac{1}{2}(A+B)}{\cot\frac{1}{2}C}$$

THE TERRESTRIAL SPHERE



A **Meridian** is a great circle passing through the North and South Poles.

The **Equator** is a great circle perpendicular to the meridians.

The **Parallels** or **Latitudes** are small circles parallel to the equator. Its measure is from 0° to 90°.

The **Prime Meridian** is the meridian passing through Greenwich England.

GMT - Greenwich Mean Time

The earth rotates 360° in 24 hours or 15° every hour. Therefore, every 15° interval of longitude has a time difference of one hour.

The mean radius of the earth is 6373 km (3959) miles, usually taken as 6400 km (4000 miles).

One Nautical Mile = 6080 feet. This is the length of arc on the surface of the earth subtended by one (1) minute of an arc of the great circle.

The Philippines (Manila) is located at 121° 05′ E Longitude and 14° 36′ N latitude with time zone of GMT + 8:00.

Problems - Set 10 Angles, Trigonometric Identities and Equations

PROBLEM 10 - 1

Find the supplement of an angle whose compliment is 62°

A. 28°
B. 118°
C. 152°
D. none of these

PROBLEM 10 - 2
CE May 1994

A certain angle has a supplement five times its compliment. Find the angle.

A. 67.5°
B. 157.5°
C. 168.5°
D. 186°

PROBLEM 10 - 3

The sum of the two interior angles of the triangle is equal to the third angle and the difference of the two angles is equal to 2/3 of the third angle. Find the third angle.

A. 15°
B. 75°
C. 90°
D. 120°

PROBLEM 10 - 4
EE April 1994

The measure of 1½ revolutions counter-clockwise is:

A. 540°
B. 520°
C. +90°
D. -90°

PROBLEM 10 - 5
ECE Nov. 1997

The measure of 2.25 revolutions counterclockwise is:

A. -835 degrees
B. -810 degrees
C. 805 degrees
D. 810 degrees

PROBLEM 10 - 6

Solve for θ: $\sin\theta - \sec\theta + \csc\theta - \tan 2\theta = -0.0866$

A. 40°
B. 41°
C. 47°
D. 43°

PROBLEM 10 - 7
ME April 1996

What are the exact values of the cosine and tangent trigonometric functions of acute angle A, given that $\sin A = 3/7$?

A. $\cos A = 7/2\sqrt{10}$; $\tan A = 2\sqrt{10}/3$
B. $\cos A = 2\sqrt{10}/7$; $\tan A = 3\sqrt{10}/20$
C. $\cos A = 2\sqrt{10}/3$; $\tan A = 7/2\sqrt{10}$
D. $\cos A = 2\sqrt{10}/3$; $\tan A = 7\sqrt{10}/20$

PROBLEM 10 - 8

Given three angles A, B, and C whose sum is 180°. If the $\tan A + \tan B + \tan C = x$, find the value of $\tan A \times \tan B \times \tan C$.

A. $1 - x$
B. $\sqrt{x}$
C. $x/2$
D. x

PROBLEM 10 - 9
ME April 1998

What is the sine of 820°?

A. 0.984
B. -0.866
C. 0.866
D. -0.500

PROBLEM 10 - 10
EE April 1994

Csc 270° = ?

A. $-\sqrt{3}$
B. -1
C. $\sqrt{3}$
D. 1

PROBLEM 10 - 11
CE May 1994

If coversine θ is 0.134, find the value of θ.

A. 60°
B. 45°
C. 30°
D. 20°

PROBLEM 10 - 12
ME Oct. 1997

Solve for cos 72° if the given relationship is $\cos 2A = 2\cos^2 A - 1$.

A. 0.309
B. 0.258
C. 0.268
D. 0.315

PROBLEM 10 - 13
CE Nov. 1993

If $\sin 3A = \cos 6B$ then:

A. $A + B = 180°$
B. $A + 2B = 30°$
C. $A - 2B = 30°$
D. $A + B = 30°$

PROBLEM 10 - 14
CE Nov. 1992

Find the value of sin (arccos 15/17).

A. 8/17
B. 17/9
C. 8/21
D. 8/9

PROBLEM 10 - 15
CE May 1993

Find the value of $\cos[\arcsin(1/3) + \arctan(2/\sqrt{5})]$

A. $(2/9)(1 + \sqrt{10})$
B. $(2/9)(\sqrt{10} - 11)$
C. $(2/9)(\sqrt{10} + 1)$
D. $(2/9)(\sqrt{10} - 1)$

PROBLEM 10 - 16
CE May 1993

If $\sin 40° + \sin 20° = \sin\theta$, find the value of θ.

A. 20°
B. 80°
C. 120°
D. 60°

PROBLEM 10 - 17

How many different value of x from 0° to 180° for the equation $(2\sin x - 1)(\cos x + 1) = 0$?

A. 3
B. 0
C. 1
D. 2

PROBLEM 10 - 18
ME April 1997

For what value of θ (less than 2π) will the following equation be satisfied?

$$\sin^2\theta + 4\sin\theta + 3 = 0$$

A. π C. $3\pi/2$
B. $\pi/4$ D. $\pi/2$

PROBLEM 10 - 19
ECE Nov. 1997

Find the value of x in the equation $\csc x + \cot x = 3$.

A. $\pi/4$ C. $\pi/2$
B. $\pi/3$ D. $\pi/5$

PROBLEM 10 - 20

If $\sec^2 A$ is 5/2, the quantity $1 - \sin^2 A$ is equivalent to:

A. 2.5 C. 1.5
B. 0.6 D. 0.4

PROBLEM 10 - 21

Find $\sin x$ if $2\sin x + 3\cos x - 2 = 0$.

A. 1 & -5/13 C. 1 & 5/13
B. -1 & 5/13 D. -1 & -5/13

PROBLEM 10 - 22
ECE Nov. 1996

If $\sin A = 4/5$, A in quadrant II, $\sin B = 7/25$, B in quadrant I, find $\sin(A + B)$.

A. 3/5 C. 3/4
B. 2/5 D. 4/5

PROBLEM 10 - 23
ECE Nov. 1996

If $\sin A = 2.571x$, $\cos A = 3.06x$, and $\sin 2A = 3.939x$, find the value of x.

A. 0.350 C. 0.100
B. 0.250 D. 0.150

PROBLEM 10 - 24

If $\cos\theta = \sqrt{3}/2$, then find the value of x if $x = 1 - \tan^2\theta$.

A. -2 C. 4/3
B. -1/3 D. 2/3

PROBLEM 10 - 25

If $\sin\theta - \cos\theta = -1/3$, what is the value of $\sin 2\theta$?

A. 1/3 C. 8/9
B. 1/9 D. 4/9

PROBLEM 10 - 26

If $x\cos\theta + y\sin\theta = 1$ and $x\sin\theta - y\cos\theta = 3$, what is the relationship between x and y?

A. $x^2 + y^2 = 20$ C. $x^2 + y^2 = 16$
B. $x^2 - y^2 = 5$ D. $x^2 + y^2 = 10$

PROBLEM 10 - 27

If $\sin x + 1 / \sin x = \sqrt{2}$, then $\sin^2 x + 1 / \sin^2 x$ is equal to:

A. $\sqrt{2}$ C. 2
B. 1 D. 0

PROBLEM 10 - 28

The equation $2\sin\theta + 2\cos\theta - 1 = \sqrt{3}$ is:

A. an identity C. a conditional equation
B. a parametric equation D. a quadratic equation

PROBLEM 10 - 29

If $x + y = 90°$, then $\dfrac{\sin x \tan y}{\sin y \tan x}$ is equal to:

A. $\tan x$ C. $\cot x$
B. $\cos x$ D. $\sin x$

PROBLEM 10 - 30

If $\cos\theta = x / 2$ then $1 - \tan^2\theta$ is equal to:

A. $(2x^2 + 4) / x^2$ C. $(2x^2 - 4) / x$
B. $(4 - 2x^2) / x^2$ D. $(2x^2 - 4) / x^2$

PROBLEM 10 - 31
CE May 1996

Find the value in degrees of arccos (tan 24°).

A. 61.48 C. 63.56
B. 62.35 D. 60.84

PROBLEM 10 - 32
ECE April 1998

$\arctan[2\cos(\arcsin(\sqrt{3}/2)]$ is equal to:

A. $\pi/3$ C. $\pi/6$
B. $\pi/4$ D. $\pi/2$

PROBLEM 10 - 33
ECE March 1996

Solve for x in the equation: $\arctan(2x) + \arctan(x) = \pi/4$

A. 0.821 C. 0.281
B. 0.218 D. 0.182

PROBLEM 10 - 34
CE Nov. 1991,
Nov. 1993,
May 1994

Solve for x from the given trigonometric equation:

$$\arctan(1 - x) + \arctan(1 + x) = \arctan 1/8$$

A. 4 C. 8
B. 6 D. 2

PROBLEM 10 - 35

Solve for y if $y = (1/\sin x - 1/\tan x)(1 + \cos x)$

A. $\sin x$ C. $\tan x$
B. $\cos x$ D. $\sec^2 x$

PROBLEM 10 - 36
CE May 1994

Solve for x: $x = (\tan\theta + \cot\theta)^2 \sin^2\theta - \tan^2\theta$.

A. $\sin\theta$ C. 1
B. $\cos\theta$ D. 2

PROBLEM 10 - 37

Solve for x: $x = 1 - (\sin\theta - \cos\theta)^2$.

A. $\sin\theta\cos\theta$ C. $\cos 2\theta$
B. $-2\cos\theta$ D. $\sin 2\theta$

PROBLEM 10 - 38

Simplify $\cos^4\theta - \sin^4\theta$.

A. 2 C. $2\sin^2\theta + 1$
B. 1 D. $2\cos^2\theta - 1$

PROBLEM 10 - 39

Solve for x: $x = \dfrac{1-\tan^2 a}{1+\tan^2 a}$

A. $\cos a$
B. $\sin 2a$
C. $\cos 2a$
D. $\sin a$

PROBLEM 10 - 40

Which of the following is different from the others?

A. $2\cos 2x - 1$
B. $\cos 4x - \sin 4x$
C. $\cos 3x - \sin 3x$
D. $1 - 2\sin 2x$

PROBLEM 10 - 41
CE Nov. 1993

Find the value of y: $y = (1 + \cos 2\theta)\tan\theta$.

A. $\cos\theta$
B. $\sin\theta$
C. $\sin 2\theta$
D. $\cos 2\theta$

PROBLEM 10 - 42

The equation $2\sinh x\cosh x$ is equal to:

A. e^x
B. e^{-x}
C. $\sinh 2x$
D. $\cosh x$

PROBLEM 10 - 43
ME April 1996

Simplifying the equation $\sin^2\theta\,(1 + \cot^2\theta)$ gives:

A. 1
B. $\sin^2\theta$
C. $\sin^2\theta\sec^2\theta$
D. $\cos^2\theta$

PROBLEM 10 - 44
CE Nov. 1996

Find the value of $\sin(90° + A)$

A. $\cos A$
B. $-\cos A$
C. $\sin A$
D. $-\sin A$

PROBLEM 10 - 45
ME Oct. 1997

Which of the following expression is equivalent to $\sin 2\theta$?

A. $2\sin\theta\cos\theta$
B. $\sin^2\theta + \cos^2\theta$
C. $\cos^2\theta - \sin^2\theta$
D. $\sin\theta\cos\theta$

PROBLEM 10 - 46

If $\tan\theta = x^2$, which of the following is incorrect?

A. $\sin\theta = 1 / \sqrt{1+x^4}$
B. $\sec\theta = \sqrt{1+x^4}$
C. $\cos\theta = 1 / \sqrt{1+x^4}$
D. $\csc\theta = \sqrt{1+x^4}\ / x^2$

PROBLEM 10 - 47

In an isosceles right triangle, the hypotenuse is how much longer than its sides?

A. 2 times
B. $\sqrt{2}$ times
C. 1.5 times
D. none of these

PROBLEM 10 - 48
ECE April 1998

Find the angle in mils subtended by a line 10 yards long at a distance of 5000 yards.

A. 2.5 mils
B. 2 mils
C. 4 mils
D. 1 mil

PROBLEM 10 - 49
ECE April 1998

The angle or inclination of ascend of a road having 8.25% grade is ____ degrees.

A. 5.12 degrees
B. 4.72 degrees
C. 1.86 degrees
D. 4.27 degrees

PROBLEM 10 - 50

The sides of a right triangle is in arithmetic progression whose common difference if 6 cm. Its area is:

A. 216 cm²
B. 270 cm²
C. 360 cm²
D. 144 cm²

ANSWER SHEET

1. Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
2. Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
3. Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
4. Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
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57. Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
58. Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
59. Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
60. Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ

Solutions to Set 10 Angles, Trigonometric Identities and Equations

SOLUTION 10 - 1
Ans: C

Let x be the angle, then

Complement of x: $90° - x = 62°$, $x = 28°$
Supplement of x: $180° - 28° = \mathbf{152°}$

SOLUTION 10 - 2
Ans: A

Let x be the angle, then

Supplement of x: $180° - x$
Complement of x: $90° - x$
Then, $180° - x = 5(90° - x)$; $x = \mathbf{67.5°}$

SOLUTION 10 - 3
Ans: C

Let x, y, and z be 1st, 2nd, and 3rd angles respectively
Then;

$x + y + z = 180°$ → (1)
$x + y = z$ → (2)
$x - y = (2/3)z$ → (3)

Substitute Eq. (2) to Eq. (1):
$z + z = 180°$, $z = \mathbf{90°}$

SOLUTION 10 - 4
Ans: A

One revolution is 360°.
$1.5(360°) = \mathbf{540°}$

SOLUTION 10 - 5
Ans: D

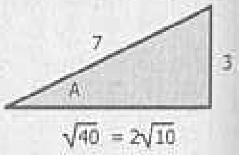
Angle = $2.25(360) = \mathbf{+810°}$ (+ for counterclockwise)

SOLUTION 10 - 6
Ans: D

This problem can be solved by substituting each of the choices to the given equation, $\theta = \mathbf{43°}$

SOLUTION 10 - 7
Ans: B

Given: $\sin A = 3/7$
From the triangle shown:



$$\cos A = \frac{2\sqrt{10}}{7}$$

$$\tan A = \frac{3}{2\sqrt{10}} \times \frac{\sqrt{10}}{\sqrt{10}} = \frac{3\sqrt{10}}{20}$$

SOLUTION 10 - 8
Ans: D

$A + B + C = 180°$
$A + B = 180° - C$, taking the tangent of both sides

$$\tan(A + B) = \tan(180° - C)$$

$$\frac{\tan A + \tan B}{1 - \tan A \tan B} = -\tan C$$
$$\tan A + \tan B = -\tan C + \tan A \tan B \tan C$$

Simplify:
$$\tan A \tan B \tan C = \tan A + \tan B + \tan C = x$$

SOLUTION 10 - 9
Ans: A

Using the calculator, $\sin 820° = \mathbf{0.984}$

SOLUTION 10 - 10
Ans: B

Using calculator, $\csc 270° = \dfrac{1}{\sin 270°} = \dfrac{1}{-1} = \mathbf{-1}$

SOLUTION 10 - 11
Ans: A

Note: coversine = versed cosine $= 1 - \sin\theta$
$1 - \sin\theta = 0.134$; $\sin\theta = 0.866$; $\theta = \mathbf{60°}$

SOLUTION 10 - 12
Ans: A

Using the calculator, $\cos 72° = \mathbf{0.309}$

SOLUTION 10 - 13
Ans: B

$\sin 3A = \cos 6B$ Note: $\sin\theta = \cos(90 - \theta)$

$\cos(90 - 3A) = \cos 6B$ (But if $\cos x = \cos y$, $x = y$)

$90° - 3A = 6B$
$3A + 6B = 90°$ Divide both sides by 3

$A + 2B = \mathbf{30°}$

SOLUTION 10 - 14
Ans: A

Using the calculator, arccos $15/17 = 20.072487$
Sin $20.072487 = 0.470588 = \mathbf{8/17}$

SOLUTION 10 - 15
Ans: D

Using calculator:
$\arcsin(1/3) = 19.471°$
$\arctan(2/\sqrt{5}) = 41.8103°$
$\cos(19.471° + 41.8103°) = 0.480506 = \mathbf{\frac{2}{9}(\sqrt{10} - 1)}$

SOLUTION 10 - 16
Ans: B

$\sin\theta = \sin 40° + \sin 20° = 0.6427876 + 0.34202$
$\sin\theta = 0.98480$; $\theta = \mathbf{80°}$

SOLUTION 10 - 17
Ans: A

$(2\sin x - 1)(\cos x + 1) = 0$

Then;
$\sin x = ½$ and $x = \underline{30°}$ & $\underline{120°}$
$\cos x = -1$ and $x = \underline{180°}$

Therefore, there are **3** values of x.

SOLUTION 10 - 18
Ans: C

By substitution, using $\theta = 3\pi/2 = 270°$:

$$\sin^2 270° + 4 \sin 270° + 3 = 0$$

SOLUTION 10 - 19
Ans: D

This can be solve by substitution: $x = \pi/5 = 36°$

$\csc 36° + \cot 36° = 3.0777 \approx 3$

SOLUTION 10 - 20
Ans: D

$\sec^2 A = 5/2 = 1 / \cos^2 A$
$\cos^2 A = 2/5$

$1 - \sin^2 A = \cos^2 A = 2/5 = \mathbf{0.4}$

SOLUTION 10 - 21
Ans: A

$2\sin x + 3\cos x - 2 = 0$
$3 \cos x = 2 - 2\sin x$
$3\cos x = 2(1 - \sin x)$ square both sides

$$9\cos^2 x = 4(1 - \sin x)^2$$
$$9(1 - \sin^2 x) = 4(1 - \sin x)^2$$
$$9(1 - \sin x)(1 + \sin x) = 4(1 - \sin x)^2$$
$$9(1 - \sin x)(1 + \sin x) - 4(1 - \sin x)^2 = 0$$
$$(1 - \sin x)(9 + 9\sin x - 4 + 4\sin x) = 0$$
$$(1 - \sin x)(13\sin x + 5) = 0$$
$\sin x = 1$ and $-5/13$

SOLUTION 10 - 22
Ans: A

$\sin A = 4/5$; $A = 53.13°$ and $\underline{126.87°}$

$\sin B = 7/25$; $B = \underline{16.26°}$

then $\sin (A + B) = \sin (126.87 + 16.26) = 0.6$
$\sin (A + B) = \mathbf{3/5}$

SOLUTION 10 - 23
Ans: B

Given: $\sin A = 2.571x$
$\cos A = 3.06x$

Dividing the two equations:

$$\frac{\sin A}{\cos A} = \frac{2.571x}{3.06x} = 0.8402$$

$\tan A = 0.8402$; $A = 40.037°$
$\sin 40.037° = 2.571x$; $x = \mathbf{0.250}$

SOLUTION 10 - 24
Ans: D

If $\cos \theta = \sqrt{3}/2$, then $\sec \theta = 2/\sqrt{3}$

$x = 1 - \tan^2 \theta = 1 - (\sec^2 \theta - 1) = 2 - \sec^2 \theta$
$x = 2 - (2/\sqrt{3})^2 = 2 - 4/3 = \mathbf{2/3}$

Another Solution:

$\cos \theta = \sqrt{3}/2$; $\theta = 30°$
$x = 1 - \tan^2 30° = 0.6666$ or $\mathbf{2/3}$

SOLUTION 10 - 25
Ans: C

$\sin \theta - \cos \theta = -1/3$

Square both sides
$\sin^2 \theta - 2 \sin \theta \cos \theta + \cos^2 \theta = 1/9$
$\sin^2 \theta + \cos^2 \theta - \sin 2\theta = 1/9$

But $\sin^2 \theta + \cos^2 \theta = 1$:
$1 - 1/9 = \sin 2\theta = \mathbf{8/9}$

SOLUTION 10 - 26
Ans: D

$x \cos \theta + y \sin \theta = 1$ square both sides

$$x^2 \cos^2 \theta + 2xy \sin \theta \cos \theta + y^2 \sin^2 \theta = 1 \quad \rightarrow (1)$$

$x \sin \theta - y \cos \theta = 3$ square both sides

$$x^2 \sin^2 \theta - 2xy \sin \theta \cos \theta + y^2 \cos^2 \theta = 9 \quad \rightarrow (2)$$

Add: Eq. (1) + Eq. (2)
$x^2 \cos^2 \theta + x^2 \sin^2 \theta + y^2 \sin^2 \theta + y^2 \cos^2 \theta = 10$
$x^2 (\sin^2 \theta + \cos^2 \theta) + y^2 (\sin^2 \theta + \cos^2 \theta) = 10$ but $\sin^2 \theta + \cos^2 \theta = 1$
$\mathbf{x^2 + y^2 = 10}$

SOLUTION 10 - 27
Ans: D

$\sin x + 1/\sin x = \sqrt{2}$ square both sides

$\sin^2 x + 2 \sin x (1/\sin x) + 1/\sin^2 x = 2$
$\sin^2 x + 2 + 1/\sin^2 x = 2$
$\sin^2 x + 1/\sin^2 x = 0$

SOLUTION 10 - 28
Ans: C

$2\sin \theta + 2\cos \theta - 1 = \sqrt{3}$

This equation will be satisfied only for some specific values of θ, therefore it is *a conditional equation*.

SOLUTION 10 - 29
Ans: C

$x + y = 90°$; $y = 90 - x$

$$\frac{\sin x \tan y}{\sin y \tan x} = \frac{\sin x \tan(90 - x)}{\sin(90 - x) \tan x}$$

But $\tan (90 - x) = \cot x$ and $\sin (90 - x) = \cos x$

$$\frac{\sin x \tan y}{\sin y \tan x} = \frac{\sin x \cot x}{\cos x \tan x} = \frac{\tan x \cot x}{\tan x}$$
$$= \cot x$$

SOLUTION 10 - 30
Ans: D

If $\cos\theta = x/2$, then $\sec\theta = 2/x$

$$1 - \tan^2\theta = 1 - (\sec^2\theta - 1) = 1 - (2/x)^2 + 1 = 2 - 4/x^2$$
$$= (2x^2 - 4)/x^2$$

SOLUTION 10 - 31
Ans: C

This problem can be solved using calculator
$\arccos(\tan 24°) = 63.56°$

SOLUTION 10 - 32
Ans: B

Using calculator:

$$\arctan[2\cos(\arcsin(\sqrt{3}/2)] = \arctan[2\cos 60°]$$
$$= \arctan(1) = 45°$$
$$= \pi/4$$

SOLUTION 10 - 33
Ans: C

$\arctan(2x) + \arctan(x) = \pi/4$

Let $A = \arctan(x)$; $\tan A = x$
Let $B = \arctan(2x)$; $\tan B = 2x$

$B + A = \pi/4 = 45°$ Taking the tangents of both sides:

$$\tan(A + B) = \tan(45°)$$
$$\frac{\tan A + \tan B}{1 - \tan A\tan B} = 1$$
$$\tan A + \tan B = 1 - \tan A\tan B$$
$$x + 2x = 1 - (x)(2x)$$
$$2x^2 + 3x - 1 = 0$$
$$x = \frac{-3 \pm \sqrt{(3)^2 - 4(2)(-1)}}{2(2)} = \mathbf{0.281}$$

Note: This can be solved by trial and error using the choices.

SOLUTION 10 - 34
Ans: A

Let $A = \arctan(1 - x)$, then $\tan A = 1 - x$
Let $B = \arctan(1 + x)$, then $\tan B = 1 + x$
Let $C = \arctan 1/8$, then $\tan C = 1/8$

$A + B = C$ Taking the tangents of both sides:

$$\tan(A + B) = \tan C$$
$$\frac{\tan A + \tan B}{1 - \tan A\tan B} = \tan C$$
$$\frac{(1-x)+(1+x)}{1-(1-x)(1+x)} = \frac{1}{8}$$

$$\frac{2}{1-(1-x^2)} = \frac{1}{8}$$
$$16 = 1 - (1 - x^2) = x^2;\ x = \pm 4$$

Note: This can be solved by trial and error using the choices.

SOLUTION 10 - 35
Ans: A

$$y = \left(\frac{1}{\sin x} - \frac{1}{\tan x}\right)(1 + \cos x) = \left(\frac{1}{\sin x} - \frac{\cos x}{\sin x}\right)(1 + \cos x)$$
$$y = \left(\frac{1 - \cos x}{\sin x}\right)(1 + \cos x) = \frac{1 - \cos^2 x}{\sin x} = \frac{\sin^2 x}{\sin x} = \mathbf{\sin x}$$

SOLUTION 10 - 36
Ans: C

$$x = (\tan\theta + \cot\theta)\sin^2\theta - \tan^2\theta$$
$$= \left(\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right)^2 \sin^2\theta - \frac{\sin^2\theta}{\cos^2\theta}$$
$$= \left(\frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}\right)^2 \sin^2\theta - \frac{\sin^2\theta}{\cos^2\theta}$$
$$= \frac{1^2}{\sin^2\theta\cos^2\theta}\sin^2\theta - \frac{\sin^2\theta}{\cos^2\theta} = \frac{1 - \sin^2\theta}{\cos^2\theta}$$
$$= \frac{\cos^2\theta}{\cos^2\theta} = \mathbf{1}$$

SOLUTION 10 - 37
Ans: D

$$x = 1 - (\sin\theta - \cos\theta)^2$$
$$= 1 - (\sin^2\theta - 2\sin\theta\cos\theta + \cos^2\theta)$$
$$= 1 - [(\sin^2\theta + \cos^2\theta) - 2\sin\theta\cos\theta]$$
$$= 1 - [1 - 2\sin\theta\cos\theta]$$
$$x = \mathbf{\sin 2\theta}$$

SOLUTION 10 - 38
Ans: D

$$\cos^4\theta - \sin^4\theta = (\cos^2\theta + \sin^2\theta)(\cos^2\theta - \sin^2\theta)$$
$$= (1)[\cos^2\theta - (1 - \cos^2\theta)]$$
$$= \cos^2\theta - 1 + \cos^2\theta$$
$$= \mathbf{2\cos^2\theta - 1}$$

Another solution: (Set $\theta = 1°$)
Note: Whenever we simplify any expression, the resulting expression should be the same (or identical) with the given expression.

$\cos^4 1° - \sin^4 1° = 0.99390827$ (choices *A* and *B* are incorrect)

Choice *C*: $2\sin^2 1° + 1 = 1.000609173$ (incorrect)
Choice *D*: $2\cos^2 1° - 1 = 0.99390827$ **(OK)**

SOLUTION 10 - 39
Ans: C

$$x = \frac{1-\tan^2 a}{1+\tan^2 a} = \frac{1}{\sec^2 a}\left(1-\frac{\sin^2 a}{\cos^2 a}\right)$$

$$x = \cos^2 a\left(1-\frac{\sin^2 a}{\cos^2 a}\right)$$

$$x = \cos^2 a - \sin^2 a = \mathbf{\cos 2a}$$

SOLUTION 10 - 40
Ans: C

A. $\underline{2\cos^2 x - 1}$
B. $\cos^4 x - \sin^4 x = (\cos^2 x - \sin^2 x)(\cos^2 x - \sin^2 x)$
$= \cos^2 x - (1 - \cos^2 x)$
$= \underline{2\cos^2 x - 1}$
C. $\cos^3 x - \sin^3 x = (\cos x - \sin x)(\cos^2 x + \cos x \sin x + \sin^2 x)$
D. $1 - 2\sin^2 x = 1 - 2(1 - \cos^2 x)$
$= 1 - 2 + 2\cos^2 x$
$= \underline{2\cos^2 x - 1}$

Note: this can also be solved by assuming a value of x and substitute to each of the choices.

SOLUTION 10 - 41
Ans: C

$$y = (1 + \cos 2\theta)\tan\theta = (1 + \cos^2\theta - \sin^2\theta)\frac{\sin\theta}{\cos\theta}$$

$$y = [(1 - \sin^2\theta) + \cos^2\theta]\frac{\sin\theta}{\cos\theta}$$

$$y = (\cos^2\theta + \cos^2\theta)\frac{\sin\theta}{\cos\theta} = 2\sin\theta\cos\theta = \mathbf{\sin 2\theta}$$

SOLUTION 10 - 42
Ans: C

$2\sinh x\cosh x = \mathbf{\sinh 2x}$

SOLUTION 10 - 43
Ans: A

$\sin^2\theta\,(1 + \cot^2\theta) = \sin^2\theta\,(1 + \cos^2\theta / \sin^2\theta)$
$= \sin^2\theta + \cos^2\theta = \mathbf{1}$

SOLUTION 10 - 44
Ans: A

$\sin(90° + A) = \sin 90°\cos A + \cos 90°\sin A$
but $\cos 90° = 0$ and $\sin 90° = 1$

$\sin(90° + A) = \mathbf{\cos A}$

This problem can also be solved by assuming a value of A, say 1°.

Then $\sin(90 + 1) = 0.999847695$

Substitute $A = 1°$ in the choices:
Choice A: $\cos 1° = 0.99984765$ (OK)

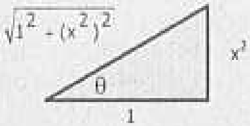
SOLUTION 10 - 45
Ans: A

$\sin 2\theta = \mathbf{2\sin\theta\cos\theta}$

SOLUTION 10 - 46
Ans: A

$\tan\theta = x^2 = x^2/1$

From the right triangle shown:



$\sin\theta = x^2 / \sqrt{1+x^4}$
$\cos\theta = 1 / \sqrt{1+x^4}$
$\sec\theta = \sqrt{1+x^4}$
$\csc\theta = \sqrt{1+x^4} / x^2$

The incorrect relationship is choice A:

SOLUTION 10 - 47
Ans: B

For a right triangle, $c^2 = a^2 + b^2$, and for an isosceles right triangle, $a = b$

Then, $c = \sqrt{a^2 + a^2} = a\sqrt{2}$

Thus the hypotenuse is $\sqrt{2}$ times longer than the sides.

SOLUTION 10 - 48
Ans: B

The angle is:
$\tan\theta = 10 / 5000$
$\theta = 0.11459° \times 6400 \text{ mil} / 360° = \mathbf{2.04\ mil}$

SOLUTION 10 - 49
Ans: B

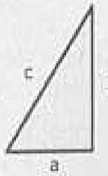
The grade of the road is the tangent of the angle of inclination.
$\tan\theta = 8.25\% = 0.0825$

$\theta = \mathbf{4.716°}$

SOLUTION 10 - 50
Ans: A

Let x be the smallest side of the triangle, then
$a = x$, $b = x + 6$, and $c = x + 12$.

By Pythagorean Theorem:



$c^2 = a^2 + b^2$
$(x + 12)^2 = x^2 + (x + 6)^2$
$x^2 + 24x + 144 = x^2 + x^2 + 12x + 36$
$x^2 - 12x - 108 = 0,$
$(x - 18)(x + 6) = 0,\ x = 18$ cm

$a = 18,$
$b = 18 + 6 = 24$
$c = 18 + 12 = 30$ cm.

Area of triangle = ½ $a \times b$ = ½ (18)(24)
Area of triangle = **216 cm²**

Problems - Set 11 Triangles, Angle of Elevation & Depression

PROBLEM 11 - 1
ECE March 1996

The hypotenuse of a right triangle is 34 cm. Find the length of the shortest leg if it is 14 cm shorter than the other leg.

A. 15 cm   C. 17 cm
B. 16 cm   D. 18 cm

PROBLEM 11 - 2
CE Nov. 1998

A truck travels from point M northward for 30 min. then eastward for one hour, then shifted N 30° W. If the constant speed is 40 kph, how far directly from M, in km. will be it after 2 hours?

A. 43.5   C. 47.9
B. 45.2   D. 41.6

PROBLEM 11 - 3

Two sides of a triangle measures 6 cm. and 8 cm. and their included angle is 40°. Find the third side.

A. 5.144 cm   C. 4.256 cm
B. 5.263 cm   D. 5.645 cm

PROBLEM 11 - 4

Given a triangle: $C = 100°$, $a = 15$, $b = 20$. Find c:

A. 34   C. 43
B. 27   D. 35

PROBLEM 11 - 5
ME Oct. 1997

Given angle $A = 32°$, angle $B = 70°$, and side $c = 27$ units. Solve for side a of the triangle.

A. 24 units   C. 14.63 units
B. 10 units   D. 12 units

PROBLEM 11 - 6
ME April 1997

In a triangle, find the side c if angle $C = 100°$, side $b = 20$, and side $a = 15$.

A. 28   C. 29
B. 27   D. 26

PROBLEM 11 - 7
CE Nov. 1994

In triangle ABC, $A = 45$ degrees and angle $C = 70$ degrees. The side opposite angle C is 40 m. long. What is the side opposite angle A?

A. 29.10 meters   C. 30.10 meters
B. 32.25 meters   D. 31.25 meters

PROBLEM 11 - 8
CE May 1996

Two sides of a triangle are 50 m. and 60 m. long. The angle included between these sides is 30 degrees. What is the interior angle (in degrees) opposite the longest side?

A. 92.74   C. 94.74
B. 93.74   D. 91.74

PROBLEM 11 - 9
CE May 1999

The sides of a triangle ABC are $AB = 15$ cm, $BC = 18$ cm, and $CA = 24$ cm. Determine the distance from the point of intersection of the angular bisectors to side AB.

A. 5.21 cm   C. 4.73 cm
B. 3.78 cm   D. 6.25 cm

PROBLEM 11 - 10
CE Nov. 1995

If $AB = 15$ m, $BC = 18$ m and $CA = 24$ m, find the point intersection of the angular bisector from the vertex C.

A. 11.3   C. 13.4
B. 12.1   D. 14.3

PROBLEM 11 - 11
CE May 1995

In triangle ABC, angle $C = 70$ degrees; angle $A = 45$ degrees; $AB = 40$ m. What is the length of the median drawn from vertex A to side BC?

A. 36.8 meters   C. 36.3 meters
B. 37.1 meters   D. 37.4 meters

PROBLEM 11 - 12

The area of the triangle whose angles are 61°9'32", 34°14'46", and 84°35'42" is 680.60. The length of the longest side is:

A. 35.53   C. 52.43
B. 54.32   D. 62.54

PROBLEM 11 - 13

Given a triangle ABC whose angles are $A = 40°$, $B = 95°$ and side $b = 30$ cm. Find the length of the bisector of angle C.

A. 21.74 cm   C. 20.45 cm
B. 22.35 cm   D. 20.98 cm

PROBLEM 11 - 14
EE Oct. 1997

The sides of a triangular lot are 130 m, 180 m, and 190 m. The lot is to be divided by a line bisecting the longest side and drawn from the opposite vertex. The length of this dividing line is:

A. 100 meters   C. 125 meters
B. 130 meters   D. 115 meters

PROBLEM 11 - 15

From a point outside of an equilateral triangle, the distance to the vertices are 10m, 10m, and 18m. Find the dimension of the triangle.

A. 25.63   C. 19.94
B. 45.68   D. 12.25

PROBLEM 11 - 16
ECE April 1998

Points A and B 1000m apart are plotted on a straight highway running East and West. From A, the bearing of a tower C is 32 degrees N of W and from B the bearing

of C is 26 degrees N of E. Approximate the shortest distance of tower C to the highway.
A. 264 meters C. 284 meters
B. 274 meters D. 294 meters

PROBLEM 11 - 17

An airplane leaves an aircraft carrier and flies South at 350 mph. The carrier travels S 30° E at 25 mph. If the wireless communication range of the airplane is 700 miles, when will it lose contact with the carrier?
A. after 4.36 hours C. after 2.13 hours
B. after 5.57 hours D. after 4.54 hours

PROBLEM 11 - 18

A statue 2 meters high stands on a column that is 3 meters high. An observer in level with the top of the statue observed that the column and the statue subtend the same angle. How far is the observer from the statue?
A. $5\sqrt{2}$ meters C. 20 meters
B. $2\sqrt{5}$ meters D. $\sqrt{10}$ meters

PROBLEM 11 - 19

From the top of a building 100 m high, the angle of depression of a point *A* due East of it is 30°. From a point *B* due South of the building, the angle of elevation of the top is 60°. Find the distance AB.
A. $100 + 3\sqrt{30}$ C. $100\sqrt{30}/3$
B. $200 - \sqrt{30}$ D. $100\sqrt{3}/30$

PROBLEM 11 - 20

An observer found the angle of elevation of the top of the tree to be 27°. After moving 10m closer (on the same vertical and horizontal plane as the tree), the angle of elevation becomes 54°. Find the height of the tree.
A. 8.65 meters C. 7.02 meters
B. 7.53 meters D. 8.09 meters

PROBLEM 11 - 21

From a point *A* at the foot of the mountain, the angle of elevation of the top *B* is 60°. After ascending the mountain one (1) mile at an inclination of 30° to the horizon, and reaching a point *C*, an observer finds that the angle *ACB* is 135°. The height of the mountain in feet is:
A. 14386 C. 11672
B. 12493 D. 11225

PROBLEM 11 - 22

A 50-meter vertical tower casts a 62.3-meter shadow when the angle of elevation of the sun is 41.6°. The inclination of the ground is:
A. 4.72° C. 5.63°
B. 4.33° D. 5.17°

PROBLEM 11 - 23

A vertical pole is 10 m from a building. When the angle of elevation of the sun is 45°, the pole cast a shadow on the building 1 m high. Find the height of the pole.
A. 0 meter C. 12 meters
B. 11 meters D. 13 meters

PROBLEM 11 - 24
ECE April 1995

A pole cast a shadow of 15 meters long when the angle of elevation of the sun is 61°. If the pole has leaned 15° from the vertical directly toward the sun, what is the length of the pole?
A. 52.43 meters C. 53.25 meters
B. 54.23 meters D. 53.24 meters

PROBLEM 11 - 25
ME April 1997

An observer wishes to determine the height of a tower. He takes sights at the top of the tower from *A* and *B*, which are 50 ft. apart, at the same elevation on a direct line with the tower. The vertical angle at point *A* is 30° and at point *B* is 40°. What is the height of the tower?
A. 85.6 feet C. 110.29 feet
B. 143.97 feet D. 92.54 feet

PROBLEM 11 - 26
CE Nov. 1997

From the top of tower *A*, the angle of elevation of the top of the tower *B* is 46°. From the foot of tower *B* the angle of elevation of the top of tower *A* is 28°. Both towers are on a level ground. If the height of tower *B* is 120m, how high is tower *A*?
A. 38.6 m C. 44.1 m
B. 42.3 m D. 40.7 m

PROBLEM 11 - 27
CE Nov. 1997

Points *A* and *B* are 100 m apart and are on the same elevation as the foot of a building. The angles of elevation of the top of the building from points *A* and *B* are 21° and 32°, respectively. How far is *A* from the building?
A. 271.6 m C. 259.2 m
B. 265.4 m D. 277.9 m

PROBLEM 11 - 28
ECE April 1998

A man finds the angle of elevation of the top of a tower to be 30 degrees. He walks 85 m. nearer the tower and finds its angle of elevation to be 60 degrees. What is the height of the tower?
A. 76.31 meters C. 73.31 meters
B. 73.61 meters D. 73.16 meters

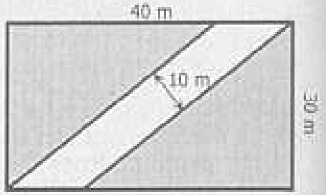
PROBLEM 11 - 29

The angle of elevation of a point C from a point *B* is 29°42'; the angle of elevation of C from another point *A* 31.2 m directly below *B* is 59°23'. How high is C from the horizontal line through *A*?
A. 47.1 meters C. 35.1 meters
B. 52.3 meters D. 66.9 meters

PROBLEM 11 - 30

A rectangular piece of land 40m × 30 m is to be crossed diagonally by a 10-m wide roadway as shown. If the land cost P1,500.00 per square meter, the cost of the roadway is:

A. P401.10
B. P60,165.00
C. P601,650.00
D. P651,500.00



PROBLEM 11 - 31

A man improvises a temporary shield from the sun using a triangular piece of wood with dimensions of 1.4m, 1.5m, and 1.3m. With the longer side lying horizontally on the ground, he props up the other corner of the triangle with a vertical pole 0.9m long. What would be the area of the shadow on the ground when the sun is vertically overhead?

A. 0.5 m²
B. 0.75 m²
C. 0.84m²
D. 0.95m²

PROBLEM 11 - 32

A rectangular piece of wood 4cm × 12cm tall is tilted at an angle of 45°. Find the vertical distance between the lower corner and the upper corner.

A. $4\sqrt{2}$ cm
B. $2\sqrt{2}$ cm
C. $8\sqrt{2}$ cm
D. $6\sqrt{2}$ cm

PROBLEM 11 - 33

A clock has a dial face 12 inches in radius. The minute hand is 9 inches long while the hour hand is 6 inches long. The plane of rotation of the minute hand is 2 inches above the plane of rotation of the hour hand. Find the distance between the tips of the hands at 5:40 AM.

A. 9.17 inches
B. 8.23 inches
C. 10.65 inches
D. 11.25 inches

PROBLEM 11 - 34

If the bearing of *A* from *B* is S 40° W, then the bearing of *B* from *A* is:

A. N 40° E
B. N 40° W
C. N 50° E
D. N 50° W

PROBLEM 11 - 35

A plane hillside is inclined at an angle of 28° with the horizontal. A man wearing skis can climb this hillside by following a straight path inclined at an angle of 12° to the horizontal, but one without skis must follow a path inclined at an angle of only 5° with the horizontal. Find the angle between the directions of the two paths.

A. 13.21°
B. 18.74°
C. 15.56°
D. 17.22°

PROBLEM 11 - 36

Calculate the area of a spherical triangle whose radius is 5 m and whose angles are 40°, 65°, and 110°.

A. 12.34 sq. m.
B. 14.89 sq. m.
C. 16.45 sq. m.
D. 15.27 sq. m.

PROBLEM 11 - 37
CE May 1997

A right spherical triangle has an angle $C = 90°$, $a = 50°$, and $c = 80°$. Find the side b.

A. 45.33°
B. 78.66°
C. 74.33°
D. 75.89°

PROBLEM 11 - 38

If the time is 8:00 a.m. GMT, what is the time in the Philippines, which is located at 120° East longitude?

A. 6 p.m.
B. 4 a.m.
C. 4 p.m.
D. 6 a.m.

PROBLEM 11 - 39

An airplane flew from Manila (14° 36'N, 121° 05'E) at a course of S 30° E maintaining a certain altitude and following a great circle path. If its groundspeed is 350 knots, after how many hours will it cross the equator?

A. 2.87 hours
B. 2.27 hours
C. 3.17 hours
D. 3.97 hours

PROBLEM 11 - 40

Find the distance in nautical miles between Manila and San Francisco. Manila is located at 14° 36' N latitude and 121° 05' E longitude. San Francisco is situated at 37° 48' N latitude and 122° 24' W longitude.

A. 7856.2 nautical miles
B. 5896.2 nautical miles
C. 6326.2 nautical miles
D. 6046.2 nautical miles

PROBLEM 11 - 41

A at certain point on the ground, the tower at the top of a 20-m high building subtends an angle of 45°. At another point on the ground 25 m closer the building, the tower subtends an angle of 45°. Find the height of the tower.

A. 124.75 m
B. 87.45 m
C. 154.32 m
D. 101.85 m

PROBLEM 11 - 42

A wooden flagpole is imbedded 3 m deep at corner *A* of a concrete horizontal slab *ABCD*, square in form and measuring 20 ft on a side. A storm broke the flagpole at a point one meter above the slab and inclined toward corner *C* in the direction of the diagonal *AC*. The vertical angles observed at the center of the slab and at corner *C* to the tip of the flagpole were 65° and 35°, respectively. What is the total length of the flagpole above the slab in yards?

A. 5.61
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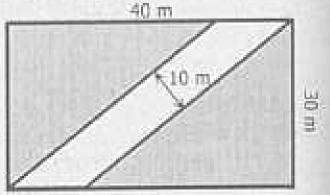

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A. 5.61 C. 4.32
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PROBLEM 11 - 43

From the third floor window of a building, the angle of depression of an object on the ground is 35° 58′, while from a sixth floor window, 9.75 m above the first point of observation the angle of depression is 58° 35′. How far is the object from the building?

A. 11.9 m C. 9.3 m
B. 10.7 m D. 15.3 m

PROBLEM 11 - 44
CE May 2003

The sides of a triangle are 18 cm, 24 cm and 34 cm, respectively. Find the length of the median to the 24-cm side, in cm.

A. 24.4 C. 23.4
B. 21.9 D. 20.4

PROBLEM 11 - 45
CE Nov. 2002

In the spherical triangle *ABC*, *A* = 116° 19′, *B* = 55° 30′, and C = 80° 37′. What is the value of side *a*.

A. 115.57° C. 119.64°
B. 113.21° D. 115.65°

ANSWER SHEET

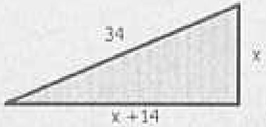
1.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
2.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
3.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
4.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
5.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
6.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
7.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
8.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
9.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
10.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
11.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
12.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
13.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
14.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
15.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
16.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
17.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
18.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
19.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
20.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
21.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
22.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
23.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
24.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
25.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
26.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
27.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
28.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
29.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
30.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
31.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
32.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
33.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
34.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
35.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
36.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
37.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
38.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
39.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
40.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
41.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
42.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
43.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
44.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
45.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ

Solutions to Set 11 Triangles, Angle of Elevation & Depression

SOLUTION 11 - 1
Ans: B

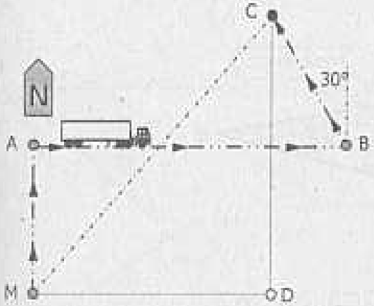
For a right triangle:

$$c^2 = a^2 + b^2$$
$$34^2 = x^2 + (x + 14)^2$$
$$1156 = x^2 + x^2 + 28x + 196$$
$$2x^2 + 28x - 960 = 0$$
$$x = \frac{28 \pm \sqrt{28^2 - 4(2)(-960)}}{2(2)} = \mathbf{16\ cm}$$



SOLUTION 11 - 2
Ans: C

$$MA = 40(30/60) = 20\text{ Km}$$
$$AB = 40(1) = 40\text{ Km}$$



After two hours, t_{BC} = 30 min

$$BC = 40(30/60) = 20\text{ km}$$

In Triangle *MDC*:

$$MD = AB - BC\sin 30°$$
$$MD = 40 - 20\sin 30° = 30\text{ km}$$
$$CD = MA + BC\cos 30°$$
$$CD = 20 + 20\cos 30° = 37.32\text{ km}$$
$$CM = \sqrt{MD^2 + CD^2}$$
$$CM = \sqrt{(30)^2 + (37.32)^2}$$
$$CM = \mathbf{47.88\ km}$$

SOLUTION 11 - 3
Ans: A

$$c^2 = a^2 + b^2 - 2ab\cos\theta$$
$$c^2 = 6^2 + 8^2 - 2(6)(8)\cos 40°; \quad c = \mathbf{5.144\ cm}$$

SOLUTION 11 - 4
Ans: B

By cosine law:

$$c^2 = a^2 + b^2 - 2ab\cos C$$
$$c^2 = 15^2 + 20^2 - 2(15)(20)\cos 100°; \quad c = \mathbf{27}$$

SOLUTION 11 - 5
Ans: C

For the given triangle, angle *C* = 180° - 32° - 70° = 78°

By sine law:

$$\frac{a}{\sin A} = \frac{c}{\sin C}; \quad \frac{a}{\sin 32°} = \frac{27}{\sin 78°}$$
$$a = \mathbf{14.63\ units}$$

SOLUTION 11 - 6
Ans: B

By cosine law:

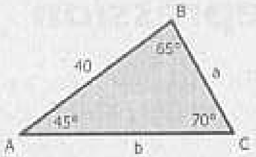
$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$c^2 = (15)^2 + (20)^2 - 2(15)(20)\cos 100°;\ c = \mathbf{27}$$

SOLUTION 11 - 7
Ans: C

$$\frac{a}{\sin 45°} = \frac{40}{\sin 70°}$$

a = **30.1 m**



SOLUTION 11 - 8
Ans: B

Solving for the third side by cosine law:

$$c^2 = 50^2 + 60^2 - 2(50)(60)\cos 30°$$

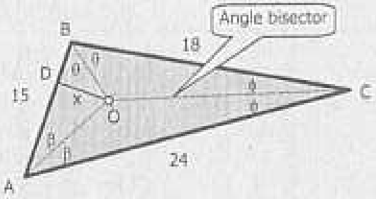
$$c = 30.064$$

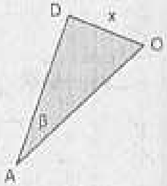
Solving for the angle opposite the 60-m side by sine law:

$$\frac{60}{\sin\theta} = \frac{30.064}{\sin 30°}$$

$\sin\theta = 0.9978;\ \theta = 86.26°$ & **93.74°**

SOLUTION 11 - 9
Ans: C





In triangle ABC:

$$18^2 = 15^2 + 24^2 - 2(15)(24)\cos A$$

$$A = 48.5°;\quad \beta = A/2 = 24.25°$$

$$24^2 = 15^2 + 18^2 - 2(15)(18)\cos B$$

$$B = 92.866°;\quad \theta = B/2 = 46.433°$$

In triangle AOB:

$$\angle O = 180 - \beta - \theta = 109.317°$$

$$\frac{OA}{\sin\theta} = \frac{15}{\sin O}$$

$$OA = \frac{15}{\sin 109.317°}\sin 46.433° = 11.517\text{ cm}$$

In triangle ADO:

$$x = OA\sin\beta = 11.517\sin 24.25°$$

x = **4.73 cm**

SOLUTION 11 - 10
Ans: D

Solving for $\angle C$ in triangle ABC:

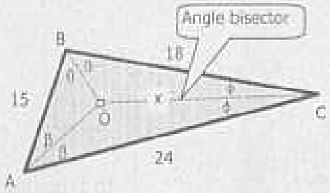
$$15^2 = 18^2 + 24^2 - 2(18)(24)\cos C$$

$$C = 38.62°;\quad \phi = C/2 = 19.31°$$

$$\frac{18}{\sin A} = \frac{15}{\sin 38.62°}$$

$$A = 48.5°$$

$$\beta = A/2 = 24.25°$$

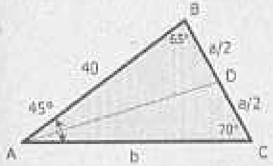


In triangle AOC:

$$\angle O = 180° - \beta - \phi = 136.44°$$

$$\frac{x}{\sin 24.25°} = \frac{24}{\sin 136.44°};\ x = \mathbf{14.3\text{ cm}}$$

SOLUTION 11 - 11
Ans: C



Solving for sides a and b in triangle ABC:

$$\frac{a}{\sin 45°} = \frac{b}{\sin 65°} = \frac{40}{\sin 70°}$$

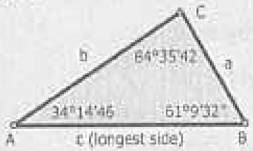
$a = 30.1$ m; $a/2 = 15.05$ m; $b = 38.56$ m

In triangle ADC:

$$AD^2 = 38.56^2 + 15.05^2 - 2(38.56)(15.05)\cos 70°$$

AD = **36.28 m**

SOLUTION 11 - 12
Ans: C



$$A = \tfrac{1}{2}ac\sin B = 680.6$$

$$\frac{a}{\sin 34°14'46"} = \frac{c}{\sin 84°35'46"}$$

$$a = 0.5653c$$

$$680.6 = \tfrac{1}{2}(0.5653c)(c)\sin 61°9'32"$$

c = **52.43 units**

SOLUTION 11 - 13
Ans: A

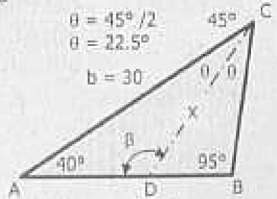
Angle $C = 180° - 40° - 95° = 45°$

In triangle ADC:

$$\beta = 180° - 40° - 22.5° = 117.5°$$

$$\frac{x}{\sin 40°} = \frac{30}{\sin 117.5°}$$

x = **21.74 cm**



SOLUTION 11 - 14
Ans: C

Solving for angle A in triangle ABC by cosine law:

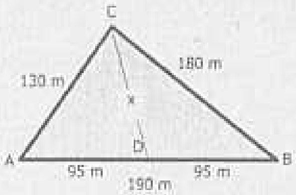
$$180^2 = 130^2 + 190^2 - 2(130)(190)\cos A$$

$$A = 65.354°$$

Solving for x in triangle ACD by cosine law:

$$x^2 = 130^2 + 95^2 - 2(130)(95)\cos 65.354°$$

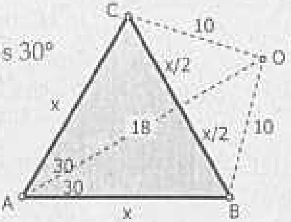
$x = 125$ m



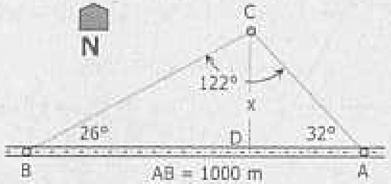
SOLUTION 11 - 15
Ans: C

In triangle *OAB* shown:

$$10^2 = x^2 + 18^2 - 2(x)(18)\cos 30°$$
$$x^2 - 31.177x + 224 = 0$$
$$x = 11.23\text{m} \ \& \ 19.94\text{m}$$



SOLUTION 11 - 16
Ans: B



Angle *ACB* = 180° - 26° - 32° = 122°

Solve for side *BC* in triangle *ABC* by sine law:

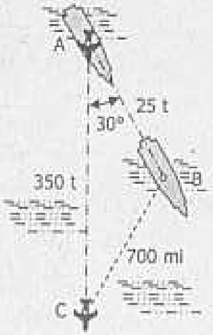
$$\frac{BC}{\sin 32°} = \frac{1000}{\sin 122°}; \ BC = 624.87 \text{ m}$$

In the right triangle *BDC*:

$$x = BC \sin 26° = 624.87 \sin 26° = \mathbf{273.92 \text{ m}}$$

SOLUTION 11 - 17
Ans: C

Let t be the time elapsed after the airplane left the aircraft carrier.



In triangle *ABC*:

$$700^2 = (350t)^2 + (25t)^2 - 2(350t)(25t)\cos 30°$$
$$700^2 = 107969.56\, t^2$$
$$t = \mathbf{2.13 \text{ hrs.}}$$

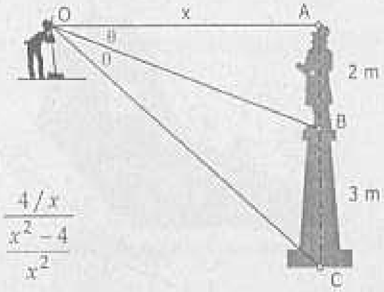
SOLUTION 11 - 18
Ans: B

In right triangle *OAB*:

$$\tan\theta = 2/x$$

In right triangle *OAC*:

$$\tan 2\theta = 5/x$$



$$\text{but } \tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$$

$$5/x = \frac{2(2/x)}{1-(2/x)^2} = \frac{4/x}{\frac{x^2-4}{x^2}}$$

$$5 = \frac{4x^2}{x^2-4} =; 5x^2 - 20 = 4x^2$$

$$x^2 = 20; x = \mathbf{2\sqrt{5} \text{ m}}$$

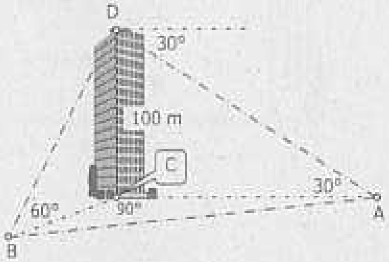
SOLUTION 11 - 19
Ans: C

In right triangle *ACD*,

$$AC = 100 \cot 30°$$
$$AC = 173.2 \text{ m}$$

In right triangle *BCD*,

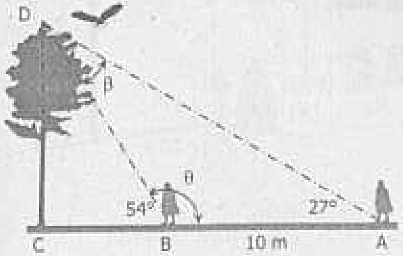
$$BC = 100 \cot 60°$$
$$BC = 57.735 \text{ m}$$



In right triangle *ABC*,

$$AB = \sqrt{(173.2)^2 + (57.735)^2}$$
$$AB = 182.57 \text{ m} = 100\sqrt{30}/3$$

SOLUTION 11 - 20
Ans: D



$$\theta = 180° - 54° = 126°$$

In triangle *ABD*,

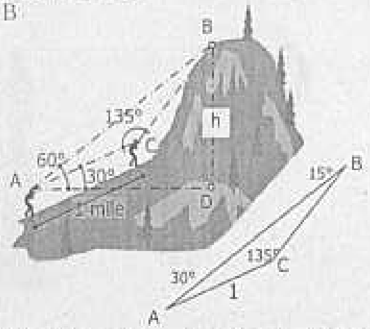
$$\beta = 180° - 27° - 126° = 27°$$

Thus, triangle *ABD* is an isosceles triangle and *BD* = *AB* = 10 m.

In right triangle *BCD*,

$$h = 10 \times \sin 54°$$
$$h = 8.09 \text{ m}$$

SOLUTION 11 - 21
Ans: B



In triangle *ACB*:

$$\frac{1}{\sin 15°} = \frac{AB}{\sin 135°}$$

$AB = 2.732 \text{ mile} \times 5280 \text{ ft/mile}$

$AB = 14{,}425.23$ feet

In right triangle *ADB*:

$h = AB \times \sin 60°$

$h = 14{,}425.23 \sin 60°$

$h = 12{,}493$ **feet**

SOLUTION 11 - 22
Ans: A

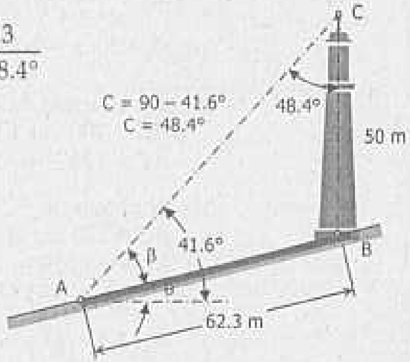
In triangle *ABC*:

$$\frac{50}{\sin\beta} = \frac{62.3}{\sin 48.4°}$$

$\beta = 36.88°$

$\theta = 41.6 - \beta$

$\theta = \mathbf{4.72°}$



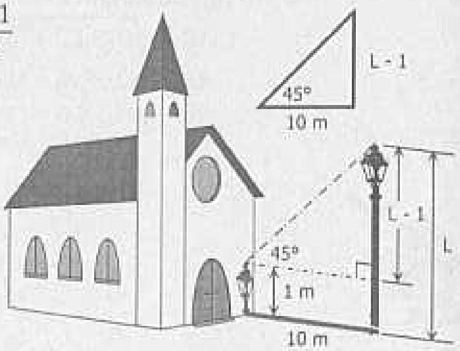
SOLUTION 11 - 23
Ans: B

$$\tan 45° = \frac{L-1}{10}$$

$$1 = \frac{L-1}{10}$$

$10 = L - 1$

$L = \mathbf{11\ m}$



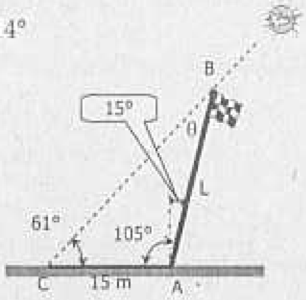
SOLUTION 11 - 24
Ans: B

In triangle *ABC*:

$\theta = 180° - 105° - 61° = 14°$

$$\frac{L}{\sin(61°)} = \frac{15}{\sin(14°)}$$

$L = \mathbf{54.23\ m}$

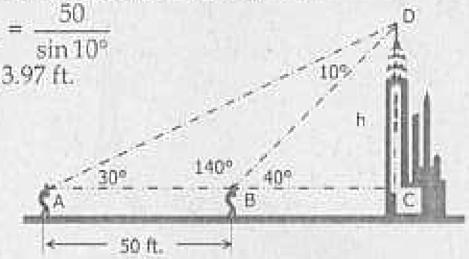


SOLUTION 11 - 25
Ans: D

Solve for *BD* in triangle *ABD* by sine law:

$$\frac{BD}{\sin 30°} = \frac{50}{\sin 10°}$$

$BD = 143.97$ ft.



In triangle *BCD*:

$$\sin 40° = \frac{h}{BD}$$

$h = 143.97 \sin 40°$

$h = \mathbf{92.54\ ft.}$

SOLUTION 11 - 26
Ans: D

In triangle *BCD*:

$$\frac{BC}{\sin 44°} = \frac{120}{\sin 74°}$$

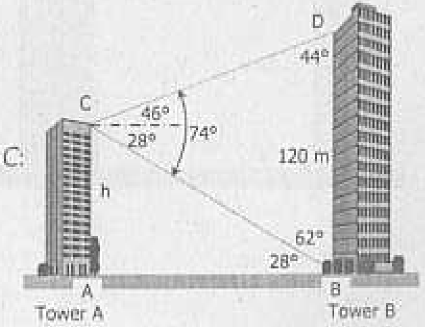
$BC = 86.718$ m

In right triangle *BAC*:

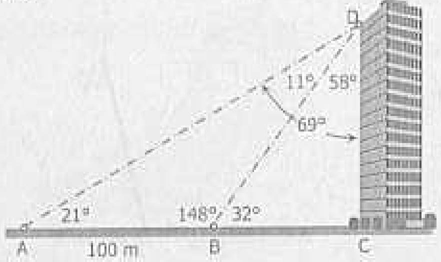
$h = BC \sin B$

$h = 86.718 \sin 28°$

$h = \mathbf{40.71\ m}$



SOLUTION 11 - 27
Ans: C



Applying sine law in triangle *ABD*:

$$\frac{AD}{\sin 148°} = \frac{100}{\sin 11°}$$

$AD = 277.72$ m

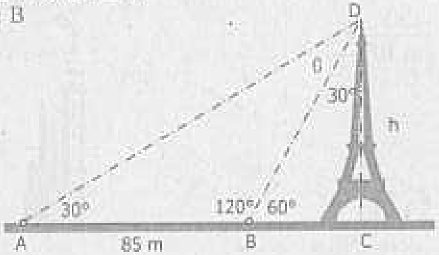
In right triangle *ACD*:

$AC = AD \cos A$

$AC = 277.72 \cos 21°$

$AC =$ **259.3 m**

SOLUTION 11 - 28
Ans: B



In triangle *ABD*:

$\theta = 180° - 30° - 120°$

$\theta = 30°$

Thus, triangle *ABD* is an isosceles triangle with $BD = AB = 85$ m

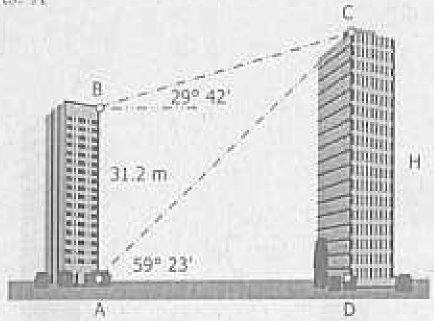
In right triangle *BCD*:

$h = BD \sin 60°$

$h = 85 \sin 60°$

$h =$ **73.61 m**

SOLUTION 11 - 29
Ans: A



In triangle *ABC*:

$\angle A = 90° - 59° 23'$

$\angle A = 30° 37'$

$\angle B = 90 + 29° 42'$

$\angle B = 119° 42'$

$\angle C = 180 - 30° 37' - 119° 42'$

$\angle C = 29° 41'$

$$\frac{AC}{\sin 119°42'} = \frac{31.2}{\sin 29°41'}$$

$AC = 54.727$ m

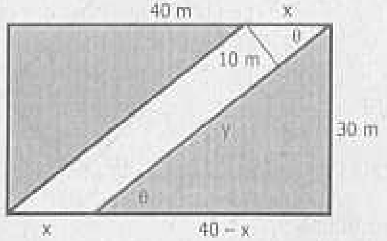
In right triangle *ACD*:

$H = AC \sin A$

$H = 54.727 \sin 59° 23'$

$H =$ **47.1 m**

SOLUTION 11 - 30
Ans: C



From the figure shown:

$\sin\theta = 30/y = 10/x$

$y = 3x$

$y^2 = 30^2 + (40 - x)^2$

$(3x)^2 = 900 + 1600 - 80x + x^2$

$9x^2 - x^2 + 80x - 2500 = 0$

$2x^2 + 20x - 625 = 0$

$x = 13.37$ m

$y = 3(13.37) = 40.11$ m

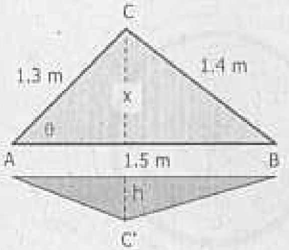
Area of road $= y(10)$

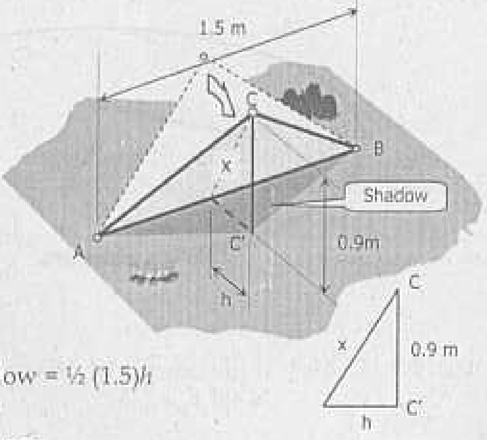
Area of road $= 40.11(10) = 401.1\ \text{m}^2$

Cost of roadway = 401.1(1500)

Cost of roadway = **P601,650.00**

SOLUTION 11 - 31
Ans: A





Area of shadow = ½ (1.5)*h*

Area = 0.75*h*

In triangle *ABC*:

$1.4^2 = 1.3^2 + 1.5^2 - 2(1.3)(1.5)\cos\theta$

$\theta = 59.49°$

$x = 1.3 \sin\theta = 1.3 \sin 59.49° = 1.12$ m

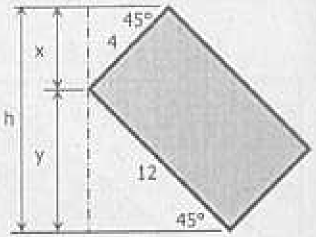
$h^2 = x^2 - (0.9)^2 = (1.12)^2 - 0.81$; $h = 0.667$ m

Area of shadow = 0.75(0.667) = **0.5 m²**

SOLUTION 11 - 32
Ans: C

$x = 4 \sin 45° = 4/\sqrt{2}$
$y = 12 \sin 45° = 12/\sqrt{2}$

$h = x + y$
$h = 4/\sqrt{2} + 12/\sqrt{2}$
$h = 16/\sqrt{2} = 8(2)/\sqrt{2}$
$h = 8\sqrt{2}$

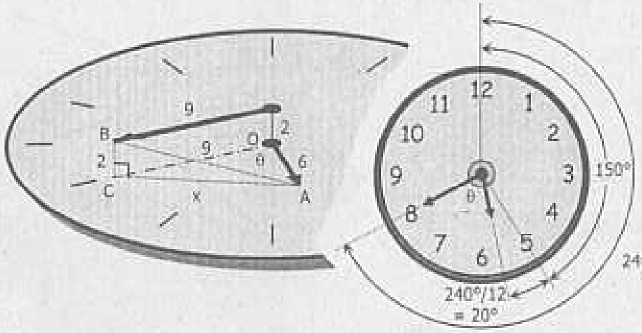


SOLUTION 11 - 33
Ans: A

From the figure shown below:
$\theta = 240° - 150° - 20° = 70°$

In triangle *AOC* (figure above)
$x^2 = 6^2 + 9^2 - 2(6)(9) \cos 70°$; $x = 8.948$ inches

In right triangle *ABC*:
$AB^2 = 2^2 + x^2 = 4 + 8.948^2$; $AB =$ **9.17 inches**



SOLUTION 11 - 34
Ans: A

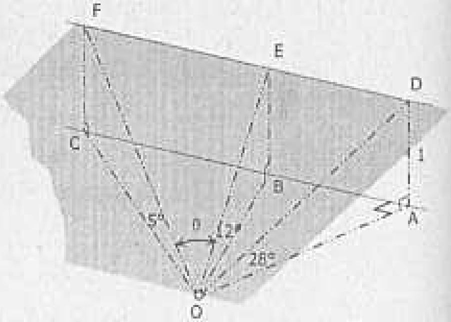
If the bearing of *AB* is S 40° W, then the bearing of *BA* is **N 40° E**

SOLUTION 11 - 35
Ans: C

In triangle *OAD*:
$OA = 1/\tan 28°$
$OA = 1.881$

In triangle *OBE*:
$OB = 1/\tan 12°$
$OB = 4.705$

$OE = 1/\sin 12°$
$OE = 4.81$



In triangle *OCF*:
$OC = 1/\tan 5° = 11.43$
$OF = 1/\sin 5° = 11.474$

In triangle *OAB*:
$AB = \sqrt{4.705^2 - 1.881^2} = 4.313$

In triangle *OAC*:
$AC = \sqrt{11.43^2 - 1.881^2} = 11.274$
$EF = BC = AC - AB = 6.961$

In triangle *OEF*:
$(6.961)^2 = (4.81)^2 + (11.474)^2 - 2(4.81)(11.474) \cos \theta$
$\theta =$ **15.56°**

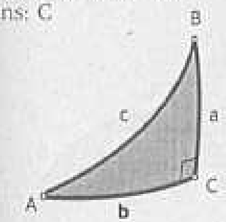
SOLUTION 11 - 36
Ans: D

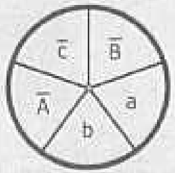
Area of spherical triangle, $A = \dfrac{\pi R^2 E}{180°}$

Radius of sphere, $R = 5$ m
Spherical Excess, $E = A + B + C - 180°$
$E = 40° + 65° + 110° - 180° = 35°$

Area, $A = \dfrac{\pi(5)^2(35°)}{180°} =$ **15.27 sq. m.**

SOLUTION 11 - 37
Ans: C





Napier's Circle

From the Napier's circle, taking $\bar{c}$ as the middle part:

$\sin \bar{c} = \cos a \times \cos b$
$\cos c = \cos a \times \cos b$
$\cos 80° = \cos 50° \times \cos b$
$\cos b = \cos 80° / \cos 50°$
$\cos b = 0.27$
$b =$ **74.33°**

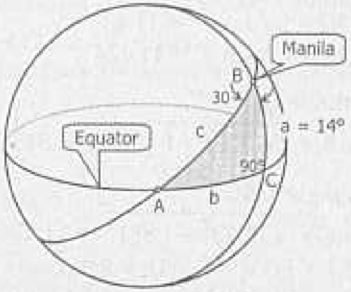
SOLUTION 11 - 38
Ans: C

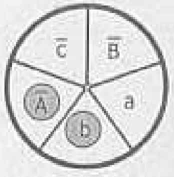
The time in the places located at East longitude is ahead of GMT. Since the earth revolves 360° in one day (24 hours), then in one hour, the earth revolves 360/24 = 15° or 1 hour every 15°.

Therefore, the time in the Philippines is:

Time = 8 + 120/15
Time = 16 (1600 hrs) or **4 p.m.**

SOLUTION 11 - 39
Ans: A





Napier's Circle

$\sin \bar{A} = \tan b \tan \bar{c}$
$\cos A = \tan b \cot c$

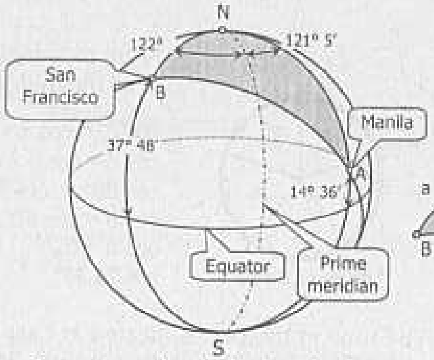
$\cos 30° = \tan 14° 36' \dfrac{1}{\tan c}$

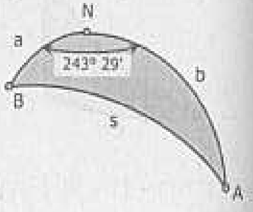
$\tan c = 0.3008$

$c = 16.74° \times \dfrac{60 \text{ nautical miles}}{1°} = 1004.4 \text{ nautical miles}$

$\text{time} = \dfrac{\text{distance}}{\text{speed}} = \dfrac{1004.4}{350} = \textbf{2.87 hours}$

SOLUTION 11 - 40
Ans: D





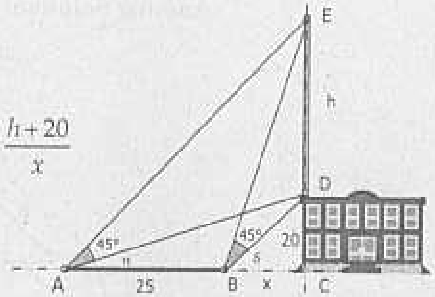
From the spherical triangle shown:
$a = 90° - 37° 48' = 52° 12'$
$b = 90 - 14° 36' = 75° 24'$

Using cosine law for sides:
$\cos s = \cos a \cos b + \sin a \sin b \cos N$
$\cos s = \cos 52° 12' \cos 75° 24' + \sin 52° 12' \sin 75° 24' \cos 243° 29'$
$\cos s = -0.1869$
$s = 100.77°$

$\text{Distance} = 100.77° \times \dfrac{60 \text{ nautical miles}}{1°}$

Distance = **6046.2 nautical miles**

SOLUTION 11 - 41
Ans: D



In triangle *BCD*:
$\tan \delta = \dfrac{20}{x}$
In trianlge *BCE*:
$\tan (45 + \delta) = \dfrac{h+20}{x}$

But $\tan (45° + \delta) = \dfrac{\tan 45° + \tan \delta}{1 - \tan 45° \tan \delta}$

$\dfrac{h+20}{x} = \dfrac{1+(20/x)}{1-(20/x)} = \dfrac{x+20}{x-20}$

$h + 20 = \dfrac{x(x+20)}{x-20} \quad \to (1)$

In triangle *ACD*: $\tan \alpha = \dfrac{20}{25+x}$

In triangle *ACE*: $\tan (45 + \alpha) = \dfrac{h+20}{25+x}$

But $\tan (45° + \delta) = \dfrac{\tan 45° + \tan \alpha}{1 - \tan 45° \tan \alpha}$

$\dfrac{h+20}{25+x} = \dfrac{1+\dfrac{20}{25+x}}{1-\dfrac{20}{25+x}} = \dfrac{45+x}{5+x}$

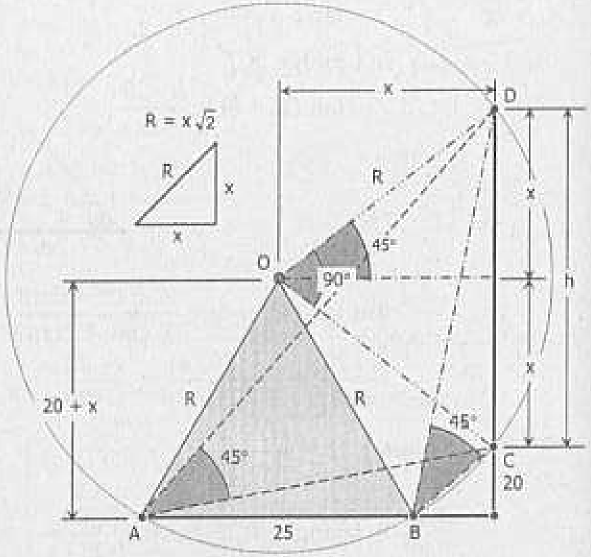
$h + 20 = \dfrac{(25+x)(45+x)}{5-x} \quad \to (2)$

Compare Eq. (1) and Eq. (2)
$\dfrac{x(x+20)}{x-20} = \dfrac{(25+x)(45+x)}{5+x}$
$(x^2 + 20x)(5 + x) = (1125 + 70x + x^2)(x - 20)$
$x^3 + 25x^2 + 100x = x^3 + 50 x^2 - 275x - 22500$
$25x^2 - 375x - 22500 = 0$
$x^2 - 15x - 900 = 0; \quad x = 38.423$

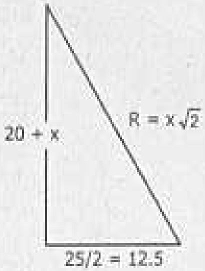
In Eq. (1) $h + 20 = \dfrac{38.423(38.423 + 20)}{38.423 - 20}$

$h = \mathbf{101.85\ m}$

Another Solution:



Since ∠DAC = ∠DBC, points A, B, C, and D lie on the same circle (concyclic), with center of circle at O and radius R.



From the figure:

$(x\sqrt{2})^2 = (20 + x)^2 + (12.5)^2$

$2x^2 = 400 + 40x + x^2 + 156.25$

$x^2 - 40x - 556.25 = 0$

$x = 50.92$ m

$h = 2x = \mathbf{101.84\ m}$

SOLUTION 11 - 42
Ans: A

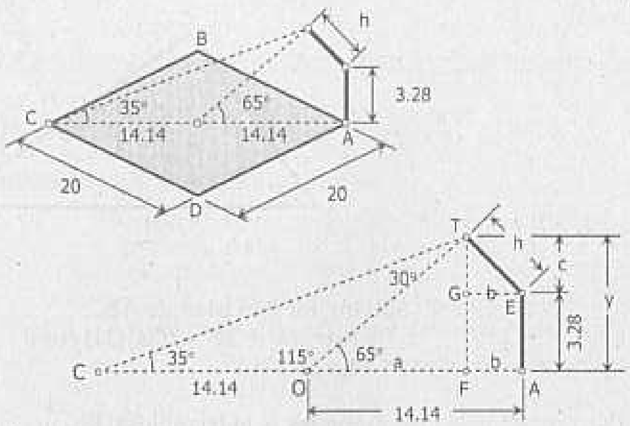
In triangle COT:

$$\frac{OT}{\sin 35^\circ} = \frac{14.14}{\sin 30^\circ};\ OT = 16.22'$$

In right triangle OFT:

$a = OT \cos 65^\circ = (16.22) \cos 65^\circ = 6.855'$

$y = OT \sin 65^\circ = (16.22) \sin 65^\circ = 14.70'$



In right triangle EGT:

$b = 14.14 - a = 7.285'$

$c = y - 3.28 = 11.42'$

$h = \sqrt{b^2 + c^2} = 13.55'$

Height of pole = 3.28 + 13.55 = 16.83′

Height of pole = **5.61 yards**

SOLUTION 11 - 43
Ans: B

In triangle OBC:

$\theta = 58^\circ 35' - 35^\circ 58'$

$\theta = 22^\circ 37'$

$\alpha = 90^\circ - 58^\circ 35' = 31^\circ 25'$

$$\frac{OB}{\sin 31^\circ 25'} = \frac{9.75}{\sin 22^\circ 37'}$$

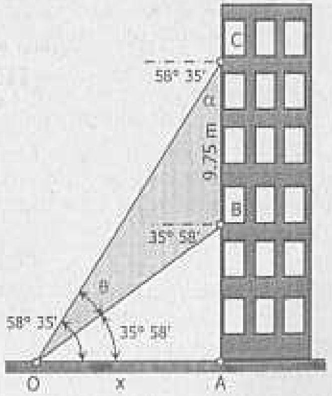
$OB = 13.2156$ m

In triangle OAB:

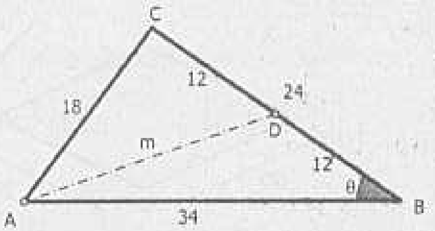
$\cos 35^\circ 58' = \dfrac{x}{OB}$

$x = 13.2156 \cos 35^\circ 58'$

$x = \mathbf{10.7\ m}$



SOLUTION 11 - 44
Ans: A



Solving for θ in triangle *ABC*:

$18^2 = 34^2 + 24^2 - 2(34)(24)\cos\theta$

$\theta = 30.37°$

Solving for *m* in triangle *ADB*:

$m^2 = 34^2 + 12^2 - 2(34)(12)\cos 30.37°$

m = **24.41 cm**

SOLUTION 11 - 45
Ans:

Given angles of spherical triangle:
$A = 116°\,19'$, $B = 55°\,30'$, and $C = 80°\,37'$

Using cosine law for angles:

$$\cos A = -\cos B\cos C + \sin B\sin C\cos a$$

$$\cos 116°\,19' = -\cos 55°\,30'\cos 80°\,37' + \sin 55°\,30'\sin 80°\,37'\cos a$$

$\cos a = -0.431664$

a = **115.573°**

Part 3
PLANE GEOMETRY

DEFINITIONS

Altitude of a Triangle - An altitude of a triangle is a perpendicular from any vertex to the side opposite, produced if necessary.

Angle - A plane angle is the opening between two straight lines drawn from the same point.

Apothem - The apothem of a polygon is the radius of its inscribed circle.

Area - The area of a plane figure is the number which expresses the ratio between its surface and the surface of the unit square.

Center of Polygon - The center of a regular polygon is the common center of its inscribed and circumscribed circles.

Circle - A circle is a closed plane curve every point of which is equally distant from a point in the plane of the curve.

Complementary Angles. Two angles are called complementary when their sum is equal to a right angle; and each is called the complement of the other.

Concurrent Lines - Three or more lines which have one point in common are said to be concurrent.

Definition of π - The number π used in calculations on the circle, is the number obtained by dividing the circumference of a circle by its diameter; that is, $\pi = C/D$. Hence, $C = \pi D$ or $C = 2\pi r$. $\pi = 3.1416$ (to 4 decimal places).

Diagonal - A diagonal of a polygon is a line joining any two nonconsecutive vertices.

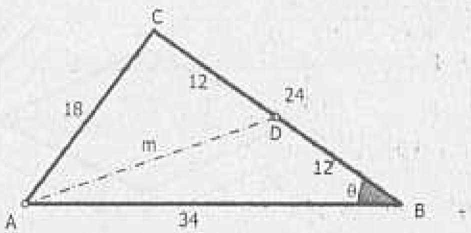
Hypotenuse - The hypotenuse of a right triangle is the side opposite the right angle.

Isosceles Triangle - An isosceles triangle is a triangle which has two equal sides.

Locus - A locus is a figure containing all the points, and only those points, which fulfill a given requirement.

Parallel Lines - Parallel lines are lines that lie in the same plane and do not meet however far they are

SOLUTION 11 - 44
Ans: A



Solving for θ in triangle *ABC*:

$$18^2 = 34^2 + 24^2 - 2(34)(24)\cos\theta$$
$$\theta = 30.37°$$

Solving for *m* in triangle *ADB*:

$$m^2 = 34^2 + 12^2 - 2(34)(12)\cos 30.37°$$
$$m = \mathbf{24.41\ cm}$$

SOLUTION 11 - 45
Ans:

Given angles of spherical triangle:
$A = 116°\,19'$, $B = 55°\,30'$, and $C = 80°\,37'$

Using cosine law for angles:

$$\cos A = -\cos B\cos C + \sin B\sin C\cos a$$
$$\cos 116°\,19' = -\cos 55°\,30'\cos 80°\,37' + \sin 55°\,30'\sin 80°\,37'\cos a$$
$$\cos a = -0.431664$$
$$a = \mathbf{115.573°}$$

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Parallel Lines - Parallel lines are lines that lie in the same plane and do not meet however far they are produced.

Parallelogram - A parallelogram is a quadrilateral whose opposite sides are parallel.

Perpendicular - If one straight line cuts another so as to make any two adjacent angles equal, each line is perpendicular to the other.

Quadrilateral - A quadrilateral is a portion of a plane bounded by four straight lines.

Rectangle - A rectangle is a parallelogram whose angles are right angles.

Regular Polygon - A regular polygon is a polygon all of whose angles are equal and all of whose sides are equal.

Similar Polygons - Two polygons are similar if their corresponding angles are equal and their corresponding sides are proportional.

Supplementary Angles - One angle is the supplement of another if their sum equals two right angles (or 180°).

Tangent - A tangent to a circle is a straight line which, however far it may be produced, has only one point in common with the circle.

Trapezoid - A trapezoid is a quadrilateral two and only two of whose sides are parallel.

Triangle - A triangle is a portion of a plane bounded by three straight lines.

Vertical Angles - When two angles have the same vertex, and the sides of one are the prolongations of the sides of the other, they are called vertical angles.

TRIANGLE

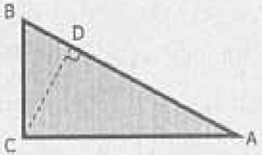
Theorems and Properties of Triangles

1. *The sum of the three angles of a triangle is equal to two right angles or 180°.*
2. *The sum of two sides of a triangle is greater than the third side, and their difference is less than the third side.*
3. *If two sides of a triangle are unequal, the angles opposite are unequal, and the greater angle is opposite the greater side; and conversely.*
4. *If two sides of a triangle are equal (an isosceles triangle), the angles opposite these sides are equal; and conversely.*
5. *The perpendicular bisectors of the sides, and the bisectors of the angles of a triangle, meet in points which are the centers of the circumscribed circle and the inscribed circle, respectively.*

6. *The altitudes of a triangle meet in a point (called* ***orthocenter****).*
7. *The medians of a triangle are concurrent at a point which is two-thirds of the distance from any vertex to the midpoint of the opposite side. The point of concurrency is the* ***centroid*** *of the triangle.*
8. *Two triangles are congruent if two angles and the included side of the one are equal, respectively, to two angles and the included side of the other.*
9. *Two triangles are congruent if two sides and the included angle of the one are equal, respectively, to two sides and the included angle of the other.*
10. *Two triangles are congruent if the three sides of the one are equal, respectively, to the three sides of the other.*

Right Triangles

1. ***Theorem of Pythagoras.*** *In any right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.*
2. *Two right triangles are equal if a side and the hypotenuse of the one are equal, respectively, to a side and the hypotenuse of the other.*
3. *Two right triangles are equal if the hypotenuse and an adjacent angle of one are equal, respectively, to the hypotenuse and an adjacent angle of the other.*
4. *If a perpendicular is drawn from the vertex of the right angle to the hypotenuse of a right triangle: (1) the two triangles formed are similar to each other and to the given triangle; (2) the perpendicular is a mean proportional between the segments of the hypotenuse; and (3) the square of either side about the right angle equals the product of the whole hypotenuse and the segment adjacent to that side.*



(1) ΔABC, ΔBDC, and ΔADC are proportional

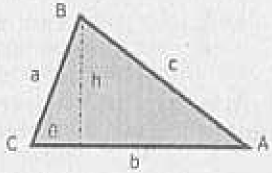
(2) $BD : CD = CD : AD$ or $(CD)^2 = (BD)(AD)$

(3) $(BC)^2 = (AB)(BD)$ and $(AC)^2 = (AB)(AD)$

Similar Triangles

1. *Two triangles are similar if the angles of one are respectively equal to the angles of the other; or if two angles of one are respectively equal to two angles of the other.*
2. *Two triangles are similar if an angle of one equals an angle of the other and the sides including these angles are proportional.*
3. *Two triangles are similar if their sides are in the same ratio.*
4. *If two triangles have their sides respectively parallel, or respectively perpendicular, each to each, they are similar.*

Area of Triangle



Given base b and altitude h

$$\text{Area} = \tfrac{1}{2}\,b\,h$$

Given two sides a and b and included angle θ.

$$\text{Area} = \tfrac{1}{2}\,a\,b\sin\theta$$

Given three sides a, b, and c: (Hero's Formula)

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

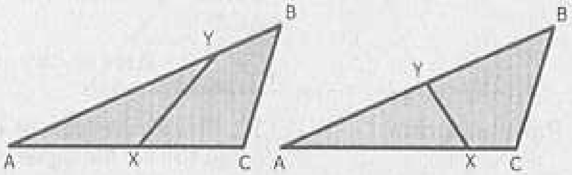
$$\text{Semi-perimeter, } s = \frac{a+b+c}{2}$$

The area under this condition can also be solved by finding one angle using cosine law and apply the formula for two sides and included angle.

Given three angles A, B, and C and one side a:

$$A = \frac{a^2 \sin B \sin C}{2 \sin A}$$

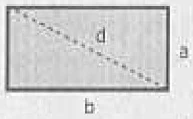
The area under this condition can also be solved by finding one side using sine law and apply the formula for two sides and included angle.



$$\frac{\text{Area of } ABC}{\text{Area of } AXY} = \frac{(AB)(AC)}{(AY)(AX)}$$

QUADRILATERALS

Rectangle

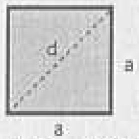


$$\text{Area} = a\,b$$

$$\text{Perimeter, } P = 2(a+b)$$

$$\text{Diagonal, } d = \sqrt{a^2+b^2}$$

Square



$$\text{Area} = a^2$$

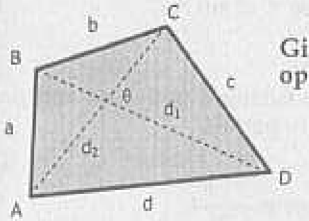
$$\text{Perimeter, } P = 4a$$

$$\text{Diagonal, } d = a\sqrt{2}$$

General Quadrilateral

Given diagonals d_1 and d_2 and included angle θ:

$$\text{Area} = \tfrac{1}{2}\,d_1\,d_2\sin\theta$$



Given four sides a, b, c, d, and sum of two opposite angles:

$$\text{Area} = \sqrt{(s-a)(s-b)(s-c)(s-d) - abcd\cos^2\theta}$$

$$\text{Semi-perimeter, } s = \frac{a+b+c+d}{2}$$

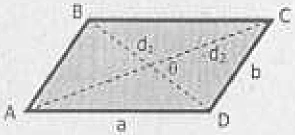
$$\theta = \frac{\angle A + \angle C}{2} \text{ or } \theta = \frac{\angle B + \angle D}{2}$$

Given four sides a, b, c, d, & two opposite angles B and D:

Divide the area into two triangles, ΔABC and ΔCAD:

$$\text{Area} = \tfrac{1}{2} a\, b \sin B + \tfrac{1}{2}\, c\, d \sin D$$

Parallelogram



1. *The opposite sides of a parallelogram are equal, and so also are the opposite angles.*
2. *The diagonals of a parallelogram bisect each other.*
3. *If two sides of a quadrilateral are equal and parallel, then the other two sides are equal and parallel, and the figure is a parallelogram.*

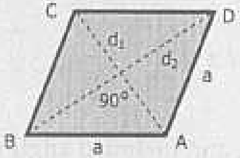
Given diagonals d_1 and d_2 and included angle θ:

$$\text{Area} = \tfrac{1}{2}\, d_1\, d_2 \sin \theta$$

Given two sides a and b and one angle A:

$$\text{Area} = a\, b \sin A$$

Rhombus



A rhombus is a parallelogram with four equal sides. The diagonals of a rhombus bisect each other at an angle of 90°.

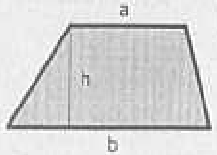
Given diagonals d_1 and d_2:

$$\text{Area} = \tfrac{1}{2}\, d_1\, d_2$$

Given side a and one angle A:

$$\text{Area} = a^2 \sin A$$

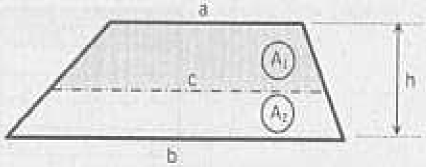
Trapezoid



A trapezoid is a quadrilateral with two and only two of whose sides are parallel.

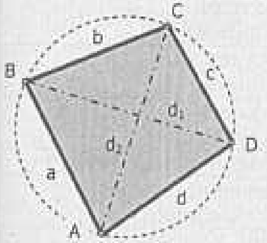
$$\text{Area} = \frac{a+b}{2} h$$

Length of dividing line parallel to the two parallel sides of a trapezoid:



$$c = \sqrt{\frac{m a^2 + n b^2}{m+n}}; \quad \frac{m}{n} = \frac{A_1}{A_2}$$

Cyclic Quadrilateral



A cyclic quadrilateral is a quadrilateral whose vertices lie on the circumference of a circle.

$$\angle A + \angle C = 180^\circ$$

$$\angle B + \angle D = 180^\circ$$

$$\text{Area} = \sqrt{(s-a)(s-b)(s-c)(s-d)}$$

$$\text{Semi-perimeter, } s = \frac{a+b+c+d}{2}$$

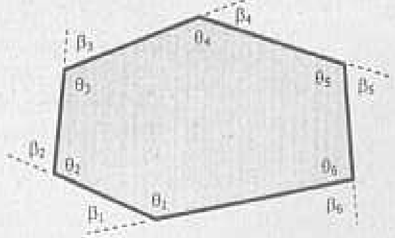
Ptolemy's Theorem

"For any cyclic quadrilateral, the product of the diagonals equals the sum of the products of the opposite sides"

$$d_1 d_2 = (a)(c) + (b)(d)$$

POLYGONS

There are two basic types of polygons, a convex and a concave polygon. A convex polygon is one in which no side, when extended, will pass inside the polygon, otherwise it called concave polygon. The following figure is a convex polygon.



Polygons are classified according to the number of sides. The following are some names of polygons.

No. of Sides	Name
2	digon
3	triangle (trigon)
4	quadrilateral (tetragon)
5	pentagon
6	hexagon
7	heptagon
8	octagon
9	nonagon (enneagon)
10	decagon
11	undecagon (hendecagon)
12	dodecagon
13	tridecagon (triskaidecagon)
14	tetradecagon (tetrakaidecagon)
15	pentadecagon (pentakaidecagon)
16	hexadecagon (hexakaidecagon)
17	heptadecagon (heptakaidecagon)
18	octadecagon (octakaidecagon)
19	enneadecagon (enneakaidecagon)
20	icosagon
30	triacontagon
40	tetracontagon
50	pentacontagon
60	hexacontagon
70	heptacontagon
80	octacontagon
90	enneacontagon
100	hectogon
1000	chilliagon
10000	myriagon

Theorems in Polygons

1. *The sum of the angles of a convex polygon of "n" sides is 2(n -2) right angles.*
2. *The exterior angles of a polygon, made by producing each of its sides in succession, are together equal to 4 right angles, or 360°.*
3. *Homologous parts of congruent figures are equal.*

Sum of Interior Angles

The sum of interior angles θ of a polygon of n sides is:

$$\text{Sum, } \Sigma\theta = (n-2) \times 180°$$

Sum of Exterior Angles

The sum of exterior angles β is equal to 360°.

$$\Sigma\beta = 360°$$

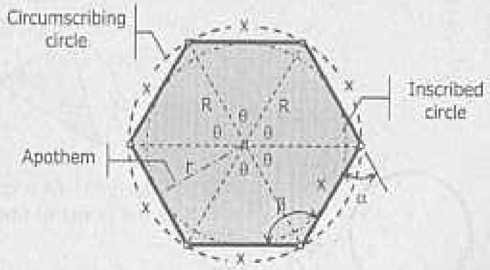
Number of Diagonals, *D*

The diagonal of a polygon is the line segment joining two non-adjacent sides. The number of diagonals is given by:

$$D = \frac{n}{2}(n-3)$$

Regular Polygons

Polygons whose sides are equal are called equilateral polygons. Polygons with equal interior angles are called equiangular polygons. Polygons that are both equilateral and equiangular are called regular polygons. The area of a regular polygon can be found by considering one segment, which has the form of an isosceles triangle.



x =side
θ =angle subtended by the side from the center
R = radius of circumscribing circle
r = radius of inscribed circle, also called the apothem
n =number of sides

$$\theta = \frac{360°}{n}$$

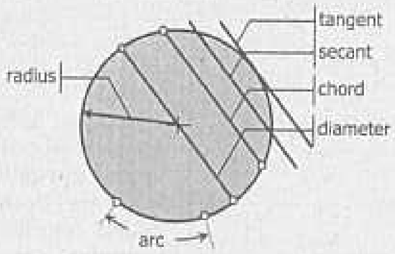
$$\text{Area} = \frac{n}{2}R^2 \sin\theta = \frac{n}{2}(x\,r)$$

$$\text{Perimeter, } P = (n)(x)$$

$$\text{Interior angle} = \beta = \frac{n-2}{n} \times 180°$$

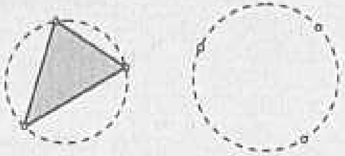
$$\text{Exterior angle} = \alpha = \theta = \frac{360°}{n}$$

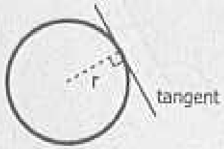
CIRCLE



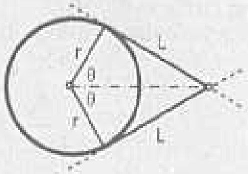
Theorems on Circles

1. *Through three points not in a straight line one circle, and only one, can be drawn.*



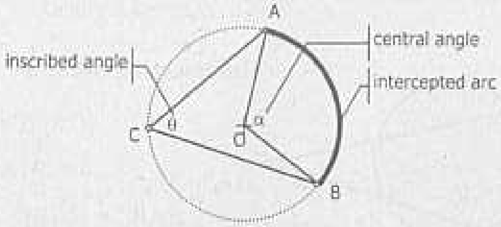


2. *A tangent to a circle is perpendicular to the radius at the point of tangency; and conversely.*

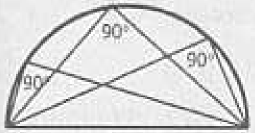


3. *The tangents to a circle drawn from an external point are equal, and make equal angles with the line joining the point to the center.*

4. *An inscribed angle is measured by one-half the intercepted arc.*



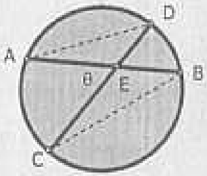
$$\theta = \frac{1}{2}\alpha \quad \text{or} \quad \alpha = 2\theta$$



5. *An angle inscribed in a semicircle is a right angle.*

   Thus, if a **right triangle** is inscribed in a circle, its **hypotenuse is the diameter** of the circle.

6. *An angle formed by two chords intersecting within the circle is measured by half the sum of the intercepted arcs.*



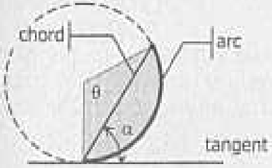
7. *If two chords intersect in a circle, the product of the segments of one is equal to the product of the segments of the other.*

$$(AE)\,(BE) = (CE)\,(DE)$$

$$\theta = \frac{1}{2}(\text{Arc}_{AC} + \text{Arc}_{BD})$$

$$\angle ADC = \angle ABC$$

$$\angle BAD = \angle BCD$$



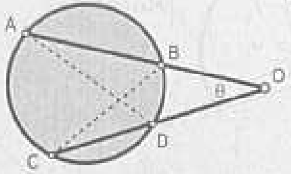
8. *An angle included by a tangent and a chord drawn from the point of contact is measured by half the intercepted arc.*

$$\alpha = \frac{1}{2}\theta$$

9. *An angle formed by two secants, two tangents, or a tangent and a secant, drawn to a circle from an external point, is measured by half the difference of the intercepted arcs.*

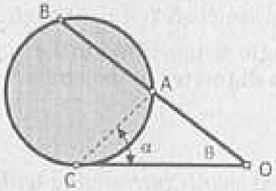
10. *If from a point outside a circle a secant and a tangent are drawn, the tangent is the mean proportional between the whole secant and its external segment.*

Intersecting Secants



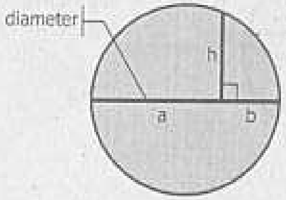
$$(OA)\,(OB) = (OC)\,(OD)$$
$$\theta = \frac{1}{2}(\text{Arc}_{AC} - \text{Arc}_{BD})$$
$$\angle ABC = \angle ADC \quad \text{and} \quad \angle BCD = \angle BAD$$

Intersecting Tangent and Secant



$$(OC)^2 = (OA)\,(OB)$$
$$\theta = \frac{1}{2}(\text{Arc}_{BC} - \text{Arc}_{AC})$$
$$\alpha = \frac{1}{2}\text{Arc}_{AC}$$

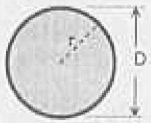
11. *A perpendicular from a point on the Circumference to a diameter of a circle is a mean proportional between the segment of the diameter.*



$$h^2 = a\,b$$

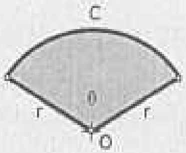
12. *The circumferences of two circles are in the same ratio as their radii, and the arcs of two circles subtended by equal central angles are in the same ratio as their radii.*

Area of Circle



$$\text{Circumference} = 2\pi\, r = \pi D$$
$$\text{Area} = \pi\, r^2 = \tfrac{\pi}{4}\, D^2$$

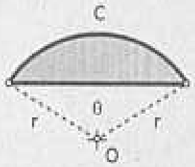
Sector of a Circle



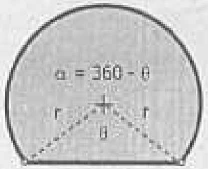
$$\text{Length of Arc, } C = r\,\theta_{\text{radians}} = \frac{\pi r\theta}{180°}$$
$$\text{Area} = \frac{1}{2} r^2\, \theta_{\text{radians}} = \frac{\pi r^2 \theta}{360°}$$
$$\text{Area} = \frac{1}{2} C\, r$$

Note: 1 radian is the angle θ such that $C = r$.

Segment of a Circle



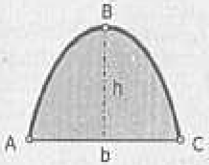
$$\text{Area} = A_{\text{sector}} - A_{\text{triangle}}$$
$$\text{Area} = \tfrac{1}{2}\, r^2\, \theta_r - \tfrac{1}{2}\, r^2 \sin\theta$$
$$\text{Area} = \tfrac{1}{2}\, r^2\, (\theta_r - \sin\theta)$$



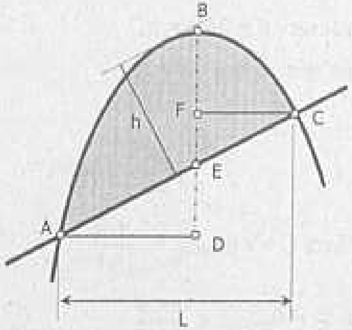
$$\text{Area} = A_{\text{sector}} + A_{\text{triangle}}$$
$$\text{Area} = \tfrac{1}{2}\, r^2\, (\alpha_r + \sin\theta)$$

where θ_r and α_r are angles in radians

PARABOLIC SEGMENT



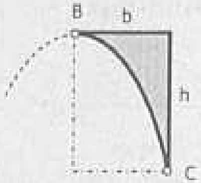
$$\text{Area} = \tfrac{2}{3}\, bh$$
$$\text{Length of } ABC = \frac{b^2}{8h}\left[m e + \ln(m + e)\right]$$
$$m = \frac{4h}{b}; \quad e = \sqrt{1 + m^2}$$



$$\text{Area}_{\text{shaded}} = A_{ADB} - A_{ADE} + A_{BFC} + A_{CFE}$$

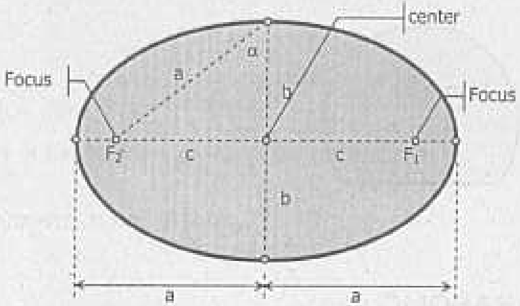
$$\text{Area}_{\text{shaded}} = \frac{2}{3}(AC)(h)$$

Spandrel of a Parabolic Segment



$$\text{Area} = \tfrac{1}{3}bh$$

ELLIPSE



$$\text{Area} = \pi a b$$

$$\text{Perimeter} = 2\pi\sqrt{\frac{a^2 + b^2}{2}}$$

$$a^2 = b^2 + c^2$$

$$\text{Eccentricity (first eccentricity), } e = \frac{c}{a}$$

$$\text{Second eccentricity, } e' = \frac{c}{b}$$

$$\text{Angular eccentricity, } \alpha = \frac{c}{a}$$

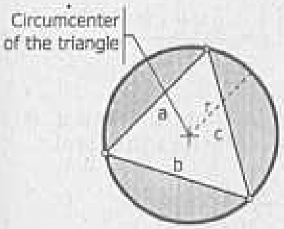
$$\text{Ellipse flatness, } f = \frac{a-b}{a}$$

$$\text{Second flatness, } f' = \frac{a-b}{b}$$

RADIUS OF CIRCLES

Circle Circumscribed about a Triangle (Circumcircle)

A circle is circumscribed about a triangle if it passes through the vertices of the triangle.

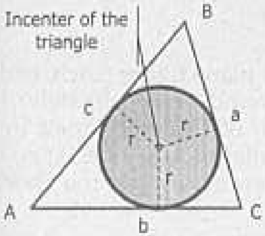


$$r = \frac{abc}{4A_T}$$

where A_T = area of the triangle

Circle Inscribed in a Triangle (Incircle)

A circle is inscribed in a triangle if it is tangent to the three sides of the triangle.

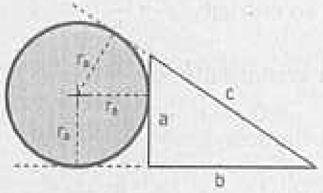


$$r = \frac{A_T}{s}$$

$$s = \frac{a+b+c}{2}$$

Circles Escribed about a Triangle (Excircles)

A circle is escribed about a triangle if it is tangent to one side and to the prolongation of the other two sides. A triangle has three escribed circles.

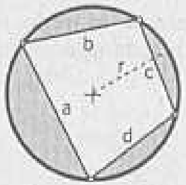


$$r_a = \frac{A_T}{s-a}$$

$$r_b = \frac{A_T}{s-b} \qquad r_c = \frac{A_T}{s-c}$$

Circle Circumscribed about a Quadrilateral

A circle is circumscribed about a quadrilateral if it passes through the vertices of the quadrilateral.



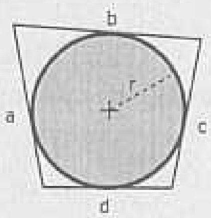
$$r = \frac{\sqrt{(ab+cd)(ac+bd)(ad+bc)}}{4A_{\text{quad}}}$$

$$A_{\text{quad}} = \sqrt{(s-a)(s-b)(s-c)(s-d)}$$

$$\text{Semi-perimeter, } s = \frac{a+b+c+d}{2}$$

Circle Inscribed in a Quadrilateral

A circle is inscribed in a quadrilateral if it is tangent to the three sides of the quadrilateral.

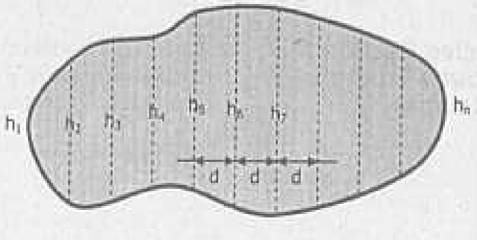


$$r = \frac{A_{\text{quad}}}{s}$$

$$A_{\text{quad}} = \sqrt{abcd} \qquad s = \frac{a+b+c+d}{2}$$

AREA BY APPROXIMATION

The area of any irregular plane figure (such as the one shown) can be found approximately by dividing it into a number of strips or panels by a series of equidistant parallel chords (offsets) h_1, h_2, ..., h_n, the common distance between the chords being d.



Area by Trapezoidal Rule

Assuming each strip as a trapezoid, then the area is:

$$\text{Area} = \frac{d}{2}\left[h_1 + 2(h_2 + h_3 + ...) + h_n\right]$$

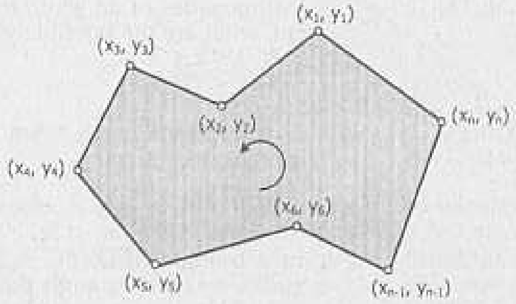
Area by Simpson's One-Third Rule

This method is more accurate than the previous because it considers the curved side. Using this rule, there must be an odd number of offsets, thus n must be odd.

$$\text{Area} = \frac{d}{3}\left[h_1 + 2\sum h_{odd} + 4\sum h_{even} + h_n\right]$$

AREA BY COORDINATES

The area of a planar (convex or concave) with vertices
(x_1, y_1), (x_2, y_2), (x_3, y_3), ..., (x_n, y_n) is:



$$\text{Area} = \frac{1}{2}\left(\begin{vmatrix} x_1 & x_2 \\ y_1 & y_2 \end{vmatrix} + \begin{vmatrix} x_2 & x_3 \\ y_2 & y_3 \end{vmatrix} + \ldots + \begin{vmatrix} x_n & x_1 \\ y_n & y_1 \end{vmatrix}\right)$$

$$\text{Area} = \frac{1}{2}\left|\begin{array}{ccccccc} x_1 & \overset{-}{x_2} & \overset{-}{x_3} & x_4 & \ldots & \overset{-}{x_n} & \overset{-}{x_1} \\ y_1 & \underset{+}{y_2} & \underset{+}{y_3} & y_4 & & \underset{+}{y_n} & \underset{+}{y_1} \end{array}\right|$$

$$\text{Area} = \frac{1}{2}\left[\, x_1 y_2 - x_2 y_1 + x_2 y_3 - x_3 y_2 + \ldots + x_n y_1 - x_1 y_n \,\right]$$

The area of a polygon is defined to be positive if the points are arranged in a counterclockwise order, and negative if they are in clockwise order.

Problems - Set 12
Triangles, Quadrilaterals, Polygons

PROBLEM 12 - 1

The sides of a right triangle have lengths $(a - b)$, a, and $(a + b)$. What is the ratio of a to b if a is greater than b and b could not be equal to zero?

A. 1 : 4   C. 1: 4
B. 3 : 1   D. 4 : 1

PROBLEM 12 - 2

Two sides of a triangle measure 8 cm and 12 cm. Find its area if its perimeter is 26 cm.

A. 21.33 sq. m.   C. 3.306 sq. in.
B. 32.56 sq. cm.   D. 32.56 sq. in.

PROBLEM 12 - 3

If three sides of an acute triangle is 3 cm, 4 cm, and "x" cm, what are the possible values of x?

A. $1 < x < 5$ C. $0 < x < 7$
B. $0 < x > 5$ D. $1 < x > 7$

PROBLEM 12 - 4
CE May 1998

In triangle ABC, AB = 8m and BC = 20m. One possible dimension of CA is:

A. 13   C. 9
B. 7   D. 11

PROBLEM 12 - 5
CE Nov. 1996, Nov. 1999

In a triangle BCD, BC = 25 m. and CD = 10 m. The perimeter of the triangle may be.

A. 72 m.   C. 69 m.
B. 70 m.   D. 71 m.

PROBLEM 12 - 6
CE May 1999

The sides of a triangle ABC are AB = 25 cm, BC = 39 cm, and AC = 40 cm. Find its area.

A. 486 sq. cm.   C. 648 sq. cm.
B. 846 sq. cm.   D. 468 sq. cm.

PROBLEM 12 - 7

The corresponding sides of two similar triangles are in the ratio 3:2. What is the ratio of their areas?

A. 3   C. 9/4
B. 2   D. 3/2

PROBLEM 12 - 8
CE Nov. 1997

Find the area of the triangle whose sides are 12, 16, and 21 units.

A. 95.45 sq. units   C. 87.45 sq. units
B. 102.36 sq. units   D. 82.78 sq. units

PROBLEM 12 - 9
ECE April 1998

The sides of a right triangle are 8, 15 and 17 units. If each side is doubled, how many square units will be the area of the new triangle?

A. 240   C. 320
B. 300   D. 420

PROBLEM 12 - 10
ECE Nov. 1997

Two triangles have equal bases. The altitude of one triangle is 3 units more than its base and the altitude of the other is 3 units less than its base. Find the altitudes, if the areas of the triangle differ by 21 square units.

A. 5 & 11   C. 6 & 12
B. 4 & 10   D. 3 & 9

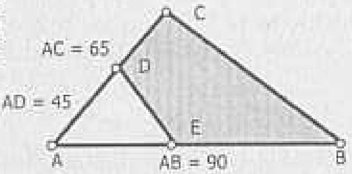
PROBLEM 12 - 11

A triangular piece of wood having a dimension 130 cm, 180 cm, and 190 cm is to be divided by a line bisecting the longest side drawn from its opposite vertex. The area of the part adjacent to the 180-cm side is:

A. 5126 sq. cm   C. 5612 sq. cm
B. 5162 sq. cm   D. 5216 sq. cm

PROBLEM 12 - 12

Find EB if the area of the inner triangle is 1/4 of the outer triangle as shown.



A. 32.5
B. 55.7
C. 56.2
D. 57.5

PROBLEM 12 - 13
ECE Nov. 1997

A piece of wire is shaped to enclose a square whose area is 169 cm². It is then reshaped to enclose a rectangle whose length is 15 cm. The area of the rectangle is:

A. 165 cm²   C. 170 cm²
B. 175 cm²   D. 156 cm²

PROBLEM 12 - 14

The diagonal of the floor of a rectangular room is 7.50 m. The shorter side of the room is 4.5 m. What is the area of the room?

A. 36 sq. m.   C. 58 sq. m.
B. 27 sq. m.   D. 24 sq. m.

PROBLEM 12 - 15

A man measuring a rectangle "x" meters by "y" meters, makes each side 15% too small. By how many percent will his estimate for the area be too small?

A. 23.55%   C. 27.75%
B. 25.67%   D. 72.25%

PROBLEM 12 - 16

The length of the side of a square is increased by 100%. Its perimeter is increased by:

A. 25%   C. 200%
B. 100%   D. 300%

PROBLEM 12 - 17

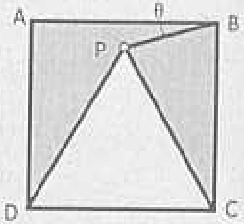
A piece of wire of length 52 cm is cut into two parts. Each part is then bent to form a square. It is found that total area of the two squares is 97 sq. cm. The dimension of the bigger square is:

A. 4   C. 3
B. 9   D. 6

PROBLEM 12 - 18

In the figure shown, *ABCD* is a square and *PDC* is an equilateral triangle. Find θ.

A. 5°
B. 15°
C. 20°
D. 25°



PROBLEM 12 - 19

One side of a parallelogram is 10 m and its diagonals are 16 m and 24 m, respectively. Its area is:

A. 156.8 sq. m.   C. 158.7 sq. m.
B. 185.6 sq. m.   D. 142.3 sq. m.

PROBLEM 12 - 20
ECE April 1998

If the sides of the parallelogram and an included angle are 6, 10 and 100 degrees respectively, find the length of the shorter diagonal.

A. 10.63   C. 10.73
B. 10.37   D. 10.23

PROBLEM 12 - 21

The area of a rhombus is 132 square cm. If its shorter diagonal is 12 cm, the length of the longer diagonal is:

A. 20 centimeter   C. 22 centimeter
B. 21 centimeter   D. 23 centimeter

PROBLEM 12 - 22

The diagonals of a rhombus are 10 cm. and 8 cm., respectively. Its area is:

A. 10 sq. cm.   C. 60 sq. cm.
B. 50 sq. cm.   D. 40 sq. cm.

PROBLEM 12 - 23

Given a cyclic quadrilateral whose sides are 4 cm, 5 cm, 8 cm, and 11 cm. Its area is:

A. 40.25 sq. cm.   C. 50.25 sq. cm.
B. 48.65 sq. cm.   D. 60.25 sq. cm.

PROBLEM 12 - 24
ECE Nov. 1995

A rectangle *ABCD* which measures 18 by 24 units is folded once, perpendicular to diagonal *AC*, so that the opposite vertices *A* and *C* coincide. Find the length of the fold.

A. 2   C. 54/2
B. 7/2   D. 45/2

PROBLEM 12 - 25
CE Nov. 1996,
CE May 1997

The sides of a quadrilateral are 10m, 8m, 16m and 20m, respectively. Two opposite interior angles have a sum of 225°. Find the area of the quadrilateral in sq. m.

A. 140.33 sq. cm.   C. 150.33 sq. cm.
B. 145.33 sq. cm.   D. 155.33 sq. cm.

PROBLEM 12 - 26
ECE Nov. 1996

A non-square rectangle is inscribed in a square so that each vertex of the rectangle is at the trisection point of the different sides of the square. Find the ratio of the area of the rectangle to the area of the square.

A. 5:9   C. 7:72
B. 2:7   D. 4:9

PROBLEM 12 - 27
ECE April 1998

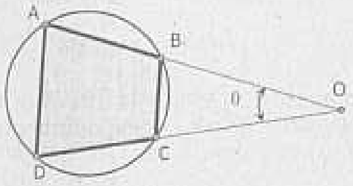
A trapezoid has an area of 36 m^2 and altitude of 2 m. Its two bases in meters have ratio of 4:5, the bases are:

A. 12, 15   C. 16, 20
B. 7, 11   D. 8, 10

PROBLEM 12 - 28

Determine the area of the quadrilateral *ABCD* shown if *OB* = 80 cm, *OA* = 120 cm, *OD* = 150 cm and θ = 25°

A. 2272 sq. cm
B. 7222 sq. cm
C. 2572 sq. cm
D. 2722 sq. cm



PROBLEM 12 - 29

A corner lot of land is 35 m on one street and 25 m on the other street. The angle between the two lines of the street being 82°. The other to two lines of the lot are respectively perpendicular to the lines of the streets. What is the worth of the lot if its unit price is P2500 per square meter?

A. P1,978,456   C. P2,234,023
B. P1,588,045   D. P1,884,050

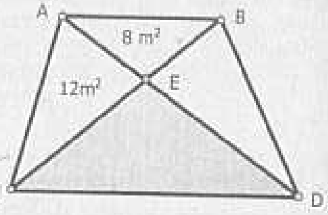
PROBLEM 12 - 30

Determine the area of the quadrilateral having (8, -2), (5, 6), (4, 1), and (-7, 4) as consecutive vertices.

A. 22 sq. units   C. 32 sq. units
B. 44 sq. units   D. 48 sq. units

PROBLEM 12 - 31

Find the area of the shaded portion shown if AB is parallel to CD.
A. 16 sq. m.
B. 18 sq. m.
C. 20 sq. m.
D. 22 sq. m.



PROBLEM 12 - 32

The deflection angles of any polygon has a sum of:
A. 360° C. 180°(n - 3)
B. 720° D. 180° n

PROBLEM 12 - 33

The sum of the interior angles of a dodecagon is:
A. 2160° C. 1800°
B. 1980° D. 2520°

PROBLEM 12 - 34

Each interior angle of a regular polygon is 165°. How many sides?
A. 23 C. 25
B. 24 D. 26

PROBLEM 12 - 35
ECE March 1996

The sum of the interior angles of a polygon is 540°. Find the number of sides.
A. 4 C. 7
B. 6 D. 5

PROBLEM 12 - 36
CE May 1997

The sum of the interior angles of a polygon of n sides is 1080°. Find the value of n.
A. 5 C. 7
B. 6 D. 8

PROBLEM 12 - 37

How many diagonals does a pentedecagon have:
A. 60 C. 80
B. 70 D. 90

PROBLEM 12 - 38

A polygon has 170 diagonals. How many sides does it have?
A. 20 C. 25
B. 18 D. 26

PROBLEM 12 - 39
CE Nov. 1999

A regular hexagon with an area of 93.53 square centimeters is inscribed in a circle. The area in the circle not covered by the hexagon is:
A. 18.38 cm² C. 19.57 cm²
B. 16.72 cm² D. 15.68 cm²

PROBLEM 12 - 40

The area of a regular decagon inscribed in a circle of 15 cm diameter is:
A. 156 sq. cm. C. 165 sq. cm.
B. 158 sq. cm. D. 177 sq. cm.

PROBLEM 12 - 41

The sum of the interior angle of a polygon is 2,520°. How many are the sides?
A. 14 C. 16
B. 15 D. 17

PROBLEM 12 - 42
ME April 1997

The area of a regular hexagon inscribed in a circle of radius 1 is:
A. 2.698 sq. units C. 3.698 sq. units
B. 2.598 sq. units D. 3.598 sq. units

PROBLEM 12 - 43

The corners of a 2-meter square are cut off to form a regular octagon. What is the length of the sides of the resulting octagon?
A. 0.525 C. 0.727
B. 0.626 D. 0.828

PROBLEM 12 - 44
ECE Nov. 1997

If a regular polygon has 27 diagonals, then it is a:
A. hexagon C. pentagon
B. nonagon D. heptagon

PROBLEM 12 - 45
ECE Nov. 1997

One side of a regular octagon is 2. Find the area of the region inside the octagon.
A. 19.3 sq. units C. 21.4 sq. units
B. 13.9 sq. units D. 31 sq. units

PROBLEM 12 - 46
ECE Nov. 1997

A regular octagon is inscribed in a circle of radius 10. Find the area of the octagon.
A. 228.2 sq. units C. 282.8 sq. units
B. 288.2 sq. units D. 238.2 sq. units

PROBLEM 12 - 47
CE May 2003

Lot $ABCDEFA$ is a closed traverse in the form of a regular hexagon with each side equal to 100 m. The bearing of AB is N 25° E. What is the bearing of CD?
A. S 35° E C. S 30° E
B. S 45° E D. S 40° E

PROBLEM 12 - 48
CE May 2001

Two sides of a parallelogram measure 68 cm and 83 cm and the shorter diagonal is 42 cm. Determine the smallest interior angle of the parallelogram.
A. 23.87° C. 49.45°
B. 30.27° D. 12.65°

PROBLEM 12 - 49
The parallel sides of a trapezoidal lot measure 160 m and 240 m and are 40 m apart. Find the length of the dividing line parallel to the two sides that will divide the lot into two equal areas.
A. 203.96
B. 214.25
C. 200
D. 186.54

PROBLEM 12 - 50
What do you call a polygon with 35 diagonals?
A. decagon
B. nonagon
C. dodecagon
D. undecagon

ANSWER SHEET

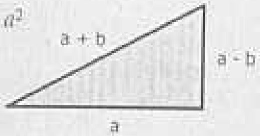
1. A B C D E
2. A B C D E
3. A B C D E
4. A B C D E
5. A B C D E
6. A B C D E
7. A B C D E
8. A B C D E
9. A B C D E
10. A B C D E
11. A B C D E
12. A B C D E
13. A B C D E
14. A B C D E
15. A B C D E
16. A B C D E
17. A B C D E
18. A B C D E
19. A B C D E
20. A B C D E
21. A B C D E
22. A B C D E
23. A B C D E
24. A B C D E
25. A B C D E
26. A B C D E
27. A B C D E
28. A B C D E
29. A B C D E
30. A B C D E
31. A B C D E
32. A B C D E
33. A B C D E
34. A B C D E
35. A B C D E
36. A B C D E
37. A B C D E
38. A B C D E
39. A B C D E
40. A B C D E
41. A B C D E
42. A B C D E
43. A B C D E
44. A B C D E
45. A B C D E
46. A B C D E
47. A B C D E
48. A B C D E
49. A B C D E
50. A B C D E
51. A B C D E
52. A B C D E
53. A B C D E
54. A B C D E
55. A B C D E
56. A B C D E
57. A B C D E
58. A B C D E
59. A B C D E
60. A B C D E

Solutions to Set 12 Triangles, Quadrilaterals, Polygons

SOLUTION 12 - 1
Ans: D

By Pythagorean theorem:

$$(a+b)^2 = (a-b)^2 + a^2$$
$$a^2 + 2ab + b^2 = a^2 - 2ab + b^2 + a^2$$
$$4ab = a^2$$
$$4 = \frac{a^2}{ab} = \frac{a}{b}$$



Thus, $a : b$ = **4:1**

SOLUTION 12 - 2
Ans: C

Given sides $a = 8$ and $b = 12$; Perimeter = 26
Perimeter = $a + b + c = 26$
$8 + 12 + c = 26; c = 6$

$$\text{Area of triangle} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$s = \frac{a+b+c}{2} = \frac{8+12+6}{2} = 13$$

$$\text{Area of triangle} = \sqrt{13(13-8)(13-12)(13-6)}$$

Area of triangle = 21.33 sq. cm
Area of triangle = 21.33 $\text{cm}^2 \times (1 \text{ in} / 2.54 \text{ cm})^2$
Area of triangle = **3.306 sq. in.**

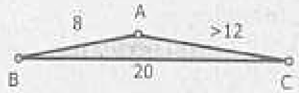
SOLUTION 12 - 3
Ans: A

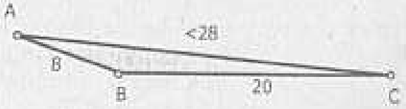
For a 3-4-x triangle, when x becomes 5, the triangle becomes a right triangle, and x should be more than 1 to form a triangle.

Thus x should be less than 5 or $1 < x < 5$

SOLUTION 12 - 4
Ans: A

The possible shapes of the triangle is as shown:

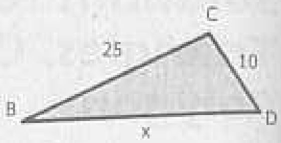




Obviously, CA must be greater than 12 and less than 28. Among the choices, the possible dimension is 13.

SOLUTION 12 - 5
Ans: C

From the triangle shown; the length of x is should be less than 35 and should be more than 15.



Therefore; the perimeter should be less than 70 and should be more than 50 or:

$$50 < \text{Perimeter} < 70$$

In the choices given, the possible perimeter is 69

SOLUTION 12 - 6
Ans: D

$$\text{Area} = \sqrt{s(s-a)(s-)(s-c)}$$

$$s = \tfrac{1}{2}(a+b+c) = \tfrac{1}{2}(26+39+40) = 52$$

$$\text{Area} = \sqrt{52(52-25)(52-39)(52-40)} = \mathbf{468\ sq.\ cm.}$$

SOLUTION 12 - 7
Ans: C

If two triangles are similar, the ratio of their area is equal to the square of the ratio of their corresponding sides.

$$\frac{A_1}{A_2} = \left(\frac{x_1}{x_2}\right)^2 = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$$

SOLUTION 12 - 8
Ans: A

By Hero's formula:

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

$$s = \frac{a+b+c}{2} = \frac{12+16+21}{2} = 24.5$$

$$A = \sqrt{24.5(24.5-12)(24.5-16)(24.5-21)} = \mathbf{95.45\ sq.\ units}$$

SOLUTION 12 - 9
Ans: A

A triangle with sides 8, 15, and 17 is a right triangle. ($17^2 = 8^2 + 15^2$)

If each side is doubled, it remains a right triangle with sides 16, 30, and 34.

The area is: $A = \tfrac{1}{2}(30)(16) =$ **240 square units**

SOLUTION 12 - 10
Ans: B

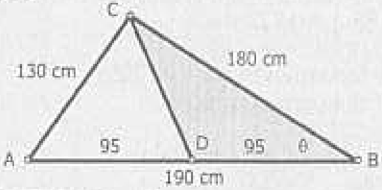
Let b be the base of the triangles.
h_1 = height of triangle 1 = $b + 3$
h_2 = height of triangle 2 = $b - 3$

$$A_1 - A_2 = 21$$
$$\tfrac{1}{2} b h_1 - \tfrac{1}{2} b h_2 = 21$$
$$\tfrac{1}{2} b(b+3) - \tfrac{1}{2} b(b-3) = 21$$
$$b^2 + 3b - b^2 + 3b = 21(2) = 42;\ b = 7$$

Thus;

$h_1 = 7 + 3 =$ **10 units**
$h_2 = 7 - 3 =$ **4 units**

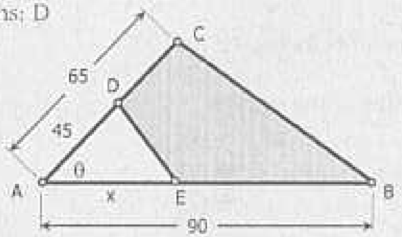
SOLUTION 12 - 11
Ans: C



$\text{Area} = \tfrac{1}{2}(180)(95)\sin\theta = 8550\sin\theta$

In triangle ABC:
$130^2 = 190^2 + 180^2 - 2(190)(180)\cos\theta$
$\theta = 41.03°$
$\text{Area} = 8550\sin 41.03°$
Area = **5612 cm²**

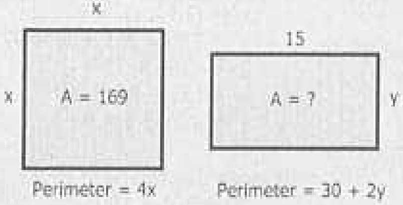
SOLUTION 12 - 12
Ans: D



$A_{ADE} = \tfrac{1}{4} A_{ABC}$
$\tfrac{1}{2}(45)(x)\sin\theta = \tfrac{1}{4}\left[\tfrac{1}{2}(65)(90)\sin\theta\right]$
$x = 32.5$

$EB = 90 - x =$ **57.5**

SOLUTION 12 - 13
Ans: A



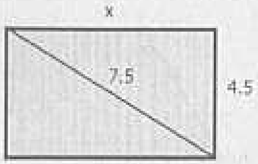
For the square:
$A = x^2 = 169,\ x = 13$ cm
Perimeter = $4x = 4(13) = 52$ cm

For the rectangle:
Perimeter = $30 + 2y = 52$
$y = 11$
Area = $15y = 15(11) =$ **165 sq. cm.**

SOLUTION 12 - 14
Ans: B

From the figure shown:

$x = \sqrt{(7.5)^2 - (4.5)^2} = 6$
Area = $4.5 \times x = 4.5(6)$
Area = **27 m²**



SOLUTION 12 - 15
Ans: C

Let x and y be the correct measure of the sides of the rectangle.

The correct area is then xy. The measured sides are $x - 0.15x = 0.85x$ and $y - 0.15y = 0.85y$, and the measured area is then $(0.85x)(0.85y) = 0.7225xy$

The measured area is too small by $xy - 0.7225xy = 0.2775xy$ or 27.75% of the correct area.

SOLUTION 12 - 16
Ans: B

Let "a" be the original side of the square.
Original perimeter = $4a$
New side = a + 100%a = $2a$
New perimeter = $4(2a) = 8a$
Therefore; the perimeter is increased by 100%

SOLUTION 12 - 17
Ans: B

Let x and y be the sides of the square.

Then;

$$4x + 4y = 52 \text{ or } y = 13 - x \quad \rightarrow (1)$$
$$x^2 + y^2 = 97 \quad \rightarrow (2)$$

Substitute y in Eq. (1) to Eq. (2):

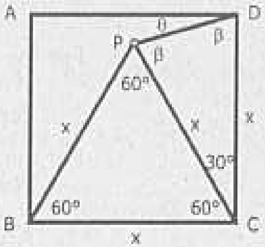
$$x^2 + (13 - x)^2 = 97$$
$$x^2 + 169 - 26x + x^2 = 97$$
$$2x^2 - 26x + 72 = 0$$
$$x^2 - 13x + 36 = 0$$
$$x = 4 \text{ and } y = 9$$

SOLUTION 12 - 18
Ans: B

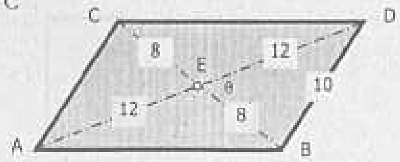
In isosceles triangle PDC:

$$2\beta + 30° = 180°$$
$$\beta = 75°$$

$$\theta = 90° - \beta = 90° - 75°$$
$$\theta = 15°$$



SOLUTION 12 - 19
Ans: C



Area = ½ $d_1 \times d_2 \sin\theta$

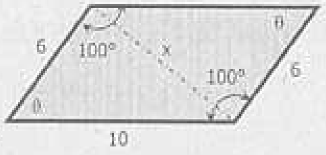
Solving for θ in triangle BED

$$10^2 = 8^2 + 12^2 - 2(8)(12)\cos\theta$$
$$\theta = 55.77°$$

Area = ½ (24)(16) sin 55.77°
Area = 158.74 m²

SOLUTION 12 - 20
Ans: C

The sum of the interior angles of a parallelogram (quadrilateral) is 360°.



$$2\theta + 2(100°) = 360°;\ \theta = 80°$$

By cosine law:

$$d^2 = 6^2 + 10^2 - 2(6)(10)\cos 80°$$
$$d = \mathbf{10.73\ units}$$

SOLUTION 12 - 21
Ans: C

The area of a rhombus is, A = ½ $d_1 \times d_2$, where d_1 and d_2 are the diagonals.
A = ½ (12) d_2 = 132; d_2 = **22 cm**

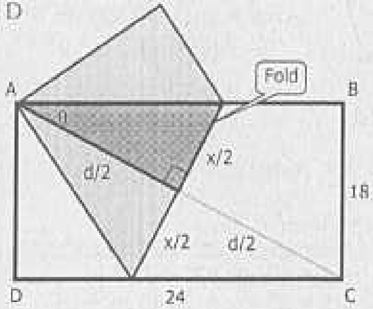
SOLUTION 12 - 22
Ans: D

A = ½ $d_1 d_2$ = ½(10)(8)
A = **40 sq. cm.**

SOLUTION 12 - 23
Ans: A

$$A = \sqrt{(s-a)(s-b)(s-c)(s-d)}$$
$$s = \frac{a+b+c+d}{2} = \frac{4+5+8+11}{2} = 14 \text{ cm}$$
$$A = \sqrt{(14-4)(14-5)(14-8)(14-11)} = \mathbf{40.25\ sq.\ cm.}$$

SOLUTION 12 - 24
Ans: D



$$d^2 = 18^2 + 24^2$$

diagonal, $d = 30$

$$\tan\theta = \frac{18}{24} = (x/2)/(d/2) = \frac{x}{d}$$

$$x = \frac{18}{24}d = \frac{18}{24}(30)$$
$$x = \mathbf{45/2\ units}$$

SOLUTION 12 - 25
Ans: B

Area of quadrilateral:

$$A = \sqrt{(s-a)(s-b)(s-c)(s-d) - abcd\cos^2\theta}$$
$$a = 10;\ b = 8;\ c = 16;\ d = 20$$
$$\theta = 225°/2$$
$$\theta = 112.5° \text{ (one-half the sum of any two opposite angles)}$$

$$s = \frac{a+b+c+d}{2} = \frac{10+8+16+20}{2} = 27$$
$$A = \sqrt{(27-10)(27-8)(27-16)(27-20) - 10(8)(16)(20)\cos^2 1125°}$$
$$A = \mathbf{145.33\ sq.\ cm.}$$

SOLUTION 12 - 26
Ans: D

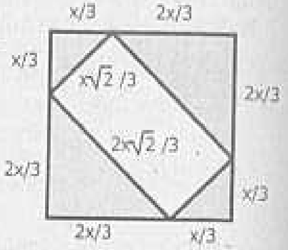
Area of square = x^2

Area of rectangle = $\frac{x\sqrt{2}}{3}\times\frac{2x\sqrt{2}}{3}$

Area of rectangle = $\frac{4}{9}x^2$

Ratio = $\frac{A_{rectangle}}{A_{square}} = \frac{\frac{4}{9}x^2}{x^2} = \frac{4}{9}$

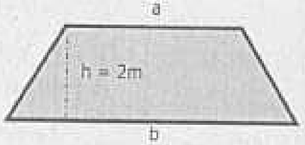
Therefore, the ratio is **4:9**



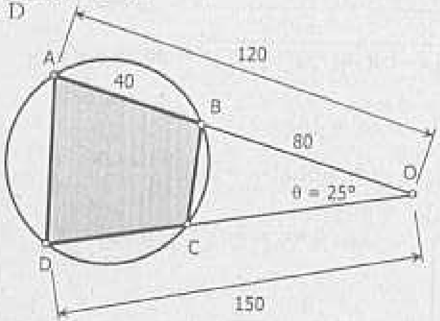
SOLUTION 12 - 27
Ans: C

$a/b = 4/5$; $a = 0.8b$

Area = ½$(a + b)\,h$
36 = ½ (0.8b + b) 2
b = 20 m
a = 0.8(20) = **16 m**



SOLUTION 12 - 28
Ans: D



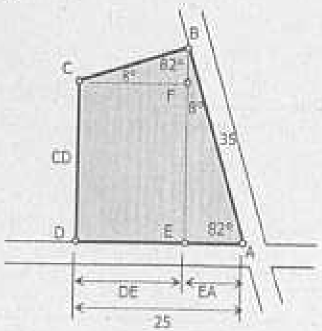
By principle:
$OB \times OA = OC \times OD$
$80(120) = OC(150)$
OC = 64 cm
Area = $A_{AOD} - A_{BOC}$

Area = ½$(OA)(OD)$ sin θ
- ½$(OB)(OC)$ sin θ

Area = ½(120)(150) sin 25°
- ½(80)(64) sin 25°

Area = **2722 cm²**

SOLUTION 12 - 29
Ans: D



$A_{lot} = A_{BCDE} + A_{ABE}$
BE = 35 sin 82° = 34.66 m
EA = 35 cos 82° = 4.87 m
DE = 25 – x = 20.13 m = CF
BF = 20.13 tan 8° = 2.83 m
CD = 34.66 – 2.83 = 31.83 m

$A_{lot} = \frac{CD + BE}{2}(DE) + ½(EA)(BE)$

$A_{lot} = \frac{31.83 + 34.66}{2}(20.13)$
+ ½ (4.87)(34.66)
A_{lot} = 753.62 sq. m.

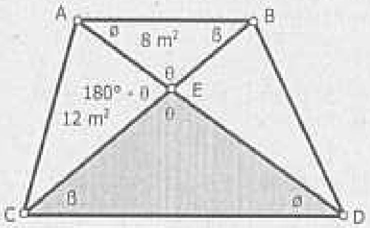
Cost = 753.62(2500) = **P1,884,050**

SOLUTION 12 - 30
Ans: A

$$A = ½\left|\frac{x_1}{y_1}\,\frac{x_2}{y_2}\dots\frac{x_n}{y_n}\,\frac{x_1}{y_1}\right| = ½\left|\frac{8}{-2}\,\frac{5}{6}\,\frac{4}{1}\,\frac{-7}{4}\,\frac{8}{-2}\right|$$

A = ½ {[8(6) + 5(1) + 4(4) + (-7)(-2)]
– [(-2)5 + 6(4) + 1(-7) + 4(8)]}
A = ½ [(48 + 5 + 16 + 14) – (-10 + 24 – 7 + 32)]
A = **22 square units**

SOLUTION 12 - 31
Ans: B



In the figure shown:
ΔABE is similar to ΔCED

$\frac{A_{ABE}}{A_{CED}} = \frac{BE^2}{CE^2}$ → (1)

$\frac{A_{ABE}}{A_{AEC}} = \frac{\frac{1}{2}AE \times BE \times \sin\theta}{\frac{1}{2}AE \times CE \times \sin(180-\theta)}$

$\frac{8}{12} = \frac{BE\sin\theta}{CE\sin(180-\theta)}$

But sin θ = sin (180 - θ),

$\frac{BE}{CE} = \frac{8}{12}$ → (2)

Substitute BE/CE of Eq. (2) to Eq. (1)

$\frac{A_{ABE}}{A_{CED}} = \left(\frac{8}{12}\right)^2 = \frac{8}{A_{CED}}$

A_{CED} = **18 sq. m.**

SOLUTION 12 - 32
Ans: A

The sum of the exterior (deflection) angles of any polygon is 360°

SOLUTION 12 - 33
Ans: C

The sum of the interior angles of a polygon is $(n - 2)180°$.
A dodecagon has 12 sides, then
Sum of interior angles = (12 – 2)(180°)
Sum of interior angles = **1800°**

SOLUTION 12 - 34
Ans: B

The sum of all interior angles of a polygon is $(n - 2)180°$. Since all interior angles of a regular polygon are equal, then the value of each angle is:

Interior angle = $\frac{(n-2)180°}{n}$

$165 = \frac{(n-2)180°}{n}$
$165n = 180n - 360$
n = **24**

SOLUTION 12 - 35
Ans: D

The sum of interior angles of a polygon is given by the formula:

$$\text{sum} = 180(n - 2)$$
$$540° = 180°(n - 2)$$
$$n = 5$$

SOLUTION 12 - 36
Ans: D

Sum of interior angles = $180(n - 2)$
Sum of interior angles = $180(n - 2) = 1080°$
$n = 8$

SOLUTION 12 - 37
Ans: A

The number of diagonals of a polygon of n sides is,

$$D = \frac{n}{2}(n-3)$$

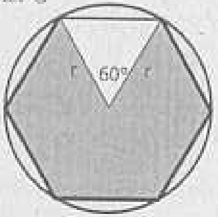
For a pentedecagon, $n = 15$, then $D = \frac{15}{2}(15-3) = 60$

SOLUTION 12 - 38
Ans: A

Number of diagonals, $D = 170 = \frac{n}{2}(n-3)$

$$340 = n^2 - 3n$$
$$n^2 - 3n - 340 = 0;\ n = 20$$

SOLUTION 12 - 39
Ans: C

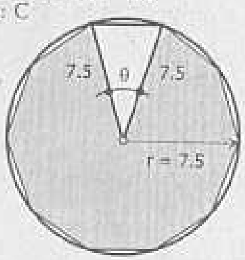


$A_{\text{hexagon}} = ½\, r^2 \sin 60° \times 6 = 93.53$
$r = 6$ cm

$A_{\text{circle}} = \pi r^2 = \pi\,(6)^2 = 113.097$

$A_{\text{required}} = A_{\text{circle}} - A_{\text{hexagon}}$
$A_{\text{required}} = 113.097 - 93.53$
A_{required} = **19.567 sq. cm.**

SOLUTION 12 - 40
Ans: C



A decagon has 10 sides ($n = 10$)
$\theta = 360°/10 = 36°$
$A_1 = ½\,(7.5)(7.5) \sin 36°$
$A_1 = 16.53$ cm²
$A_{\text{total}} = n\,A_1 = 10(16.53)$
A_{total} = **165.3 sq. cm.**

SOLUTION 12 - 41
Ans: C

Sum of interior angles of a polygon = $(n - 2)180°$, where n is the number of sides.

Sum of interior angles = $(n - 2)\,180° = 2520°$
$n = 16$

SOLUTION 12 - 42
Ans: B

The area of a regular polygon inscribed in a circle of radius r is:

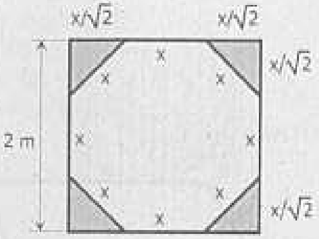
$$A = \frac{n}{2} r^2 \sin \frac{360}{n}, \text{ where } n = 6 \text{ for hexagon, and } r = 1.$$

$$A = \tfrac{6}{2}(1)^2 \sin \frac{360}{6} = 2.598 \text{ sq. units}$$

SOLUTION 12 - 43
Ans: D

From the figure shown:

$$x + \frac{x}{\sqrt{2}} + \frac{x}{\sqrt{2}} = 2$$
$x =$ **0.828 m**



SOLUTION 12 - 44
Ans: B

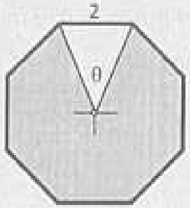
A polygon is classified according to the number of its sides n.

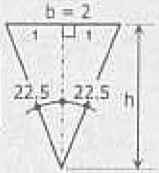
Solving for n from the relationship, $D = \frac{n}{2}(n - 3)$, where D is the number of diagonals = 27.

$$27 = \frac{n}{2}(n - 3);\ 54 = n^2 - 3n$$
$$0 = n^2 - 3n - 54 = (n - 9)(n + 6)$$

Thus, **$n = 9$ (a nonagon)**

SOLUTION 12 - 45
Ans: A





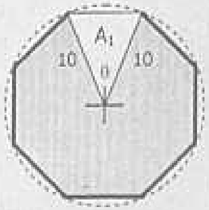
Octagon = 8 sides
$\theta = 360°/8 = 45°$

From the figure shown:
$h = 1 \cot 22.5°$
$h = 2.4142$ units
$A_{\text{triangle}} = ½\, bh$
$A_{\text{triangle}} = ½\,(2)\,(2.4142) = 2.4142$
$A_{\text{octagon}} = 8 \times A_{\text{triangle}}$
$A_{\text{octagon}} = 8 \times 2.4142 =$ **19.314 sq. units**

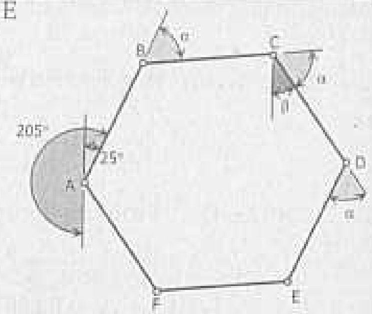
SOLUTION 12 - 46
Ans: C

From the figure shown:
$\theta = 360 / 8 = 45°$
$A_1 = ½\,(10)(10) \sin 45°$
$A_1 = 35.355$ sq. units

$A_{\text{total}} = 8 \times A_1 = 8 \times 35.355$
A_{total} = **282.84 sq. units**



SOLUTION 12 - 47
Ans: A

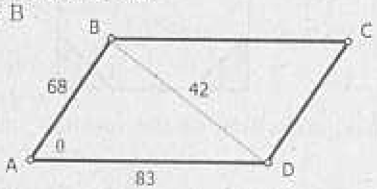


Bearing of AB = N 25° E
Azimuth of AB = 205°
$\alpha = 360°/6 = 60°$

Azimuth of CD:
$= 205° + 2\alpha$
$= 325°$

Bearing of CD
$\beta = 360° - 325°$
$\beta = 35°$ = **S 35° E**

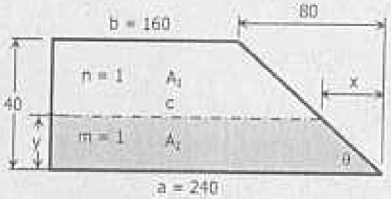
SOLUTION 12 - 48
Ans: B



From the figure, the smallest interior angle is θ.

By cosine law:
$42^2 = 68^2 + 83^2 - 2(68)(83)\cos\theta$
$\cos\theta = 0.86366$
$\theta = \mathbf{30.27°}$

SOLUTION 12 - 49
Ans: A



$$\text{Area of lot} = \frac{160 + 240}{2}(40)$$
Area of lot = 8,000 m²
$A_1 = A_2 = 4{,}000\text{ m}^2$
$$\frac{x}{y} = \frac{80}{40};\ y = 0.5x$$
$A_2 = 240(y) - ½(x)(y) = 4{,}000$
$240(0.5x) - 0.5(x)(0.5x) = 4{,}000$
$120x - 0.25x^2 = 4000$
$x^2 - 480x + 16{,}000 = 0;\ x = 36.039\text{ m}$

Length of dividing line, $c = 240 - x =$ **203.96 m**

Using the formula:
$$c = \sqrt{\frac{ma^2 + nb^2}{m+n}}\text{, where } \frac{m}{n} = \frac{A_2}{A_1}$$
$$c = \sqrt{\frac{1(240)^2 + 1(160)^2}{1+1}} = 203.96$$

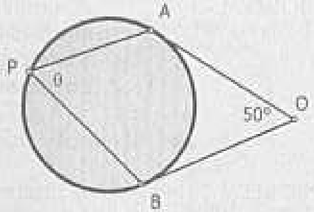
SOLUTION 12 - 50
Ans: A

$$\text{Number of diagonals, } D = \frac{n}{2}(n-3)$$
$$35 = \frac{n}{2}(n-3);\quad n^2 - 3n - 70 = 0$$
$(n-10)(n+7) = 0;\ n = $ **10 (decagon)**

Problems - Set 13
Circles, Parabola, Ellipse, & Miscellaneous Figures

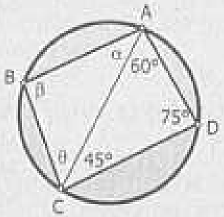
PROBLEM 13 - 1

In the figure shown OA and OB are tangent to the circle. If $\angle AOB$ is 50°, find the $\angle APB$.
A. 45°
B. 50°
C. 60°
D. 65°



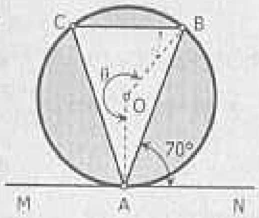
PROBLEM 13 - 2

In the figure shown, arc BC is half the length of arc CD. Solve for θ.
A. 30°
B. 40°
C. 45°
D. 50°



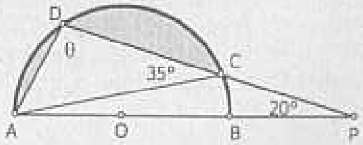
PROBLEM 13 - 3

In the figure shown, MAN is a tangent to the circle of center O. If $\angle BAN = 70°$ find θ.
A. 220°
B. 210°
C. 140°
D. 250°



PROBLEM 13 - 4

In the figure shown, AB is the diameter of the circle. Find angle θ.
A. 100°
B. 105°
C. 110°
D. 210°

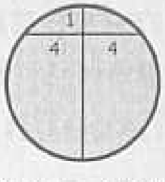


PROBLEM 13 - 5
ME April 1996

The area of a circle is 89.42 square inches. What is the circumference?
A. 35.33 inches
B. 32.25 inches
C. 33.52 inches
D. 35.55 inches

PROBLEM 13 - 6

Find the area of the circle shown.
A. 152.53 sq. units
B. 193.30 sq. units
C. 215.30 sq. units
D. 226.98 sq. unit



PROBLEM 13 - 7
CE May 1998

A circle whose area is 452 cm square is cut into two segment by a chord whose distance from the center of the circle is 6 cm. Find the area of the larger segment in cm square.
A. 372.5 C. 368.4
B. 363.6 D. 377.6

PROBLEM 13 - 8
CE Nov. 1998

A circle is divided into two parts by a chord, 3 cm a way from the center. Find the area of the smaller part, in cm square, if the circle has an area of 201 cm square.
A. 51.4 C. 55.2
B. 57.8 D. 53.7

PROBLEM 13 - 9

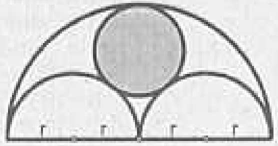
A quadrilateral *ABCD* is inscribed in a semi-circle with side *AD* coinciding with the diameter of the circle. If sides *AB*, *BC*, and *CD* are 8cm, 10cm, and 12cm long, respectively, find the area of the circle.
A. 317 sq. cm. C. 456 sq. cm.
B. 356 sq. cm. D. 486 sq. cm.

PROBLEM 13 - 10

A semi-circle of radius 14 cm is formed from a piece of wire. If it is bent into a rectangle whose length is 1cm more than its width, find the area of the rectangle.
A. 256.25 sq. cm. C. 386.54 sq. cm.
B. 323.57 sq. cm. D. 452.24 sq. cm.

PROBLEM 13 - 11

Find the area of the shaded circle shown.
A. $\frac{2}{3}\pi r^2$
B. $\frac{2}{9}\pi r^2$
C. $\frac{4}{3}\pi r^2$
D. $\frac{4}{9}\pi r^2$



PROBLEM 13 - 12
ECE April 1998

The angle of a sector is 30 degrees and the radius is 15 cm. What is the area of the sector?
A. 89.5 cm² C. 59.8 cm²
B. 58.9 cm² D. 85.9 cm²

PROBLEM 13 - 13

A sector has a radius of 12 cm. If the length of its arc is 12 cm, its area is:

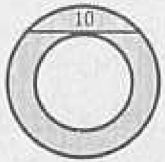
A. 66 sq. cm. B. 144 sq. cm.
B. 82 sq. cm. D. 72 sq. cm.

PROBLEM 13 - 14

The perimeter of a sector is 9 cm and its radius is 3 cm. What is the area of the sector?
A. 4 cm² C. 11/2 cm²
B. 9/2 cm² D. 27/2 cm²

PROBLEM 13 - 15

Find the area of the shaded portion.
A. 50π C. 20π
B. 25π D. 30π



PROBLEM 13 - 16

A swimming pool is to be constructed in the shape of partially overlapping identical circles. Each of the circles has a radius of 9 m, and each passes through the center of the other. Find the area of the swimming pool.
A. 302.33 m² C. 398.99 m²
B. 362.55 m² D. 409.44 m²

PROBLEM 13 - 17

Given are two concentric circles with the outer circle having a radius of 10 cm. If the area of the inner circle is half of the outer circle, find the boarder between the two circles.
A. 2.930 cm C. 3.265 cm
B. 2.856 cm D. 2.444 cm

PROBLEM 13 - 18

A circle of radius 5 cm has a chord which is 6cm long. Find the area of the circle concentric to this circle and tangent to the given chord.
A. 14π C. 9π
B. 16π D. 4π

PROBLEM 13 - 19
ECE April 1995

A reversed curve on a railroad track consist of two circular arcs. The central angle of one side is 20° with radius 2500 feet, and the central angle of the other is 25° with radius 3000 feet. Find the total lengths of the two arcs.
A. 2812 ft. C. 2821 ft.
B. 2218 ft. D. 2182 ft.

PROBLEM 13 - 20

Given a triangle whose sides are 24 cm, 30 cm, and 36 cm. Find the radius of a circle which is tangent to the shortest and longest side of the triangle, and whose center lies on the third side.
A. 9.111 cm C. 12.31 cm
B. 11.91 cm D. 18 cm

PROBLEM 13 - 21

Find the area of the largest circle that can be cut from a triangle whose sides are 10 cm, 18 cm, and 20 cm.

A. 11π cm² C. 14π cm²
B. 12π cm² D. 15π cm²

PROBLEM 13 - 22

The diameter of the circle circumscribed about a triangle *ABC* with sides a, b, c is equal to:

A. $a/\sin A$ C. $c/\sin C$
B. $b/\sin B$ D. all of the above

PROBLEM 13 - 23

The sides of a triangle are 14 cm., 15 cm., and 13 cm. Find the area of the circumscribing circle.

A. 207.4 sq. cm. C. 215.4 sq. cm.
B. 209.6 sq. cm. D. 220.5 sq. cm.

PROBLEM 13 - 24
CE May 1996

What is the radius of the circle circumscribing an isosceles right triangle having an area of 162 sq. cm.?

A. 13.52 C. 12.73
B. 14.18 D. 1564

PROBLEM 13 - 25
ECE Nov. 1996

If the radius of the circle is decreased by 20%, by how much is its area decreased?

A. 36% C. 46%
B. 26% D. 56%

PROBLEM 13 - 26
ECE April 1998

The distance between the center of the three circles which are mutually tangent to each other externally are 10, 12 and 14 units. The area of the largest circle is.

A. 72 π C. 64 π
B. 23 π D. 16 π

PROBLEM 13 - 27

The sides of a cylic quadrilateral measures 8 cm, 9 cm, 12 cm, and 7 cm, respectively. Find the area of the circumscribing circle.

A. 8.65 cm² C. 6.54 cm²
B. 186.23 cm² D. 134.37 cm

PROBLEM 13 - 28
ECE Nov. 1997

The wheel of a car revolves n times, while the car travels x km. The radius of the wheel in meter is:

A. $10{,}000\,x\,/\,(\pi\,n)$ C. $500{,}000\,x\,/\,(\pi\,n)$
B. $500\,x\,/\,(\pi\,n)$ D. $5{,}000\,x\,/\,(\pi\,n)$

PROBLEM 13 - 29

If the inside wheels of a car running a circular track are going half as fast as the outside wheel, determine the length of the track, described by the outer wheels, if the wheels are 1.5 m apart.

A. 4π C. 6π
B. 5π D. 8π

PROBLEM 13 - 30
ME Oct. 1997

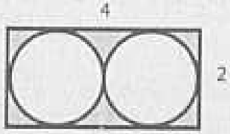
A goat is tied to a corner of a 30 ft by 35 ft building. If the rope is 40 ft long and the goat can reach 1 ft farther than the rope length, what is the maximum area the goat can cover?

A. 5281 ft² C. 3961 ft²
B. 4084 ft² D. 3970 ft²

PROBLEM 13 - 31

What is the area of the shaded portion shown?

A. 8 - 8π
B. 8 - 2π
C. 8 - 4π
D. 8 - π



PROBLEM 13 - 32

The interior angles of a triangle measures $2x$, $x + 15$, and $2x + 15$. What is the value of x?

A. 30° C. 42°
B. 66° D. 54°

PROBLEM 13 - 33

Two complementary angles are in the ratio 2:1. Find the larger angle.

A. 30° C. 75°
B. 60° D. 15°

PROBLEM 13 - 34

Two transmission towers 40 feet high is 200 feet apart. If the lowest point of the cable is 10 feet above the ground, the vertical distance from the roadway to the cable 50 feet from the center is:

A. 17.25 feet C. 17.75 feet
B. 17.5 feet D. 18 feet

PROBLEM 13 - 35
CE May 1996

What is the area bounded by the curves $y^2 = 4x$ and $x^2 = 4y$?

A. 6.0 C. 6.666
B. 7.333 D. 5.333

PROBLEM 13 - 36
ME April 1998

What is the area between $y = 0$, $y = 3x^2$, $x = 0$, and $x = 2$?

A. 8 C. 24
B. 12 D. 6

PROBLEM 13 - 37
CE May 2003

A circle having an area of 224 sq. m. is inscribed in an octagon. Find the area of the octagon.

A. 238.6 sq. m. C. 236.3 sq. m.
B. 245.2 sq. m. D. 246.7 sq. m.

PROBLEM 13 - 38
CE Nov 2002

A circle is circumscribed about a hexagon. Determine the area of the hexagon if the area outside the hexagon but inside the circle is 15 sq. cm.

A. 73.3 sq. cm. C. 71.7 sq. cm.
B. 72.4 sq. cm D. 74.8 sq. cm.

PROBLEM 13 - 39
CE May 2002

A circle of radius 8 cm is inscribed in a sector having a central angle of 80°. What is the area of the sector?

A. 195.63 cm^2 C. 321.47 cm^2
B. 291.84 cm^2 D. 475.42 cm^2

PROBLEM 13 - 40

A triangular piece of land has one side measuring 2 km. The land is to be divided into two equal areas by a dividing line parallel to the given side. What is the length of the dividing line?

A. 6 C. 7.623
B. 8.485 D. 8

PROBLEM 13 - 41
CE May 2002

A piece of wire having a total length of 72 cm was cut into two unequal segments and bent to from two unequal squares. If the total area of the squares is 180 sq. cm, what is the difference in the lengths of the two segments?

A. 24 cm C. 32 cm
B. 16 cm D. 28 cm

PROBLEM 13 - 42
CE Nov. 2003

Determine the area of a regular hexagon inscribed in a circle having an area of 170 square centimeters.

A. 169.8 C. 148.2
B. 124.1 D. 140.6

PROBLEM 13 - 43

Three circles of radii 110, 140, and 220 are tangent to one another. What is the area of the triangle formed by joining the centers of the circles?

A. 39,904 D. 32,804
B. 25,476 C. 47,124

PROBLEM 13 - 44

A circle with area 254.469 sq. cm. is circumscribed about a triangle whose area is 48.23 square cm. If one side of the triangle measure 18 cm, determine length of the shorter leg of the triangle in cm.

A. 3.625 C. 8.652
B. 4.785 D. 5.643

PROBLEM 13 - 45

A road is tangent to a circular lake. Along the road and 12 miles from the point of tangency, another road opens towards the lake. From the intersection of the two roads to the periphery of the lake, the length of the new road is 11 miles. If the new road will be prolonged across the lake, find the length of the bridge to be constructed.

A. 2.112 mi C. 2.103 mi
B. 2.091 mi D. 2.512 mi

PROBLEM 13 - 46

The center of two circles with radii of 3 m and 5 m, respectively are 4 m apart. Find the area of the portion of smaller circle outside the larger circle.

A. 11.25 m^2 C. 9.75 m^2
B. 12.15 m^2 D. 10.05 m^2

PROBLEM 13 - 47

Find the area in square centimeter of the largest square that can be cut from a sector of a circle radius 8 cm and central angle 120°.

A. 21.9 C. 33.5
B. 45.2 D. 54.8

PROBLEM 13 - 48

A circle is inscribed in a square and circumscribed about another. Determine the ratio of the area larger square to the area of the smaller square.

A. 2:1 C. 1:4
B. 1:2 D. 4:1

PROBLEM 13 - 49

One side of a rectangle, inscribed in a circle of diameter 17 cm, is 8 cm. Find the length of the other side.

A. 16 cm C. 14 cm
B. 15 cm D. 13 cm

PROBLEM 13 - 50

The sum of the sides of two polygons is 12 and the sum of their diagonals is 19. The polygons are:

A. pentagon & heptagon C. quadrilateral & octagon
B. both hexagon D. triangle & nonagon

ANSWER SHEET

1. Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
2. Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
3. Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
4. Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
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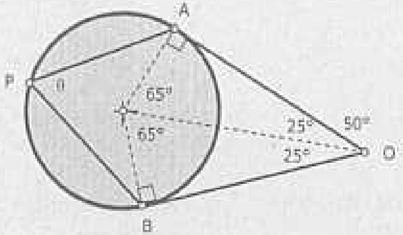
57. Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
58. Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
59. Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
60. Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ

Solutions to Set 13 Circles, Parabola, Ellipse, & Miscellaneous Figures

SOLUTION 13 - 1
Ans: D

By principle, the measure of angle θ is half the arc *AB*.

$\theta = 65°$



SOLUTION 13 - 2
Ans: C

By principle, $\angle B + \angle D = 180°$
$\beta + 75° = 180°, \beta = 105°$

If arc *BC* is ½ of arc *CD*, then
$\alpha = \frac{1}{2}(60°) = 30°$

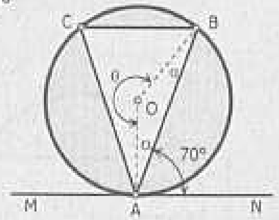
In triangle *ABC*, $\theta + \beta + \alpha = 180°$
$\theta + 105° + 30° = 180°;\ \theta = 45°$

SOLUTION 13 - 3
Ans: A

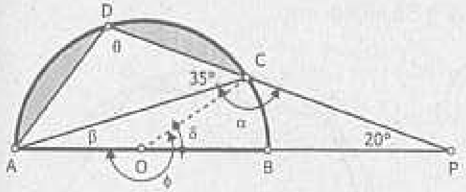
In the figure shown, $\alpha + 70° = 90°$
$\alpha = 20°$

In triangle *BOA*, $\beta + 2\alpha = 180°$
$\beta = 180° - 2(20°) = 140°$

$\theta = 360° - \beta = 360° - 140°$
$\theta = 220°$



SOLUTION 13 - 4
Ans: B



From the figure shown:
$\alpha = 180 - 35° = 145°$

In triangle *ACP*
$\beta = 180° - 20° - 145° = 15°$

By principle, $\delta = 2\beta = 2 \times 15°$
$\delta = 30°$
$\phi = 180 + \delta = 180° + 30° = 210°$

By principle, $\phi = 2\theta$
$210° = 2\theta;\ \theta = 105°$

SOLUTION 13 - 5
Ans: C

Circumference, $C = 2\pi r$

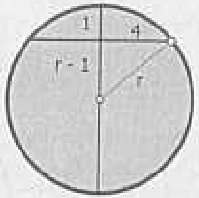
Solve for r:

$\text{Area} = \pi r^2 = 89.42;\ r = 5.335$
$C = 2\pi(5.335) = $ **33.52 inches**

SOLUTION 13 - 6
Ans: D

From the right triangle shown,

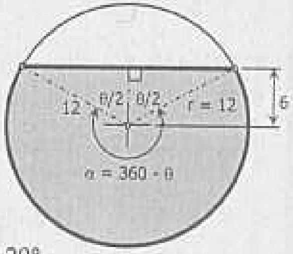
$r^2 = (r-1)^2 + 4^2$
$r^2 = r^2 - 2r + 1 + 16$
$2r = 17;\ r = 17/2$



Area of circle $= \pi r^2 = \pi(17/2)^2$
Area of circle = **226.98 sq. units**

SOLUTION 13 - 7
Ans: B

Area of circle $= \pi r^2 = 452$
$r = 12$ cm



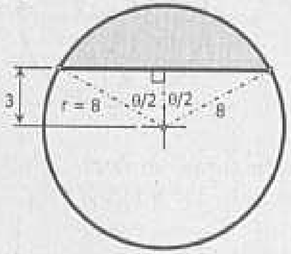
$\cos(\theta/2) = 6/12;\ \theta = 120°$
$\alpha = 360 - 120 = 240°$

$$A = \frac{\pi r^2 \alpha}{360} + \tfrac{1}{2} r^2 \sin\theta$$

$$A = \frac{\pi(12)^2(240°)}{360°} + \tfrac{1}{2}(12)^2 \sin 120°$$

$A = $ **363.95 sq. cm.**

SOLUTION 13 - 8
Ans: D



Area of circle $= \pi r^2 = 201$
$r = 8$ cm

$\cos(\theta/2) = 3/8$
$\theta = 135.95°$

$A = A_{\text{sector}} - A_{\text{triangle}}$

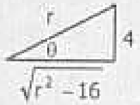
$$A = \frac{\pi(8)^2(135.95°)}{360°} - \tfrac{1}{2}(8)^2 \sin 135.95°$$

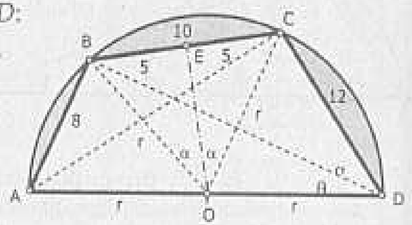
$A = $ **53.68 sq. cm.**

SOLUTION 13 - 9
Ans: A

In right triangle ABD:

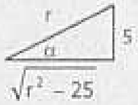
$\sin\theta = 8/2r = 4/r$





$$\cos\theta = \frac{\sqrt{r^2 - 16}}{r}$$

In right triangle OEB:



$$\sin\alpha = 5/r;\ \cos\alpha = \frac{\sqrt{r^2-25}}{r}$$

In right triangle ACD:

$\cos(\theta + \alpha) = 12/2r = 6/r$

but $\cos(\theta + \alpha) = \cos\theta \times \cos\alpha - \sin\theta \times \sin\alpha$

$$\frac{6}{r} = \frac{\sqrt{r^2-16}}{r}\,\frac{\sqrt{r^2-25}}{r} - \frac{4}{r}\,\frac{5}{r}$$

$$6r = \sqrt{(r^2-16)(r^2-25)} - 20$$

$$6r + 20 = \sqrt{(r^2-16)(r^2-25)} \qquad \text{square both side}$$

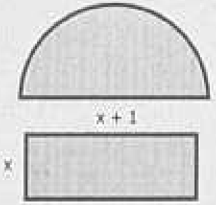
$$36r^2 + 240r + 400 = r^4 - 41r^2 + 400$$

$$0 = r^4 - 77r^2 - 240r;\quad 0 = r^3 - 77r^2 - 240$$

By trial and error: $r = 10.045$ cm
Area of the circle $= \pi r^2 = \pi(10.045)^2 = $ **317 sq. cm.**

SOLUTION 13 - 10
Ans: B

$P_{\text{semi circle}} = P_{\text{rectangle}}$
$\pi(14) + 2(14) = 2[x + (x+1)]$
$71.98 = 4x + 2;\ x = 17.49$



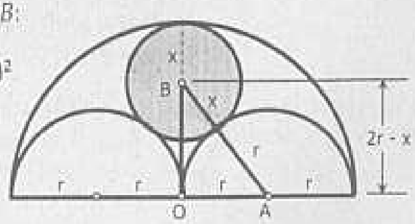
Area of rectangle:

$A = x(x+1)$
$A = 17.49(17.49 + 1)$
$A = $ **323.4 sq. cm.**

SOLUTION 13 - 11
Ans: D

In right triangle AOB:

$(r+x)^2 = r^2 + (2r-x)^2$
$r^2 + 2rx + x^2 = r^2 + 4r^2 - 4rx + x^2$
$6rx = 4r^2$
$x = \frac{2}{3} r$



Area $= \pi x^2 = \pi[(2/3)r]^2 = (4/9)\pi r^2$

SOLUTION 13 - 12
Ans: B

The area of a sector of radius r and central angle θ (in degrees) is:

$$\text{Area} = \pi r^2 \theta / 360°$$
$$\text{Area} = \pi (15)^2 (30°) / 360° = \mathbf{58.9\ cm^2}$$

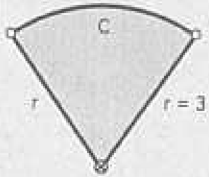
SOLUTION 13 - 13
Ans: D

Area of sector = ½ $C \times r$, where C is the length of arc.

$$\text{Area} = ½ (12)(12)$$
$$\text{Area} = \mathbf{72\ sq.\ cm.}$$

SOLUTION 13 - 14
Ans: B

$$\text{Perimeter} = 2r + C$$
$$9 = 2(3) + C;\ C = 3$$



$$\text{Area} = ½\, r\, C = ½\,(3)(3)$$
$$\text{Area} = \mathbf{9/2\ cm^2}$$

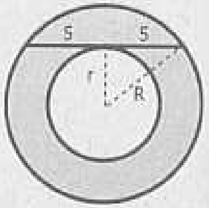
SOLUTION 13 - 15
Ans: B

$$A_{\text{shaded}} = \pi R^2 - \pi r^2 = \pi(R^2 - r^2)$$

From the right triangle shown:

$$R^2 = r^2 + 5^2$$
$$R^2 - r^2 = 25$$

$$A_{\text{shaded}} = \pi(25) = \mathbf{25\pi}$$

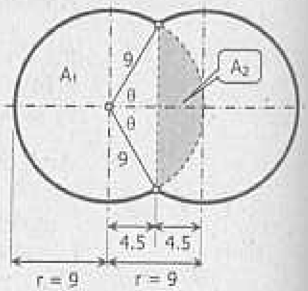


SOLUTION 13 - 16
Ans: D

$$\text{Area} = 2\,A_1$$
$$A_1 = \pi(9)^2 - A_2$$
$$\cos\theta = 4.5/9;\ \theta = 60°$$
$$2\theta = 120°$$

$$A_2 = A_{\text{sector}} - A_{\text{triangle}}$$
$$A_2 = \frac{\pi(9)^2(120°)}{360°} - ½(9)^2 \sin 120°$$
$$A_2 = 49.75\ m^2$$

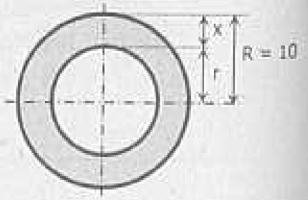
$$A_1 = \pi(9)^2 - 49.75 = 204.72\ m^2$$
$$\text{Area} = 2(204.72) = \mathbf{409.44\ m^2}$$



SOLUTION 13 - 17
Ans: A

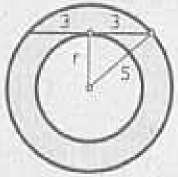
$$\text{Border},\ x = 10 - r$$
$$A_{\text{inner}} = ½\, A_{\text{outer}}$$
$$\pi r^2 = ½\, \pi(10)^2$$
$$r = 7.07\ \text{cm}$$

$$\text{Border} = 10 - 7.07$$
$$\text{Border} = \mathbf{2.93}$$



SOLUTION 13 - 18
Ans: B

$$r = \sqrt{5^2 - 3^2} = 4$$
$$A = \pi r^2 = \pi(4)^2$$
$$A = \mathbf{16\pi}$$



SOLUTION 13 - 19
Ans: D

Length of arc, $C = r \times \theta_r$, where r is the radius and θr is the central angle in radians

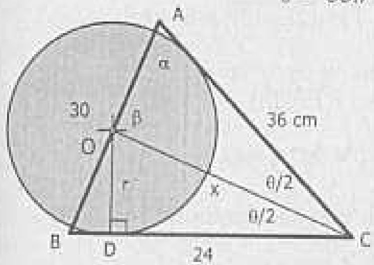
Total length of arc, $C = C_1 + C_2$

$$C = 2500 \times 20°(\pi/180°) + 3000 \times 25°(\pi/180°)$$
$$C = \mathbf{2181.66\ feet}$$

SOLUTION 13 - 20
Ans: B

Solve for θ in triangle ABC using cosine law:

$$30^2 = 36^2 + 24^2 - 2(36)(24)\cos\theta$$
$$\theta = 55.77°;\ \theta/2 = 27.866°$$



Solve for α in triangle AOC using sine law:

$$\frac{24}{\sin\alpha} = \frac{30}{\sin\theta} = \frac{30}{\sin(55.77°)}$$
$$\alpha = 41.41°$$
$$\beta = 180 - \theta/2 - \alpha = 110.704°$$

Solve for x in triangle AOC using sine law:

$$\frac{x}{\sin\alpha} = \frac{36}{\sin\beta}$$
$$x = 36 \sin(41.41°)/\sin(110.704°) = 25.456\ \text{cm}$$

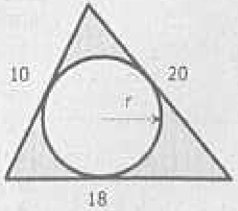
From the right triangle ODC:

$$r = x \sin(\theta/2) = 25.456 \sin(27.886°)$$
$$r = \mathbf{11.91\ cm}$$

SOLUTION 13 - 21
Ans: C

Radius of circle, $r = A_T / s$

$$s = (a + b + c)/2 = (10 + 20 + 18)/2$$
$$s = 24$$
$$A_T = \sqrt{s(s-a)(s-b)(s-c)}$$
$$A_T = \sqrt{24(14)(4)(6)}$$
$$A_T = 89.7998\ \text{cm}^2$$



$$r = 89.8 / 24 = 3.74166\ \text{cm}$$
$$A_{\text{circle}} = \pi r^2 = \pi(3.74166)^2$$
$$A_{\text{circle}} = \mathbf{14\pi\ cm^2}$$

SOLUTION 13 - 22
Ans: D

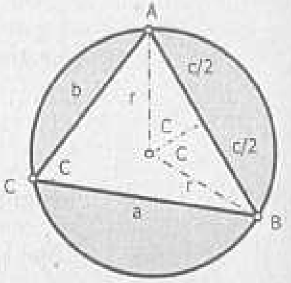
$$\sin C = \frac{c/2}{r} = \frac{c}{2r} = \frac{c}{D}$$

$$\text{Diameter, } D = \frac{c}{\sin C}$$

Similarly:

$$D = \frac{c}{\sin C}$$

$$D = \frac{a}{\sin A} = \frac{b}{\sin B} \text{ (sine law)}$$



SOLUTION 13 - 23
Ans: A

$a = 14$, $b = 15$, and $c = 13$

$s = (a + b + c)/2 = (14 + 15 + 13)/2 = 21$

$A_{Triangle} = \sqrt{s(s-a)(s-b)(s-c)}$

$A_{Triangle} = \sqrt{21(21-14)(21-15)(21-13)} = 84 \text{ cm}^2$

The radius of circumscribing circle is:

$r = abc\ /\ (4A_{Triangle}) = 14(15)(13)\ /\ (4 \times 84)$

$r = \underline{8.125 \text{ cm}}$

$A_{circle} = \pi r^2 = \pi(8.125)^2 = 207.4 \text{ cm}^3$

The diameter of the circle can also be found by using the relationship:

$D = a/\sin A = b\ /\ \sin B = c\ /\ \sin C$

Solving for A by cosine law:

$14^2 = 15^2 + 13^2 - 2(15)(13) \cos A$

$A = 59.4898°$

$D = 14\ /\ \sin 59.4898° = 16.25$, or $r = \underline{8.125 \text{ cm}}$

SOLUTION 13 - 24
Ans: C

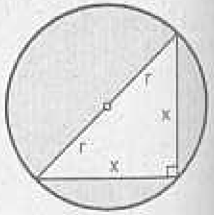
$A_{triangle} = ½\, x^2 = 162$

$x = 18$ cm

The hypotenuse of the triangle shown is the diameter of the circle.'

$2r = \sqrt{(18)^2 + (18)^2} = 25.456$ cm

$r =$ **12.728 cm**



Note: The diameter of the circle circumscribed about any right triangle always coincide with the hypotenuse of the triangle.

SOLUTION 13 - 25
Ans: A

Let r be the original radius, then the reduced radius is $0.8r$.

Since all circles are similar;

$$\frac{A_{orig}}{A_{reduced}} = \left(\frac{r_{orig}}{r_{reduced}}\right)^2 = \left(\frac{r}{0.8r}\right)^2 = \frac{1}{0.64}$$

$A_{reduced} = 0.64\, A_{orig} = 64\%$ of the original area

Therefore the area is reduced by **36%**

SOLUTION 13 - 26
Ans: C

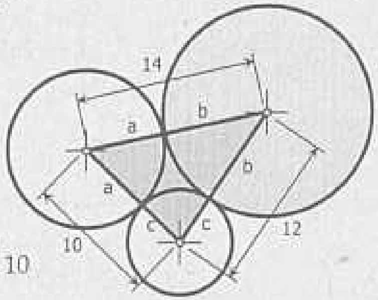
Let a, b, and c, respectively be the radius of each circle.

From the figure shown:

$a + b = 14 \rightarrow (1)$

$a + c = 10 \rightarrow (2)$

$b + c = 12 \rightarrow (3)$



Subtract: Eq. (1) - Eq. (2)

$[a + b] - [a + c] = 14 - 10$

$b - c = 4 \rightarrow (4)$

Add: Eq. (3) + Eq. (4)

$[b + c] + [b - c] = 12 + 4$

$2b = 16$; $b = 8$, $c = 4$, $a = 6$

The largest circle has a radius of 8.
The area is $\pi(8)^2 =$ **64 π sq. units**

SOLUTION 13 - 27
Ans: D

Given: $a = 8$, $b = 9$, $c = 12$, and $d = 7$.
Radius of circumscribing circle,

$$r = \frac{\sqrt{(ab+cd)(ac+bd)(ad+bc)}}{4\, A_{quadrilateral}}$$

$$A_{quadrilateral} = \sqrt{(s-a)(s-b)(s-c)(s-d)}$$

$$s = \frac{a+b+c+d}{2} = \frac{8+9+12+7}{2} = 18$$

$$A_{quadrilateral} = \sqrt{(18-8)(18-9)(18-12)(18-7)} = 77.07 \text{ cm}^2$$

$$r = \frac{\sqrt{(8\times9+12\times7)(8\times12+9\times7)(8\times7+9\times12)}}{4(77.07)} = 6.54 \text{ cm}$$

Area $= \pi r^2 =$ **134.37**

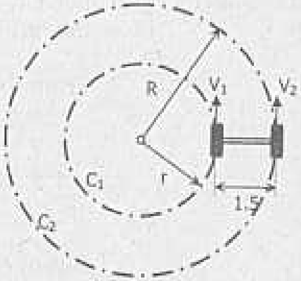
SOLUTION 13 - 28
Ans: B

The distance traveled is given by $S = r \times \theta$, where θ is the total angular displacement in radians and r is the radius of the wheel.

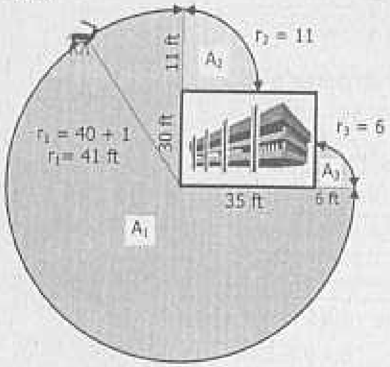
$S = x$ km $= 1000x$ meters
$\theta = n$ rev $\times 2\pi$ rad / rev $= 2\pi n$
$1000x = r \times 2\pi n$
$\mathbf{r = 500x / (\pi n)}$

SOLUTION 13 - 29
Ans: C

If $V_1 = \frac{1}{2} V_2$, then $C_1 = \frac{1}{2} C_2$
$C_1 = 2\pi r$
$C_2 = 2\pi R$
$r = R - 1.5$
$C_1 = \frac{1}{2} C_2$
$2\pi r = \frac{1}{2}(2\pi R)$
$2(R - 1.5) = R; R = 3$
$\mathbf{C_2 = 2\pi(3) = 6\pi\ m}$



SOLUTION 13 - 30
Ans: B



From the figure shown:
$A = A_1 + A_2 + A_3$
$A = (3/4)\pi r_1^2 + (1/4)\pi r_2^2 + (1/4)\pi r_3^2$
$A = (3/4)\pi (41)^2 + (1/4)\pi (11)^2 + (1/4)\pi(6)^2$
$A = 4084$ ft²

SOLUTION 13 - 31
Ans: B

Area shaded = Area of rectangle - 2(Area of circle)
Area shaded = $4(2) - 2[\pi(1)^2]$
Area shaded = **$(8 - 2\pi)$ sq. units**

SOLUTION 13 - 32
Ans: A

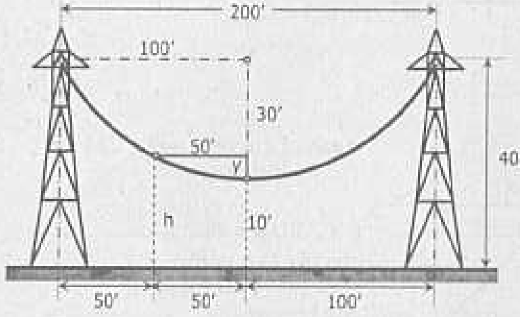
Sum of interior angles = 180°
$2x + (x + 15) + (2x + 15) = 180°$
$5x + 30° = 180°$
$5x = 150°; x = 30°$

SOLUTION 13 - 33
Ans: B

Let x and y be the angles,
then $x + y = 90°$ → (1)
and $x : y = 2 : 1$ or $x = 2y$ → (2)

Then;
$2y + y = 90° = 3y$
$y = 30°$ and $x = 60°$

SOLUTION 13 - 34
Ans: B



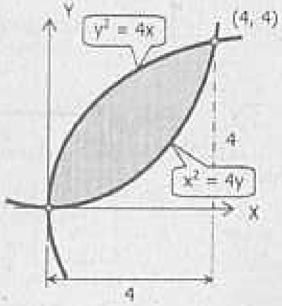
By squared property of parabola:
$$\frac{100^2}{30} = \frac{50^2}{y}$$
$y = 7.5'$

$h = 10 + y$
$\mathbf{h = 17.5'}$

SOLUTION 13 - 35
Ans: D

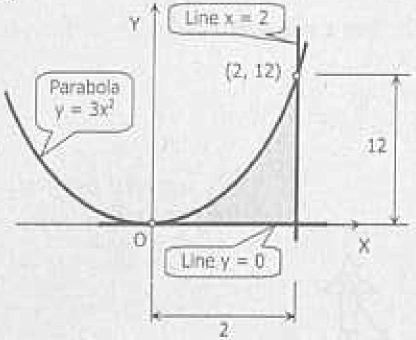
Points of intersection:
$y^2 = 4x$ square both side
$y^4 = 16x^2$
$y^4 = 16(4y) = 64y$
$y^4 - 64y = 0$
$y(y^3 - 64) = 0$
$y = 0$ and
$y = 4$

When $y = 0, x = 0$ (0, 0)
When $y = 4, x = 4$ (4, 4)



Using the formula:
Area = $A_{parabola} - A_{spandrel}$
Area = $\frac{2}{3}(4)(4) - \frac{1}{3}(4)(4)$
Area = **5.333 sq. units**

SOLUTION 13 - 36
Ans: A



The plane area has the form of a spandrel.

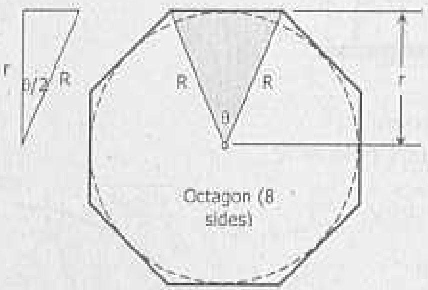
When $x = 2$, $y = 3(2)^2 = 12$

The area is given by the formula, $A = (1/3)\,bh$

Area = (1/3)(2 × 12)
Area = **8 square units**

SOLUTION 13 - 37
Ans: C

Area of circle $= \pi r^2 = 224\text{ m}^2$
$r = 8.444$



$\theta = 360/8 = 45°$
$\cos(\theta/2) = r/R$
$\cos(45°/2) = 8.444/R$
$R = 9.1397$

$A_{octagon} = 8 \times A_{triangle}$
$A_{octagon} = 8 \times ½(9.1397)^2 \sin 45°$
$A_{octagon} =$ **236.27 m²**

SOLUTION 13 - 38
Ans: C

A_1 = Area of sector – Area of triangle

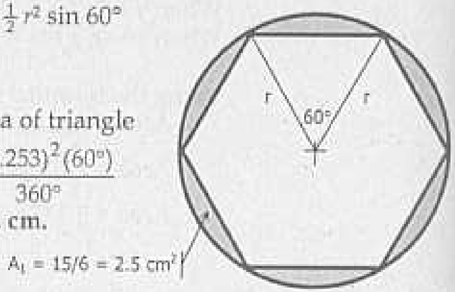
$$2.5 = \frac{\pi r^2(60°)}{360°} - \tfrac{1}{2} r^2 \sin 60°$$

$r = 5.253$ cm

$A_{hexagon}$ = 6 × Area of triangle

$$A_{hexagon} = 6 \times \frac{\pi(5.253)^2(60°)}{360°}$$

$A_{hexagon} =$ **71.7 sq. cm.**



SOLUTION 13 - 39
Ans: B

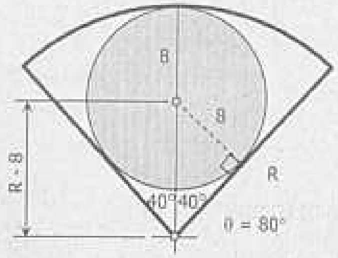
$$\sin 40° = \frac{8}{R-8}$$

$R - 8 = 12.4458$
$R = 20.4458$ cm

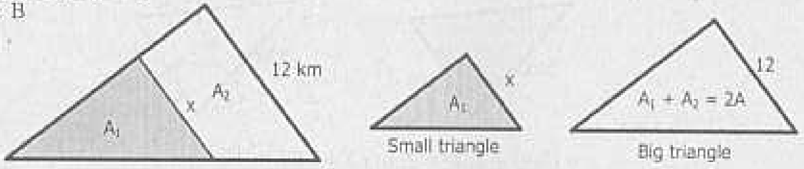
$$A_{sector} = \frac{\pi R^2 \theta}{360°}$$

$$A_{sector} = \frac{\pi(20.4458)^2(80°)}{360°}$$

$A_{sector} =$ **291.84 cm²**



SOLUTION 13 - 40
Ans: B



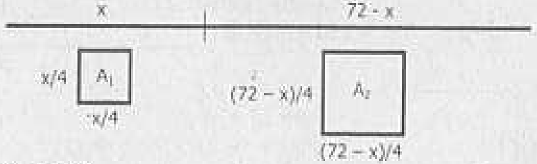
$A_1 = A_2 = A$

Since the small and big triangles are similar

$$\frac{A_{small}}{A_{big}} = \left(\frac{x}{12}\right)^2$$

$$\frac{A}{2A} = \left(\frac{x}{12}\right)^2; \quad x = 8.485\text{ km}$$

SOLUTION 13 - 41
Ans: A



$A_1 + A_2 = 180$

$$\left(\frac{x}{4}\right)^2 + \left(\frac{72-x}{4}\right)^2 = 180$$

$$\frac{x^2}{16} + \frac{5184 - 144x + x^2}{16} = 180$$

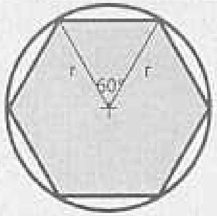
$2x^2 - 144x + 2304 = 0$
$x = 24\ \&\ 48$
$72 - x = 48\ \&\ 24$

Difference in length = 48 – 24 = 24 cm

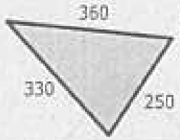
SOLUTION 13 - 42
Ans: D

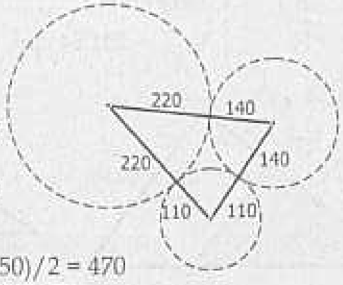
Area of circle = $\pi r^2 = 170$
$r = 7.356$

$A_{\text{hexagon}} = ½(7.356)^2 \sin 60° \times 6$
$A_{\text{hexagon}} = \mathbf{140.6\ cm^2}$



SOLUTION 13 - 43
Ans: A

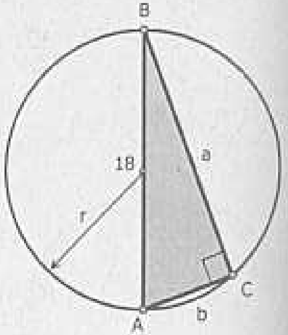




$s = (330 + 360 + 250)/2 = 470$
$A = \sqrt{s(s-a)(s-b)(s-c)}$
$A = \sqrt{470(470-330)(470-360)(470-250)}$
$A = \mathbf{39{,}904.4\ square\ units}$

SOLUTION 13 - 44
Ans: D

The diameter of the circle is 18 cm, hence one side of the triangle is the diameter of the circle and by principle this triangle is a RIGHT TRIANGLE.



$A_T = ½\,a\,b$
$48.23 = ½\,a\,b$
$a\,b = 96.46 \quad \rightarrow (1)$

$a^2 + b^2 = 18^2$
$a^2 = 324 - b^2 \rightarrow (2)$

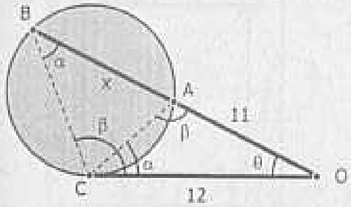
Squaring both sides of equation (1)
$a^2\,b^2 = 9304.5316$
$(324 - b^2)\,b^2 = 9304.5316$
$b^4 - 324\,b^2 + 9304.5316 = 0$

$$b^2 = \frac{324 \pm \sqrt{(-324)^2 - 4(1)(9304.5316)}}{2(1)} = 31.848$$

$b = 5.643$ cm; $a = 17.092$ cm

Thus, the shorter leg is **5.643 cm**

SOLUTION 13 - 45
Ans: B



ΔBCO is similar to ΔCAO

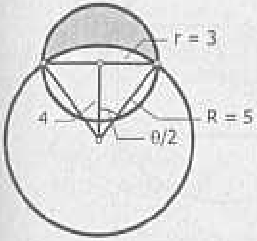
$\left[\frac{\text{opposite of }\beta}{\text{opposite of }\alpha}\right] \quad \frac{x+11}{12} = \frac{12}{11}$
$x = \frac{144}{11} - 11 = \mathbf{2.091\ mi}$

Or, using the relationship:

$(OA)(OB) = (OC)^2$

$11(11 + x) = 12^2$
$x = \frac{144}{11} - 11 = \mathbf{2.091\ mi}$

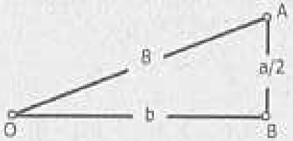
SOLUTION 13 - 46
Ans: D

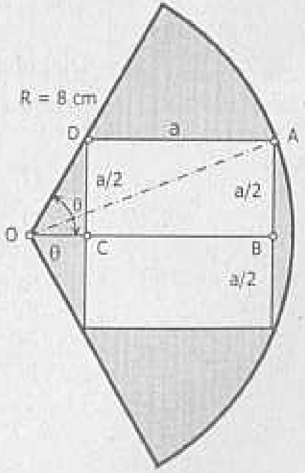


From the figure:
$\theta/2 = \arctan(3/4)$
$\theta = 73.74° = 0.4097\pi$ rad

Area of the shaded portion:
$A = \frac{1}{2}\pi r^2 - \frac{1}{2}R^2(\theta_r - \sin\theta)$
$A = \frac{1}{2}\pi(3^2) - \frac{1}{2}(5^2)(0.4097\pi - \sin 73.74)$
$A = \mathbf{10.05\ m^2}$

SOLUTION 13 - 47
Ans: C





$2\theta = 120°;\ \theta = 60°$
$OC = (a/2)\cot\theta = 0.28867a$

$b = OB = a + OC = 1.28867a$

In triangle OBA:
$8^2 = (a/2)^2 + (b)^2$
$64 = 0.25a^2 + (1.28867a)^2$
$64 = 1.9107a^2$
$a = 5.7876$ cm

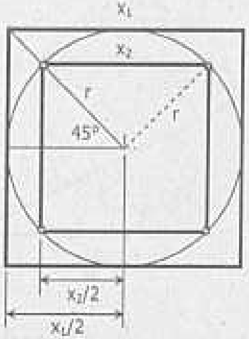
Area = a^2 = **33.5 cm²**

SOLUTION 13 - 48
Ans: A

$x_1/2 = r;\ x_1 = 2r$
$A_1 = (2r)^2 = 4r^2$

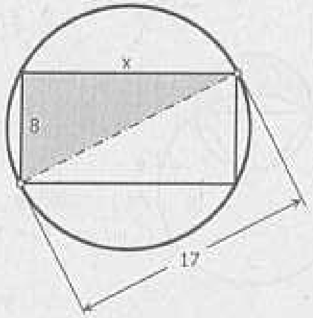
$x_2/2 = r \cos 45°$
$x_2 = 2r \cos 45°$
$A_2 = (2r \cos 45°)^2 = 2r^2$

Ratio = $A_1/A_2 = 4r^2 / 2r^2$
Ratio = **2/1 = 2:1**



SOLUTION 13 - 49
Ans: B

$x^2 = 17^2 + 8^2$
$x = 15$ cm



SOLUTION 13 - 50
Ans: A

Let n_1 and n_2 = number of sides of each polygon
$n_1 + n_2 = 12;\ n_1 = 12 - n_2$

Diagonals:

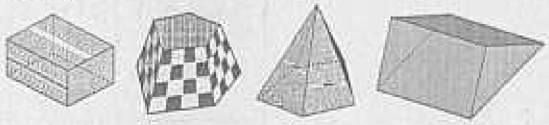
$$D = \frac{n_1}{2}(n_1 - 3) + \frac{n_2}{2}(n_2 - 3) = 19$$
$$(12 - n_2 - 3) + \frac{n_2}{2}(n_2 - 3) = 19$$
$$(12 - n_2)(\,(9 - n_2) + n_2(n_2 - 3) = 38$$
$$(108 - 21n_2 + n_2^2 + n_2^2 - 3n_2 = 38$$
$$n_2^2 - 24n_2 + 70 = 0$$
$$n_2^2 - 12n_2 + 35 = 0$$
$$(n_2 - 5)(n_2 - 7) = 0$$
$$n_2 = 5 \text{ and } 7$$

$n_1 = 7$ and 5 **(heptagon and pentagon)**

Part 4
SOLID GEOMETRY

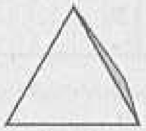
POLYHEDRONS

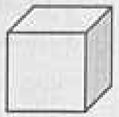
Polyhedrons are solids whose faces are plane polygons.

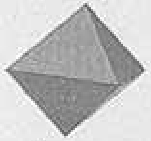


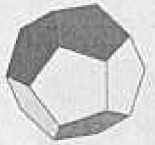
Regular Polyhedrons

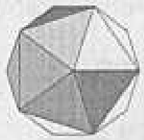
Regular polyhedrons are those which have identical faces. There are only five known regular polyhedrons, namely *tetrahedron*, *hexahedron*, *octahedron*, *dodecahedron*, and *icosahedron*. These solids are known as **Platonic solids**.


Regular Tetrahedron


Regular Hexahedron


Regular Octahedron


Regular Dodecahedron


Regular Icosahedron

Let m = number of polygons meeting at a vertex,
n = number of vertices of each polygon,
f = number of faces of the polyhedron,
e = number of edges of the polyhedron, and
v = number of vertices of the polyhedron.

SOLUTION 13 - 48
Ans: A

$x_1/2 = r; x_1 = 2r$
$A_1 = (2r)^2 = 4r^2$

$x_2/2 = r \cos 45°$
$x_2 = 2r \cos 45°$
$A_2 = (2r \cos 45°)^2 = 2r^2$

Ratio = $A_1/A_2 = 4r^2 / 2r^2$
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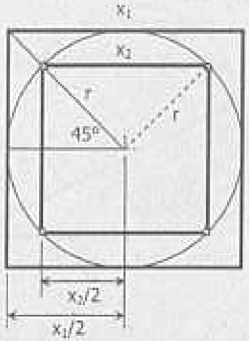
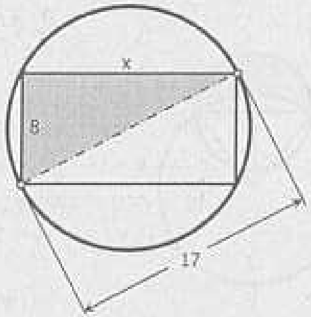


SOLUTION 13 - 49
Ans: B

$x^2 = 17^2 + 8^2$
$x = 15$ cm



SOLUTION 13 - 50
Ans: A

Let n_1 and n_2 = number of sides of each polygon
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Diagonals:

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$n_2 = 5$ and 7

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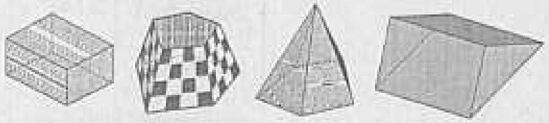

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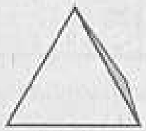
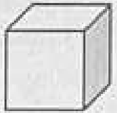
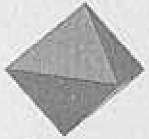
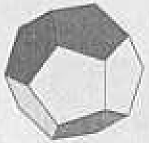
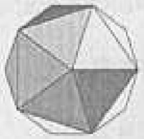

Regular Tetrahedron


Regular Hexahedron


Regular Octahedron


Regular Dodecahedron


Regular Icosahedron

Let m = number of polygons meeting at a vertex,
n = number of vertices of each polygon,
f = number of faces of the polyhedron,
e = number of edges of the polyhedron, and
v = number of vertices of the polyhedron.

For any polyhedron:

$$\text{Number of edges, } e = \frac{nf}{2}$$

$$\text{Number of vertices, } v = \frac{nf}{m}$$

$$\text{Radius of circumscribing sphere, } r = \frac{3V}{A_s}$$

Platonic Solids

Properties of Platonic Solids

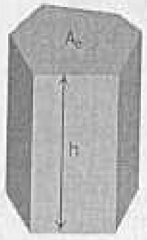
Name	f	e	v	m	Surface area A_s	Volume, V
Tetrahedron	4	6	4	3	$a^2\sqrt{3}$	$\frac{a^3}{6\sqrt{2}}$
Hexahedron (Cube)	6	12	8	3	$6a^2$	a^3
Octahedron	8	12	6	4	$2a^2\sqrt{3}$	$\frac{a^3\sqrt{2}}{3}$
Dodecahedron	12	30	20	3	$15a^2\sqrt{\frac{3+\sqrt{5}}{5-\sqrt{5}}}$	$\frac{a^3(15+7\sqrt{5})}{4}$
Icosahedron	20	30	12	5	$5a^2\sqrt{3}$	$\frac{5a^3(3+\sqrt{5})}{12}$

where a is the length of the edge.

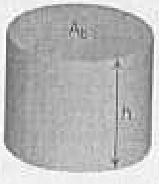
Euler's Polyhedron Theorem for all Convex Polyhedra:

$$f = 2 + e - v$$

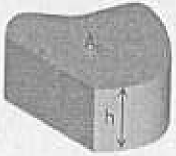
SOLIDS FOR WHICH VOLUME = BASE AREA × HEIGHT



Prism



Cylinder



Miscellaneous

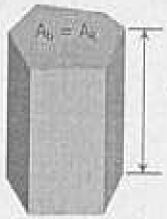
$$\text{Volume} = A_b h$$

PRISM

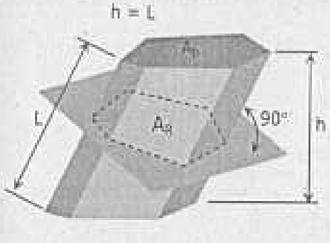
Prisms are polyhedron whose bases are equal polygons in parallel planes and whose sides are parallelograms.

Prisms are classified according to their bases. Thus, a hexagonal prism is one whose base is a hexagon, and a regular hexagonal prism has a base of a regular hexagon.

The **axis** of a prism is the line joining the centroids of the bases. A **right prism** is one whose axis is perpendicular to the base. The **altitude** "h" of a prism is the distance between the bases.



Right Prism



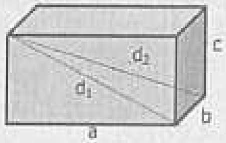
Oblique Prism

$$\text{Volume, } V = A_b h = A_R L$$

$$\text{Lateral Area, } A_L = P_R L$$

where: A_R = area of right section
L = lateral edge
A_b = area of base
P_R = perimeter of right section

Rectangular Parallelepiped



$$\text{Volume, } V = A_b h = a\,b\,c$$

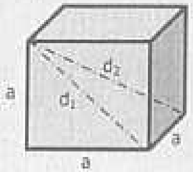
$$\text{Lateral area, } A_L = A_R h = 2(ac + bc)$$

$$\text{Total surface area, } A_s = A_L + 2A_b = 2(a\,b + b\,c + a\,c)$$

$$\text{Face diagonal, } d_1 = \sqrt{a^2 + c^2}$$

$$\text{Space diagonal, } d_2 = \sqrt{a^2 + b^2 + c^2}$$

Cube (Regular hexahedron)



$$\text{Volume, } V = A_b h = a^3$$

$$\text{Lateral area, } A_L = 4a^2$$

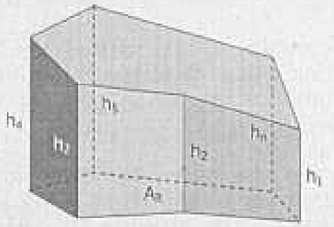
$$\text{Total surface area, } A_S = 6a^2$$

$$\text{Face diagonal, } d_1 = a\sqrt{2}$$

$$\text{Space diagonal, } d_2 = a\sqrt{3}$$

Truncated Prism

A_R = area of the right section;
n =number of sides

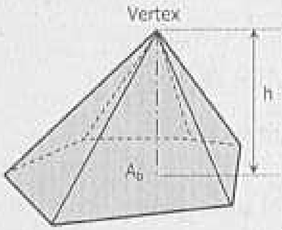


$$\text{Volume} = A_R \frac{\Sigma h}{n}$$

$$\text{Volume} = A_R \frac{h_1 + h_2 + \ldots + h_{n-1} + h_n}{n}$$

PYRAMIDS

Pyramids are polyhedron with a polygonal base and triangular faces that meet at a common point called the vertex.
Like prisms, pyramids are classified according to their bases.

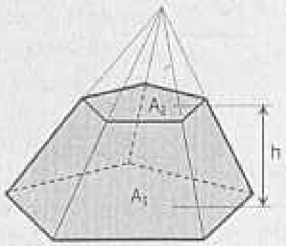


$$\text{Volume} = \frac{1}{3} A_b h$$

where:
A_b = area of the base
h = altitude, perpendicular distance from the vertex to the base

Frustum of Pyramid

Frustum of a pyramid is the portion of the pyramid between the base and a cutting plane parallel to the base.



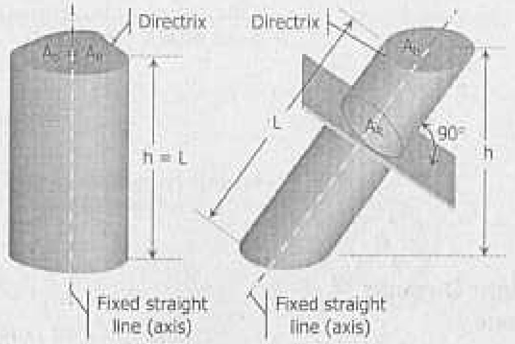
$$\text{Volume} = \frac{h}{3}\left(A_1 + A_2 + \sqrt{A_1 A_2}\right)$$

where A_1 = lower base area
A_2 = upper base area
h = altitude

CYLINDERS

A cylinder is the surface generated by a straight line intersecting and moving along a closed plane curve, the *directrix*, while remaining parallel to a *fixed straight line* (called the *axis*) that is not on or parallel to the plane of the directrix.

Like prisms, cylinders are classified according to their bases.



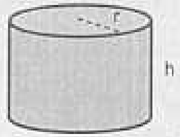
Right Cylinder Oblique Cylinder

$$\text{Volume, } V = A_b h = A_R L$$

$$\text{Lateral Area, } A_L = P_R L$$

where: A_R = area of right section
L = lateral edge
A_b = area of base
P_R = perimeter of right section

Right Circular Cylinder

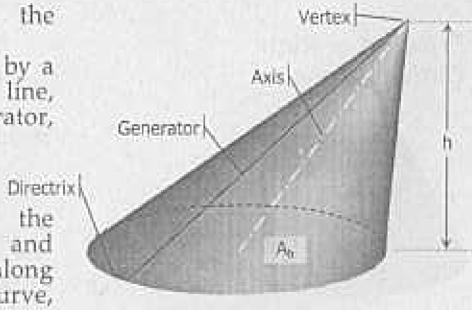


$$\text{Volume, } V = A_b h = \pi r^2 h$$

$$\text{Lateral area, } A_L = \text{Base perimeter} \times h = 2\pi r h$$

CONE

Cone is the surface generated by a straight line, the generator, passing through a fixed point, the vertex, and moving along a fixed curve,



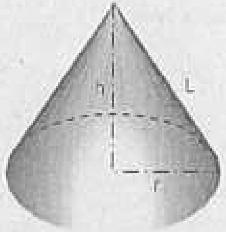
the directrix.

Like pyramids, cones are classified according to their bases.

$$\text{Volume, } V = \frac{1}{3} A_b\, h$$

where A_b = base area
h = altitude

Right Circular Cone



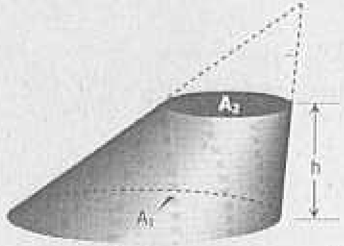
$$\text{Slant height, } L = \sqrt{r^2 + h^2}$$

$$\text{Volume, } V = \frac{1}{3} A_b\, h = \frac{1}{3} \pi r^2 h$$

$$\text{Lateral area, } A_L = \pi r L$$

where r = base radius
h = altitude

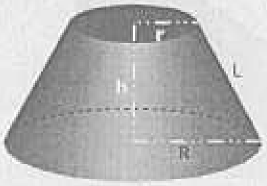
Frustum of a Cone



$$\text{Volume, } V = \frac{h}{3}\left(A_1 + A_2 + \sqrt{A_1 A_2}\right)$$

where
A_1 = lower base area
A_2 = upper base area
h = altitude

Frustum of Right Circular Cone



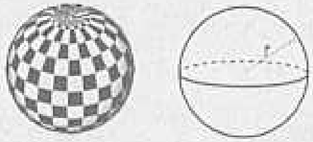
$$\text{Slant height, } L = \sqrt{h^2 + (R - r)^2}$$

$$\text{Volume, } V = \frac{\pi h}{3}\left(R^2 + r^2 + Rr\right)$$

$$\text{Lateral area, } A_L = \pi (R + r) L$$

where R = lower base radius
r = upper base radius,
h = altitude

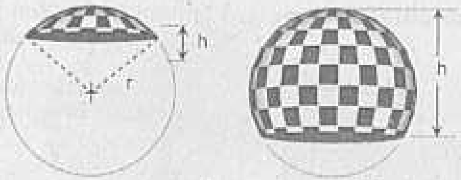
SPHERE



$$\text{Volume, } V = \frac{4}{3} \pi r^3$$

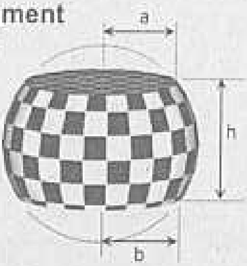
$$\text{Surface area, } A_s = 4 \pi r^2$$

Spherical Segment of One Base



$$A_{zone} = 2 \pi r h \qquad \text{Volume, } V = \frac{\pi h^2}{3}(3r - h)$$

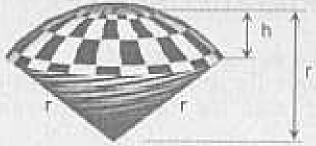
Spherical Segment of Two Bases



$$\text{Lateral Area, } A_z = 2 \pi r h$$

$$\text{Volume, } V = \frac{\pi h}{6}\left(3a^2 + 3b^2 + h^2\right)$$

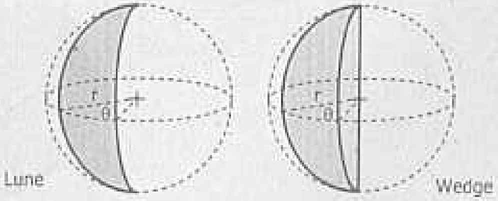
Spherical Cone or Spherical Sector



$$\text{Volume} = \frac{1}{3} A_{zone}\, r$$

$$\text{Volume} = \frac{2}{3} \pi r^2 h$$

Spherical Lune and Wedge

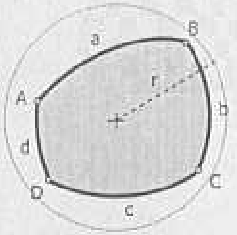


$$\frac{A_{\text{lune}}}{\theta} = \frac{4\pi r^2}{360°} \quad \text{or} \quad A_{\text{lune}} = \frac{\pi r^2\theta}{90°}$$

$$\frac{V_{\text{wedge}}}{\theta} = \frac{\frac{4}{3}\pi r^3}{360°} \quad \text{or} \quad V_{\text{wedge}} = \frac{\pi r^3\theta}{270°}$$

Spherical Polygons

A spherical polygon is a polygon on the surface of a sphere whose sides are arcs of great circles.

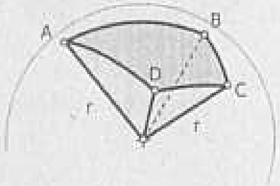


n = number of sides E = spherical excess
r = radius of sphere

$$\text{Area} = \frac{\pi r^2 E}{180°}$$

$$E = \text{sum of angles} - (n-2)\,180°$$

Spherical Pyramid

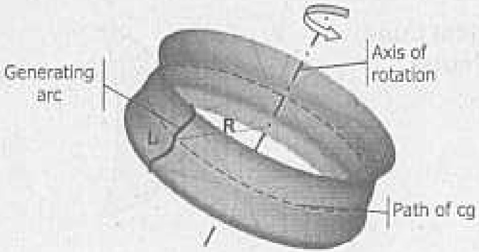


$$\text{Volume} = \frac{\pi r^3 E}{540°}$$

SOLIDS OF REVOLUTION (PAPPUS THEOREMS)

First proposition of Pappus

The surface area generated by a surface of revolution equals the product of the length of the generating arc and the distance traveled by its centroid.



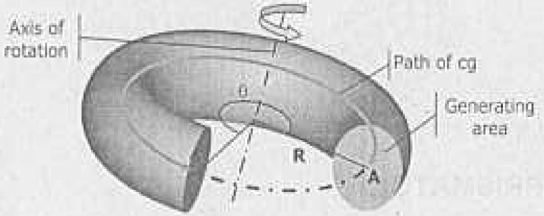
$$A_s = L\,C$$

$$C = R \times (\text{Angle of rotation in radians, } \theta)$$

$$\text{If } \theta = 360°,\ A_s = L(2\pi R)$$

Second Proposition of Pappus

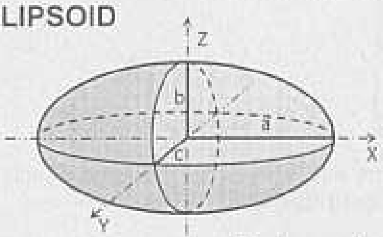
The volume area generated by a solid of revolution equals the product of the generating area and the distance traveled by its centroid.



$$\text{Volume} = A\,C = A(R\theta_{\text{radians}})$$

$$\text{If } \theta = 360°,\ \text{Volume} = A(2\pi R)$$

ELLIPSOID



$$\text{Volume} = \frac{4}{3}\pi abc$$

Prolate Spheroid

Prolate spheroid is formed by revolving the ellipse about its major (X) axis. Thus from the figure above, $c = b$, then,

$$\text{Volume} = \frac{4}{3}\pi ab^2$$

$$A_s = 2\pi b^2 + 2\pi ab\,\frac{\arcsin e}{e}$$

$$e = \frac{\sqrt{a_2 - b^2}}{a}$$

Oblate Spheroid

Prolate spheroid is formed by revolving the ellipse about its minor (Z) axis. Thus from the figure above, $c = a$, then,

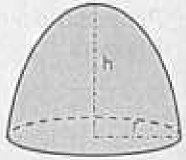
$$\text{Volume} = \frac{4}{3}\pi a^2 b$$

$$A_s = 2\pi a^2 + \frac{\pi b^2}{e}\ln\frac{1+e}{1-e}$$

PARABOLOID OF REVOLUTION

$$\text{Volume} = \frac{1}{2}\pi r^2 h$$

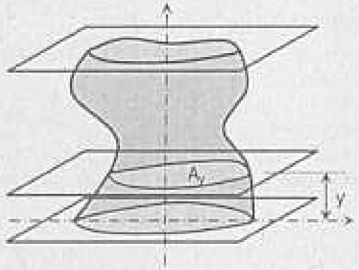
$$A_L = \frac{4\pi r}{3h^2}\left[\left(\frac{r^2}{4}+h^2\right)^{3/2} - \left(\frac{r}{2}\right)^3\right]$$



PRISMATOID

General Prismatoid

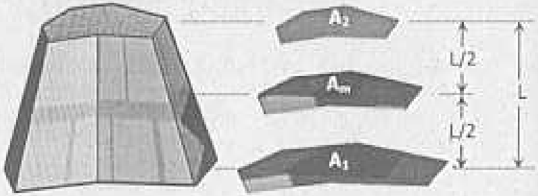
A *general prismatoid* is a solid such that the area of any section, say A_y, parallel to and distant y from a fixed plane can be expressed as a polynomial of y of degree not higher than the third.



$$A_y = ay^3 + by^2 + cy + d$$

where a, b, and c are constants which may be positive, negative, or zero.

Prismoidal Formula



$$\text{Volume} = \frac{L}{6}\left[A_1 + 4A_m + A_2\right]$$

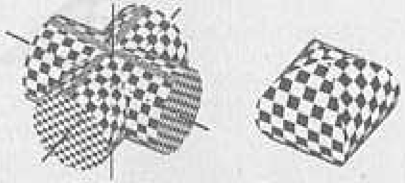
One criteria of knowing if a certain solid is a *prismatoid* is that all section parallel to a certain base are all similar.

Prismatoid Theorem

The volume of a prismatoid is equal to the algebraic sum of the volumes of a pyramid, a wedge, and a parallelepiped.

Volume of Some Prismatoid

Volume common to two equal cylinders with their axis meeting at right angles.

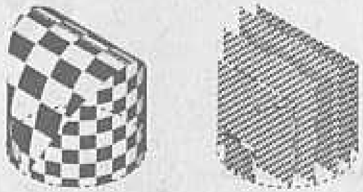


Radius = r

Common Volume

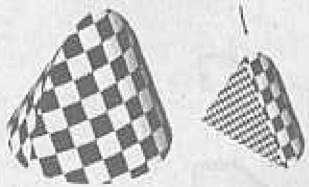
$$\text{Volume} = \frac{16}{3}r^3$$

Solid with circular base of radius r and every cutting plane perpendicular to a certain diameter is a SQUARE



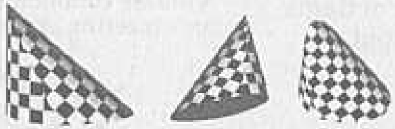
$$\text{Volume} = \frac{16}{3}r^3$$

Solid with circular base of radius r and every cutting plane perpendicular to a certain diameter is an EQUILATERAL TRIANGLE.



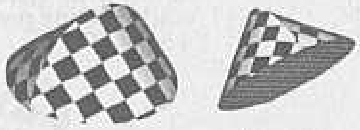
$$\text{Volume} = \frac{4r^3}{\sqrt{3}}$$

Solid with circular base of radius r and every cutting plane perpendicular to a certain diameter is an ISOSCELES RIGHT TRIANGLE with *one leg in the plane of the base.*



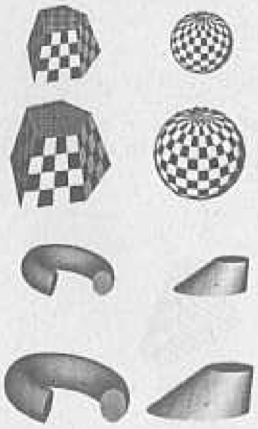
$$\text{Volume} = \frac{8}{3}r^3$$

Solid with circular base of radius r with every cutting plane perpendicular to a certain diameter is an ISOSCELES RIGHT TRIANGLE with *hypotenuse in the plane of the base*



$$\text{Volume} = \frac{4}{3}r^3$$

SIMILAR SOLIDS



Two solids are similar if any two corresponding sides or planes are proportional. All spheres, cubes are similar.

The areas of similar figures or similar surfaces (a_s, A_s) have the same ratio as the squares of any two corresponding lines (x, X).

$$\frac{a_s}{A_s} = \frac{x^2}{X^2}$$

The volumes of similar solids (v, V) have the same ratio as the cubes of two corresponding lines (x, X)

$$\frac{v}{V} = \frac{x^3}{X^3}$$

Problems - Set 14 Prisms, Pyramids, Cylinders, Cones

PROBLEM 14 - 1

If the edge of a cube is doubled, which of the following is incorrect?
A. the lateral area will be quadrupled
B. the volume is increased 8 times
C. the diagonal is doubled
D. the weight is doubled

PROBLEM 14 - 2

The volume of a cube is reduced by how much if all sides are halved?
A. 1/8 C. 6/8
B. 5/8 D. 7/8

PROBLEM 14 - 3
ECE April 1995

Each side of a cube is increased by 1%. By what percent is the volume of the cube increased?
A. 23.4% C. 3%
B. 33.1% D. 34.56%

PROBLEM 14 - 4
CE Nov. 1996

If the edge of a cube is increased by 30%, by how much is the surface area increased?
A. 67 C. 63
B. 69 D. 65

PROBLEM 14 - 5
ECE Nov. 1997

Find the approximate change in the volume of a cube of side x inches caused by increasing its side by 1%.
A. $0.3x^3$ cu. in. C. 0.02 cu. in.
B. $0.1x^3$ cu. in. D. $0.03x^3$ cu. in.

PROBLEM 14 - 6

A rectangular bin 4 feet long, 3 feet wide, and 2 feet high is solidly packed with bricks whose dimensions are 8 in. by 4 in. by 2 in. The number of bricks in the bin is:
A. 68 C. 648
B. 386 D. 956

PROBLEM 14 - 7

Find the total surface area of a cube of side 6 cm.
A. 214 sq. cm. C. 226 sq. cm.
B. 216 sq. cm. D. 236 sq. cm.

PROBLEM 14 - 8

The space diagonal of a cube is $4\sqrt{3}$ m. Find its volume.
A. 16 cubic meters C. 64 cubic meters
B. 48 cubic meters D. 86 cubic meters

PROBLEM 14 - 9
ECE Nov. 1995

A reservoir is shaped like a square prism. If the area of its base is 225 sq. cm., how many liters of water will it hold?

A. 3.375
B. 3375
C. 33.75
D. 337.5

PROBLEM 14 - 10

Find the angle formed by the intersection of a face diagonal to the diagonal of a cube drawn from the same vertex.

A. 35.26°
B. 32.56°
C. 33.69°
D. 42.23°

PROBLEM 14 - 11

The space diagonal of a cube (the diagonal joining two non-coplanar vertices) is 6 m. The total surface area of the cube is:

A. 60
B. 66
C. 72
D. 78

PROBLEM 14 - 12
CE Nov. 1997

The base edge of a regular hexagonal prism is 6 cm and its bases are 12 cm apart. Find its volume in cu. cm.

A. 1563.45 cm^3
B. 1058.45 cm^3
C. 1896.37 cm^3
D. 1122.37 cm^3

PROBLEM 14 - 13
CE Nov. 1997

The base edge of a regular pentagonal prism is 6 cm and its bases are 12 cm apart. Find its volume in cu. cm.

A. 743.22 cm^3
B. 786.89 cm^3
C. 587.45 cm^3
D. 842.12 cm^3

PROBLEM 14 - 14
CE May 1999

The bases of a right prism is a hexagon with one side 6 cm long. If the volume of the prism is 450 cc, how far apart are the bases?

A. 5.74 cm
B. 3.56 cm
C. 4.11 cm
D. 4.81 cm

PROBLEM 14 - 15
CE May 1998

A trough has an open top 0.30 m by 6 m and closed vertical ends which are equilateral triangles 30 cm on each side. It is filled with water to half its depth. Find the volume of the water in cubic meters.

A. 0.058
B. 0.046
C. 0.037
D. 0.065

PROBLEM 14 - 16
ME April 1996

Determine the volume of a right truncated prism with the following dimensions: Let the corner of the triangular base be defined by A, B, and C. The length AB = 10 feet, BC = 9 feet, and CA = 12 feet. The sides at A, B and C are perpendicular to the triangular base and have the height of 8.6 feet, 7.1 feet, and 5.5 feet, respectively.

A. 413 ft^3
B. 311 ft^3
C. 313 ft^3
D. 391 ft^3

PROBLEM 14 - 17

The volume of a regular tetrahedron of side 5 cm is:

A. 13.72 cu. cm
B. 14.73 cu. cm
C. 15.63 cu. cm
D. 17.82 cu. cm

PROBLEM 14 - 18

A regular hexagonal pyramid whose base perimeter is 60 cm has an altitude of 30 cm. The volume of the pyramid is:

A. 2958 cu. cm.
B. 2598 cu. cm.
C. 2859 cu. cm.
D. 2589 cu. cm.

PROBLEM 14 - 19
CE May 1997

A frustum of a pyramid has an upper base 100 m by 10 m and a lower base of 80 m by 8 m. If the altitude of the frustum is 5 m, find its volume.

A. 4567.67 cu. m.
B. 3873.33 cu. m.
C. 4066.67 cu. m.
D. 2345.98 cu. m.

PROBLEM 14 - 20
CE Nov. 1997

The altitude of the frustum of a regular rectangular pyramid is 5m the volume is 140 cu. m. and the upper base is 3m by 4m. What are the dimensions of the lower base in m?

A. 9 × 10
B. 6 × 8
C. 4.5 × 6
D. 7.50 × 10

PROBLEM 14 - 21
CE Nov. 1998

The frustum of a regular triangular pyramid has equilateral triangles for its bases. The lower and upper base edges are 9m and 3m, respectively. If the volume is 118.2 cu.m.., how far apart are the base?

A. 9m
B. 8m
C. 7m
D. 10m

PROBLEM 14 - 22

A cylindrical gasoline tank, lying horizontally, 0.90 m. in diameter and 3 m long is filled to a depth of 0.60 m. How many gallons of gasoline does it contain?
Hint: One cubic meter = 265 gallons

A. 250
B. 360
C. 300
D. 270

PROBLEM 14 - 23

A closed cylindrical tank is 8 feet long and 3 feet in diameter. When lying in a horizontal position, the water is 2 feet deep. If the tank is in the vertical position, the depth of water in the tank is:

A. 5.67 m
B. 5.82 m
C. 5.82 ft
D. 5.67 ft

PROBLEM 14 - 24
CE May 1996

A circular cylinder is circumscribed about a right prism having a square base one meter on an edge. The volume of the cylinder is 6.283 cu. m. Find its altitude in m.

A. 5
B. 4.5
C. 69.08
D. 4

PROBLEM 14 - 25

If 23 cubic meters of water are poured into a conical vessel, it reaches a depth of 12 cm. How much water must be added so that the depth reaches 18 cm.?

A. 95 cubic meters
B. 100 cubic meters
C. 54.6 cubic meters
D. 76.4 cubic meters

PROBLEM 14 - 26

The height of a right circular cone with circular base down is h. If it contains water to a depth of $2h/3$ the ratio of the volume of water to that of the cone is:

A. 1:27
B. 2:3
C. 8:27
D. 26:27

PROBLEM 14 - 27
CE May 1997

A right circular cone with an altitude of 9m is divided into two segments, one is a smaller circular cone having the same vertex with an altitude of 6m. Find the ratio of the volume of the two cones.

A. 19:27
B. 2:3
C. 1:3
D. 8:27

PROBLEM 14 - 28
CE May 1996

A conical vessel has a height of 24 cm. and a base diameter of 12 cm. It holds water to a depth of 18 cm. above its vertex. Find the volume of its content in cc.

A. 387.4
B. 381.7
C. 383.5
D. 385.2

PROBLEM 14 - 29
CE May 1999

A right circular cone with an altitude of 8 cm is divided into two segments. One is a smaller circular cone having the same vertex with volume equal to ¼ of the original cone. Find the altitude of the smaller cone.

A. 4.52 cm
B. 6.74 cm
C. 5.04 cm
D. 6.12 cm

PROBLEM 14 - 30
CE Nov. 1998

The slant height of a right circular cone is 5m long. The base diameter is 6m. What is the lateral area in sq. m?

A. 37.7
B. 47
C. 44
D. 40.8

PROBLEM 14 - 31

A right circular cone has a volume of $128\pi/3$ cm^3 and an altitude of 8 cm.
The lateral area is:

A. $16\sqrt{5}\pi$ cm^2
B. $12\sqrt{5}\pi$ cm^2
C. 16π cm^2
D. 15π cm^2

PROBLEM 14 - 32

The volume of a right circular cone is 36π. If its altitude is 3, find its radius.

A. 3
B. 4
C. 5
D. 6

PROBLEM 14 - 33

A cone and hemisphere share base that is a semicircle with radius 3 and the cone is inscribed inside the hemisphere. Find the volume of the region outside the cone and inside the hemisphere.

A. 24.874
B. 27.284
C. 28.274
D. 28.724

PROBLEM 14 - 34
CE Nov. 1997

A cone was formed by rolling a thin sheet of metal in the form of a sector of a circle 72 cm in diameter with a central angle of 210°. What is the volume of the cone?

A. 13,602 cc
B. 13,504 cc
C. 13,716 cc
D. 13,318 cc

PROBLEM 14 - 35
CE May 1998

A cone was formed by rolling a thin sheet of metal in the form of a sector of a circle 72 cm in diameter with a central angle of 150°. Find the volume of the cone in cc.

A. 7733
B. 7722
C. 7744
D. 7711

PROBLEM 14 - 36

A chemist's measuring glass is conical in shape. If it is 8 cm deep and 3 cm across the mouth, find the distance on the slant edge between the markings for 1 cc and 2 cc.

A. 0.82 cm
B. 0.79 cm
C. 0.74 cm
D. 0.92 cm

PROBLEM 14 - 37

The base areas of a frustum of a cone are 25 sq. cm. and 16 sq. cm., respectively. If its altitude is 6 cm., find its volume.

A. 120 cm^3
B. 122 cm^3
C. 129 cm^3
D. 133 cm^3

PROBLEM 14 - 38
CE May 2003

How far from a vertex is the opposite face of a tetrahedron if an edge is 50 cm long?

A. 38.618 cm
B. 40.825 cm
C. 39.421 cm
D. 41.214 cm

PROBLEM 14 - 39
CE May 2003

A truncated prism has a horizontal triangular base ABC, $AB = 10$ cm, $BC = 12$ cm and $CA = 8$ cm. The vertical edges through A, B, and C are 20 cm, 12 cm, and 18 cm long respectively. Determine the volume of the prism, in cc.

A. 661
B. 559
C. 685
D. 574

PROBLEM 14 - 40
CE Nov. 2002

A lateral edge of the frustum of a regular pyramid is 1.8 m long. The upper base is a square 1 m by 1 m and the lower base 2.4 m by 2.4 m square. Determine the volume of the frustum in cubic meters.

A. 4.6
B. 3.3
C. 5.7
D. 6.5

PROBLEM 14 - 41
CE Nov. 2002

A solid spherical steel ball 20 cm in diameter is placed into a tall vertical cylinder containing water, causing the water level to rise by 10 cm. What is the radius of the cylinder?

A. 12.14
B. 9.08
C. 10.28
D. 11.55

PROBLEM 14 - 42
CE Nov. 2002

A conical vessel one meter diameter at the top and 60 cm high holds salt at a depth of 36 cm from the bottom.

How many cc of salt does it contain?

A. 37,214 C. 35,896
B. 33,929 D. 31,574

PROBLEM 14 - 43
CE May 2002

An open-top cylindrical tank is made of a metal sheet having an area of 43.82 square meter. If the diameter is 2/3 the height, what is the height of the tank?

A. 3.24 m C. 4.23 m
B. 2.43 m D. 5.23 m

PROBLEM 14 - 44
CE Nov. 2003

The lateral area of a right circular cone is 386 square centimeters. If its diameter is one-half its altitude, determine its altitude in centimeters.

A. 24.7 C. 18.9
B. 17.4 D. 22.5

PROBLEM 14 - 45
CE Nov. 2003

The surface area of a regular tetrahedron is 173.2 square centimeters. What is its altitude?

A. 8.2 cm C. 7.2
B. 9.6 cm D. 6.5

PROBLEM 14 - 46

A cube of edge 25 cm is cut by a plane containing two diagonally opposite edges of the cube. Find the area of the section thus formed in sq. cm.

A. 812.3 C. 841.2
B. 912.7 D. 883.9

PROBLEM 14 - 47

A swimming pool is rectangular in shape of length 12 m and width 5.5 m. It has a sloping bottom and is 1 m deep at one end and 3.6 m deep at the other end. The water from a full cylindrical reservoir 3.6 m in diameter and 10 m deep is emptied to the pool. Find the depth of water at the deep end.

A. 2.912 m C. 2.842 m
B. 2.695 m D. 2.754 m

PROBLEM 14 - 48

Two vertical conical tanks (both inverted) have their vertices connected by a short horizontal pipe. One tank, initially full of water, has an altitude of 6 m and a diameter of base 7 m. The other tank, initially empty, has an altitude of 9 m and a diameter of base 8 m. If the water is allowed to flow through the connecting pipe, find the level to which the water will ultimately rise in the empty tank. (Neglect the water in the pipe.)

A. 4.254 m C. 4.687 m
B. 3.257 m D. 5.151 m

PROBLEM 14 - 49

A sphere of radius 5 cm and a right circular cone of radius 5 cm and height 10 cm stand on a plane. How far from the base of the cone must a cutting plane (parallel to the base of the cone) pass in order to cut the solids in

Solutions to Set 14 Prisms, Pyramids, Cylinders, Cones

SOLUTION 14 - 1
Ans: D

Since all cubes are similar, then

$$\frac{V_1}{V_2} = \left(\frac{x_1}{x_2}\right)^3 \text{ and } \frac{A_1}{A_2} = \left(\frac{x_1}{x_2}\right)^2$$

when $x_2 = 2\,x_1$

$$\frac{A_1}{A_2} = \left(\frac{x_1}{2x_1}\right)^2 = \frac{1}{4}$$

$A_2 = 4A_1$; Thus, the area is quadrupled

$$\frac{V_1}{V_2} = \left(\frac{x_1}{2x_1}\right)^3 = \frac{1}{8};\ V_2 = 8\ V_1$$

Thus, the volume is increased 8 times and so the weight is also increased 8 times. Therefore, the incorrect statement is choice D

SOLUTION 14 - 2
Ans: D

Since all cubes are similar, then

$$\frac{V_{orig}}{V_{new}} = \left(\frac{x_{orig}}{x_{new}}\right)^3 \text{; where } x_{new} = ½\ x_{orig}$$

$$\frac{V_{orig}}{V_{new}} = \left(\frac{x_{orig}}{\frac{1}{2}\times x_{orig}}\right)^3 = 8;\ V_{new} = (1/8)\ V_{orig}$$

Thus, the volume is reduced by $V_{orig} - V_{new}$ = **7/8 V_{orig}**

SOLUTION 14 - 3
Ans: B

$$\frac{v}{V} = \left(\frac{a}{1.1a}\right)^3$$

$V = (1.1)^3\,v = 1.331v$

Therefore; the volume is increased by **33.1%**

SOLUTION 14 - 4
Ans: B

Let A_1 = original surface area of the cube
A_2 = increased surface area of the cube
x_1 = original side
x_2 = increased side = $x_1 + 30\%x_1 = 1.3x_1$

Since all cubes are similar:

$$\frac{A_1}{A_2} = \left(\frac{x_1}{x_2}\right)^2 = \left(\frac{x_1}{1.3x_1}\right)^2 = \frac{1}{1.69};\ A_2 = 1.69A_1$$

Therefore; the surface area is increased by **69%.**

SOLUTION 14 - 5
Ans: D

If x is the side of the cube then its volume is x^3. If the sides are increased by 1%, it will become $x + 0.01x = 1.01x$ and the volume becomes $(1.01x)^3$ or $1.0303x^3$.

The change in volume is therefore $1.0303x^3 - x^2$
Change in volume = **$0.0303x^3$ cu. in.**

SOLUTION 14 - 6
Ans: C

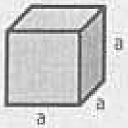
The total volume of bin in cubic inches
is (4×12)(3×12)(2×12) = 41,472

$$\text{Number of bricks required} = 41{,}472\ \text{in}^3 \times \frac{1\,\text{brick}}{8(4)(2)\,\text{in}^3}$$

Number of bricks required = **648**

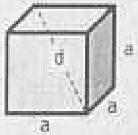
SOLUTION 14 - 7
Ans: B

$A_{Total} = 6a^2 = 6(6)^2$
A_{Total} = **216 sq. cm.**



SOLUTION 14 - 8
Ans: C

The diagonal of a cube of side a is $a\sqrt{3}$
$4\sqrt{3} = a\sqrt{3}$; a = 4 m
Volume = $a^3 = 4^3$
Volume = **64 cubic meters**



SOLUTION 14 - 9
Ans: A

Note: A square prism is a cube:

$V = a^3$; where a is the side
Base area, A = a^2 = 225; a = 15 cm

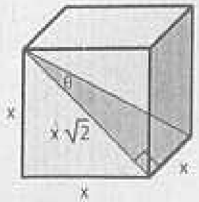
$$V = (15)^3 = 3375\ \text{cu. cm.} \times \frac{1\ \text{lit}}{1000\ \text{cu. cm.}}$$

V = **3.375 liters**

SOLUTION 14 - 10
Ans: A

$$\tan\theta = \frac{x}{x\sqrt{2}} = \frac{1}{\sqrt{2}}$$

θ = 35.26°



SOLUTION 14 - 11
Ans: C

The space diagonal of a cube is:

$d = x\sqrt{3} = 6$, where x = side of the cube

$x = 6 / \sqrt{3}$

The total surface area of a cube is:

$$A_T = 6\,x^2 = 6\left(\frac{6}{\sqrt{3}}\right)^2 = 6 \times 36/3 = \textbf{72 square meter}$$

SOLUTION 14 - 12
Ans: D

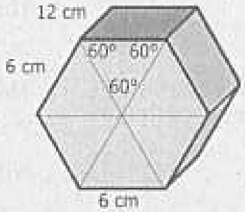
Volume = $A_b\,h$

$A_b = ½(6)(6)\sin 60° \times 6$

$A_b = 93.53\ \text{cm}^2$

Volume = 93.53 × 12

Volume = **1122.37 cm³**



SOLUTION 14 - 13
Ans: A

Volume = $A_b \times h$

$\theta = 360/5 = 72°$

apothem, $a = (6/2)\cot(72°/2)$

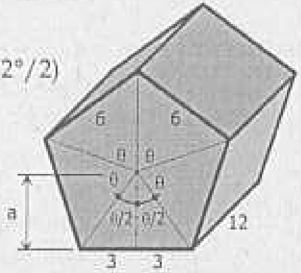
$a = 4.129$ cm

$A_b = ½\,(6)(4.129) \times 5$

$A_b = 61.935\ \text{cm}^2$

Volume = 61.935 × 12

Volume = **743.22 cm³**

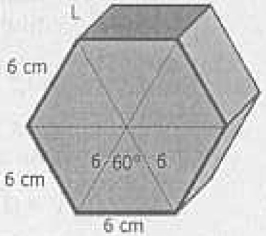


SOLUTION 14 - 14
Ans: D

$V = A_b\,L$

$450 = [½(6)(6)\sin 60° \times 6]\,L$

L = **4.81 cm**

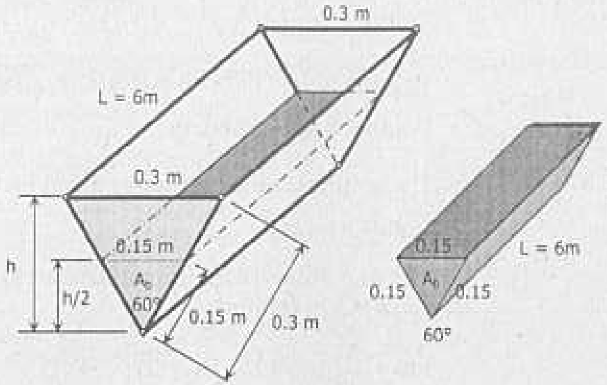


SOLUTION 14 - 15
Ans: A

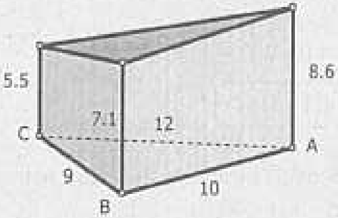
The water will assume the shape of a triangular prism.

The volume is $V = A_b \times L$, where A_b is the base area with the shape of equilateral triangle of side 0.3m.

$V = [½(0.15)(0.15)\sin 60°] \times 6 =$ **0.0584 m³**



SOLUTION 14 - 16
Ans: B



Volume, $V = A_{\text{base}} \times h_{\text{mean}}$

Solve of A_{base} using Hero's Formula:

$$A_{\text{base}} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$s = \frac{10 + 12 + 9}{2} = 15.5$$

$$A_{\text{base}} = \sqrt{15.5(5.5)(6.5)(3.5)}$$

$$A_{\text{base}} = 44.039\ \text{ft}^2$$

$$h_{\text{mean}} = \frac{8.6 + 7.1 + 5.5}{3} = 7.067\ \text{ft.}$$

$V = 44.039 \times 7.067 =$ **311.2 ft³**

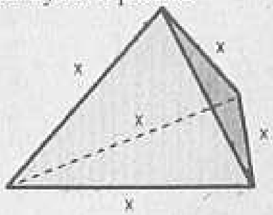
SOLUTION 14 - 17
Ans: B

A tetrahedron is a solid bounded by four planes. The volume of a regular tetrahedron of side x is:

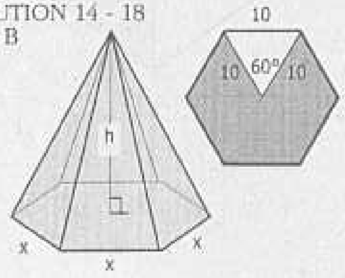
Volume = $x^3 / 6\sqrt{2}$

Volume = $5^3 / 6\sqrt{2}$

Volume = **14.73 cu. cm**



SOLUTION 14 - 18
Ans: B



Volume of pyramid = $\frac{1}{3} A_{\text{base}} \times h$

Perimeter = $6\,x = 60$

$x = 10$ cm

$A_{\text{base}} = ½\,(10)(10)\sin 60° \times 6$

$A_{\text{base}} = 259.81\ \text{cm}^2$

Volume = $\frac{1}{3}(259.81)(30)$

Volume = **2598 cu. cm.**

SOLUTION 14 - 19
Ans: C

$$\text{Volume} = \frac{h}{3}(A_1 + A_2 + \sqrt{A_1 A_2})$$

$$\text{Volume} = \frac{5}{3}\{(100)(10) + (80)(8) + \sqrt{[100(10)][80(8)]}\}$$

Volume = **4066.67 cu. m.**

SOLUTION 14 - 20
Ans: B

The volume of the frustum is given by the formula:

$$V = \frac{h}{3}\left[A_1 + A_2 + \sqrt{A_1 A_2}\right]$$

$A_2 = 3(2) = 12\text{ m}^2$ (upper base area)
$h = 5$ m (altitude)

$$140 = \frac{5}{3}\left[A_1 + 12 + \sqrt{A_1(12)}\right] \rightarrow (1)$$

$$72 = A_1 + \sqrt{12A_1}$$

$\sqrt{12A_1} = 72 - A_1$ (square both sides)

$$12A_1 = 5184 - 144A_1 + A_1^2$$
$$0 = A_1^2 - 156A_1 + 5184$$

$$A_1 = \frac{-(-156) \pm \sqrt{(-156)^2 - 4(1)(5184)}}{2(1)} = 48 \; \& \; 108$$

The acceptable value is 48 m². Therefore the dimension should be **6 × 8**

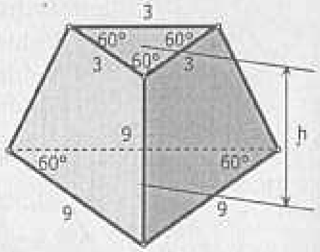
Note: This can also be solved by trial and error using Eq. (1).

SOLUTION 14 - 21
Ans: C

$$V = \frac{h}{3}\left[A_1 + A_2 + \sqrt{A_1 A_2}\right]$$

$A_1 = ½(3)(3)\sin 60°$
$A_1 = 3.897\text{ m}^2$

$A_2 = ½(9)(9)\sin 60°$
$A_2 = 35.074\text{ m}^2$



$$118.2 = \frac{h}{3}\left[3.897 + 35.074 + \sqrt{(3.897)(35.074)}\right]$$

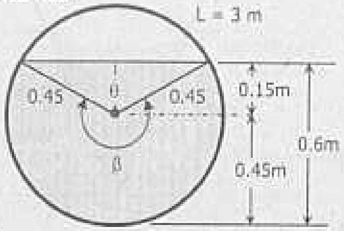
$h = 7$ m

SOLUTION 14 - 22
Ans: B

Volume = $A_s \times L$

Solving for θ amd β from the figure shown.

cos (θ/2) = 0.15/0.45, θ = 141.06°
β = 360 - θ = 218.94°



$$A_s = A_{\text{sector}} + A_{\text{triangle}}$$

$$A_s = \frac{\pi r^2 \beta}{360°} + ½ r^2 \sin\theta$$

$$A_s = \frac{\pi(0.45)^2(218.94°)}{360°} + ½(0.45)^2 \sin 141.06°$$

$A_s = 0.4505\text{ m}^2$

Volume = 0.4505 × 3 = 1.3515 m³ × 265 gal/m³
Volume = **358.15 gal**

SOLUTION 14 - 23
Ans: D

In Figure *a*:
cos θ/2 = 0.5/1.5; θ = 141.06°
β = 360 - θ = 218.94°

$$A_s = A_{\text{sector}} + A_{\text{triangle}}$$

$$A_s = \frac{\pi(1.5)^2(218.94°)}{360°} + ½(1.5)^2 \sin 141.06°$$

$A_s = 5\text{ ft}^2$

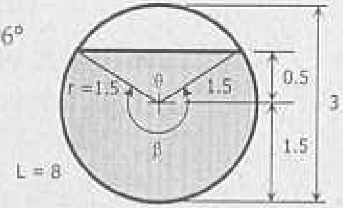


Figure a

Volume of water = $A_s \times L$ = 5(8)
Volume = 40 ft³

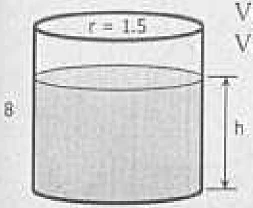


Figure b

In Figure *b*:
Volume of water = $A_b \times h$
$40 = \pi(1.5)^2 \times h$
$h = 5.66$ ft

SOLUTION 14 - 24
Ans: D

Volume of the cylinder:

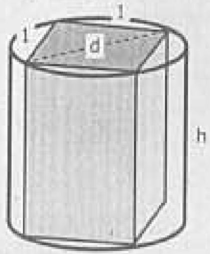
$$V = A_b h = \tfrac{\pi}{4} D^2 h$$

The diameter of the cylinder is the diagonal of the square:

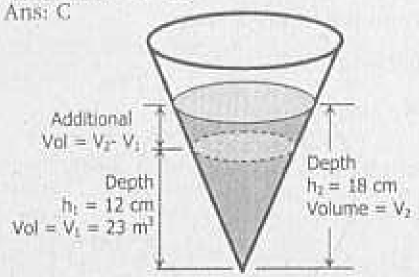
$$D = \sqrt{1^2 + 1^2} = \sqrt{2}$$

then;

$$6.283 = \tfrac{\pi}{4}(\sqrt{2})^2 h; \; h = 4\text{ m}$$



SOLUTION 14 - 25
Ans: C



The cones shown in the figure are similar.

$$\frac{V_1}{V_2} = \left(\frac{h_1}{h_2}\right)^3$$

$$\frac{23}{V_2} = \left(\frac{12}{18}\right)^3$$

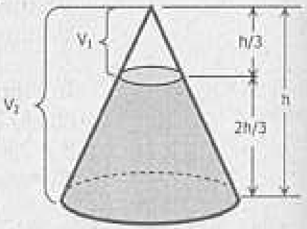
$V_2 = 77.625\ \text{m}^3$

Additional water = 77.625 – 23
Additional water = **54.625 m³**

SOLUTION 14 - 26
Ans: D

Volume of cone, $V_{cone} = V_2$
Volume of water, $V_w = V_2 - V_1$

Since V_1 and V_2 are similar:



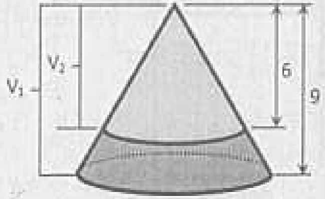
$$\frac{V_1}{V_2} = \left(\frac{h/3}{h}\right)^3 = \frac{1}{27}$$

$V_2 = 27\ V_1$

$V_w = 27V_1 - V_1 = 26\ V_1$
$V_{cone} = 27\ V_1$

$$\frac{V_w}{V_{cone}} = \frac{26V_1}{27V_1} = 26{:}27$$

SOLUTION 14 - 27
Ans: D



$$\frac{V_2}{V_1} = \left(\frac{6}{9}\right)^3$$

$$\frac{V_2}{V_1} = \left(\frac{2}{3}\right)^3 = \frac{8}{27}$$

Ratio of volume = 8 : 27

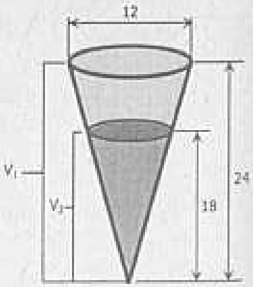
SOLUTION 14 - 28
Ans: B

Total capacity of the vessel is:

$V_1 = \frac{1}{3} \times r^2 h = \frac{1}{3} \times (6)^2\,(24)$

$V_1 = 904.78\ \text{cm}^3$

Since the solid formed by the smaller cone of height 18 cm is similar to the bigger cone then;



$$\frac{V_1}{V_2} = \frac{{h_1}^3}{{h_2}^3}$$

$$\frac{904.78}{V_2} = \frac{(24)^3}{(18)^3};\ V_2 = 381.7\ \text{cu. cm.}$$

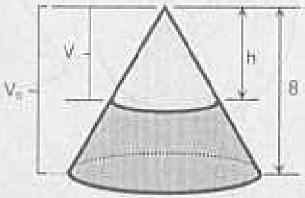
SOLUTION 14 - 29
Ans: C

$V = ¼\ V_o$

$$\frac{V}{V_o} = \left(\frac{h}{8}\right)^3$$

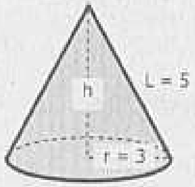
$$\frac{\frac{1}{4}V_o}{V_o} = \frac{h^3}{512}$$

$h^3 = 128;\ h = 5.04\ \text{cm}$



SOLUTION 14 - 30
Ans: B

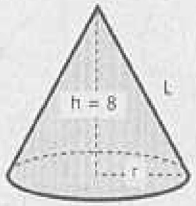
$A_L = \pi\, r\, L$
$A_L = \pi(3)(5)$
$A_L = \mathbf{47.12}$ **sq. m**



SOLUTION 14 - 31
Ans: A

Let r be the radius of the base.

$V = (1/3)\ \pi r^2 h$
$128\pi/3 = (1/3)\pi r^2\,(8);\ r = 4$
$L = \sqrt{8^2 + 4^2} = \sqrt{80} = 4\sqrt{5}$
$A_L = \pi\, r\, L = \pi(4)(\,4\sqrt{5}\,)$
$A_L = \mathbf{16\sqrt{5}\,\pi}$



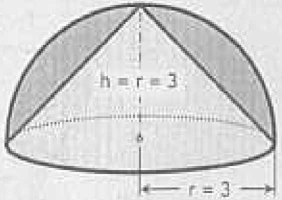
SOLUTION 14 - 32
Ans: D

$V = \frac{1}{3}\pi r^2 h$

$36\pi = \frac{1}{3}\pi r^2(3);\ r = 6$

SOLUTION 14 - 33
Ans: C

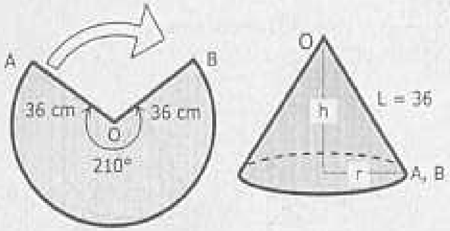
$$\begin{aligned} V_{\text{required}} &= V_{\text{hemisphere}} - V_{\text{cone}} \\ &= \tfrac{2}{3}\pi r^3 - \tfrac{1}{3}\pi r^2 h \\ &= \tfrac{2}{3}\pi\,(3)^3 - \tfrac{1}{3}\pi(3)^2\,(3) \\ &= \mathbf{28.274\ cu.\ units} \end{aligned}$$



SOLUTION 14 - 34
Ans: B

The volume of the cone is, $V = \frac{1}{3}\pi r^2 h$

By rolling the sector to form a cone, the length of arc AB becomes the perimeter of the base of the cone, then,



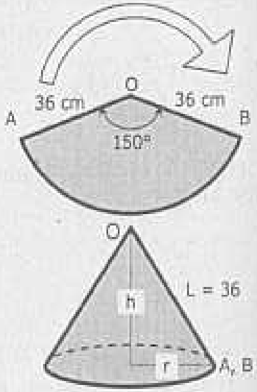
$$2\,\pi\, r = \frac{\pi 36(210^\circ)}{180^\circ}$$
$r = 21$ cm

By Pythagorean Theorem;
$36^2 = h^2 + 21^2$
$h = 29.24$ cm

Then,
$V = \frac{1}{3}\,\pi\,(21)^2\,(29.24)$
$V =$ **13,504 cm³**

SOLUTION 14 - 35
Ans: D

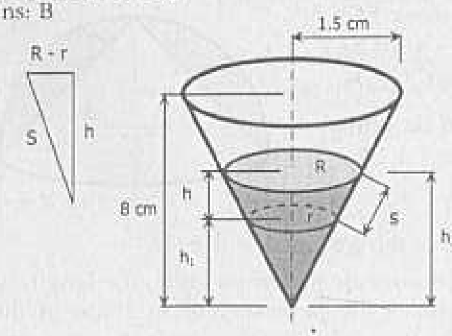
The volume of the cone is
$V = \frac{1}{3}\,\pi\, r^2\, h$
By rolling the sector to form a cone, the length of arc AB becomes the perimeter of the base of the cone, then,



$$2\,\pi\, r = \frac{\pi 36(150^\circ)}{180^\circ}$$
$r = 15$ cm

By Pythagorean Theorem;
$36^2 = h^2 + 15^2$
$h = 32.726$ cm
Then,
$V = \frac{1}{3}\,\pi\,(15)^2\,(32.726)$
$V =$ **7,711 cc**

SOLUTION 14 - 36
Ans: B



$V_1 = 1$ cc ; $V_2 = 2$ cc
$$\frac{r}{h_1} = \frac{1.5}{8}$$
$r = 0.1875\, h_1$ and
$R = 0.1875\, h_2$

$V_1 = \frac{1}{3}\,\pi\, r^2\, h_1$
$1\text{ cc} = \frac{1}{3}\,\pi(0.1875\, h_1)^2\, h_1$
$h_1{}^3 = 27.162,\ h_1 = 3.006$ cm
$r = 0.564$ cm

$V_2 = \frac{1}{3}\,\pi\, R^2\, h_2$

$2\text{ cc} = \frac{1}{3}\,\pi\,(0.1875\, h_2)^2\, h_2$
$h_2{}^3 = 54.325;\ h_2 = 3.787$ cm
$R = 0.71$ cm
$h = h_2 - h_1 = 0.781$ cm

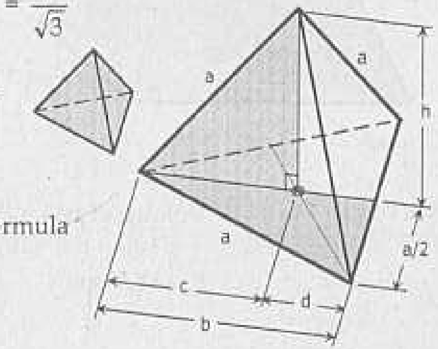
$s^2 = h^2 + (R - r)^2 = (0.781)^2 + (0.71 - 0.564)^2$
$s =$ **0.7945 cm**

SOLUTION 14 - 37
Ans: B

The volume of a frustum is:
$$V = \frac{h}{3}\left(A_1 + A_2 + \sqrt{A_1 A_2}\right) = \frac{6}{3}\left(25 + 16 + \sqrt{(25)(16)}\right)$$
$V =$ **122 cm³**

SOLUTION 14 - 38
Ans: B

$b = \sqrt{a^2 - (a/2)^2} = a\sqrt{3}/2$
$$c = \frac{2}{3}b = \frac{2}{3}\,\frac{a\sqrt{3}}{2} = \frac{a}{\sqrt{3}}$$
$h = \sqrt{a^2 - c^2}$
$$h = \sqrt{a^2 - \left(\frac{a}{\sqrt{3}}\right)^2}$$
$\boxed{h = \sqrt{\tfrac{2}{3}}\, a}$ → Formula

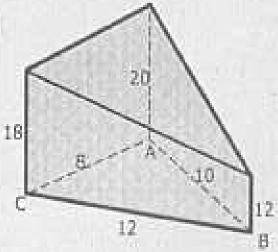


$h = \sqrt{\tfrac{2}{3}}\,(50)$
$h =$ **40.825 cm**

SOLUTION 14 - 39
Ans: A

$$\text{Volume} = A_b \times \frac{\Sigma h}{n}$$

Solving for A_b (triangle) using Heros Formula



$A_b = \sqrt{s(s-a)(s-b)(s-c)}$
$$s = \frac{a+b+c}{2} = \frac{12+8+10}{2}$$
$s = 15$
$A_b = \sqrt{(15)(15-12)(15-8)(15-10)} = 39.686$ cm²

$$\text{Volume} = 39.686\,\frac{20+12+18}{3} = \textbf{661.44 cc}$$

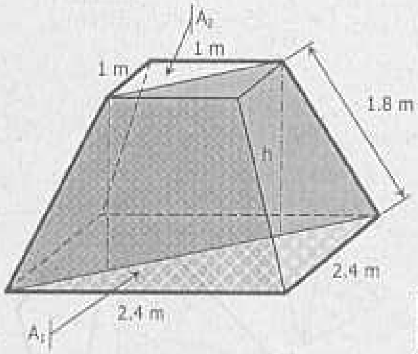
SOLUTION 14 - 40
Ans: A

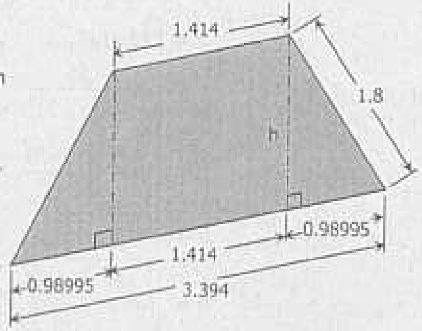
$$V = \frac{h}{3}\left[A_1 + A_2 + \sqrt{A_1 A_2}\right]$$

$A_1 = (2.4)^2 = 5.76\ \text{m}^2$
$A_2 = (1)^2 = 1\ \text{m}^2$

$h^2 = 1.8^2 - 0.98995^2;\ h = 1.503\ \text{m}$

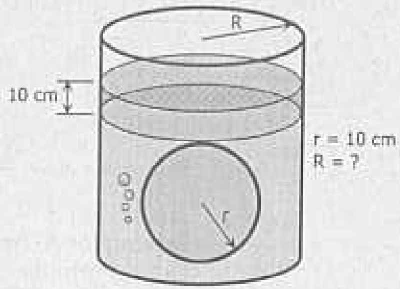
$$V = \frac{1.503}{3}\left[5.76 + 1 + \sqrt{5.76(1)}\right] = \mathbf{4.59\ m^3}$$





SOLUTION 14 - 41
Ans: D

Volume of ball = (Volume rise)$_{\text{water}}$
$\frac{4}{3}\pi(10)^3 = \pi R^2 (10)$
R = **11.55 cm**



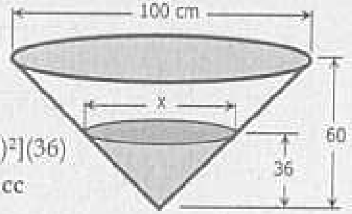
SOLUTION 14 - 42
Ans: B

Solving fo x:

$$\frac{x}{36} = \frac{100}{60}$$

$x = 60$ cm

Volume of salt = $\frac{1}{3}\left[\frac{\pi}{4}(60)^2\right](36)$
Volume of salt = **33,929.2 cc**



SOLUTION 14 - 43
Ans: C

$As = A_{\text{base}} + A_{\text{lateral}} = \frac{\pi}{4}D^2 + \pi D H$

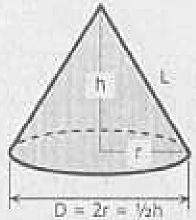
but $D = 2H/3$
$43.82 = \frac{\pi}{4}(2H/3)^2 + \pi(2H/3)H;\ H = \mathbf{4.23\ m}$

SOLUTION 14 - 44
Ans: D

Lateral area, $A_L = \pi r L = 386$
$rL = 122.87 \quad \rightarrow (1)$

$D = 2r = \frac{1}{2}h$
$h = 4r \quad \rightarrow (2)$

$L^2 = h^2 + r^2$
$L^2 = (4r)^2 + r^2 = 17r^2$
$L = r\sqrt{17} \quad \rightarrow (3)$



Substitute L in Eq. (3) to Eq. (1):
$r(r\sqrt{17}) = 122.87;\ r = 5.459$

Then, $L = 5.459\sqrt{17}$ = **22.51 cm**

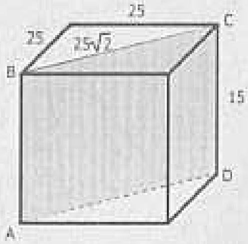
SOLUTION 14 - 45
Ans: A

Surface area = $a^2\sqrt{3} = 173.2$
$a = 10$ cm

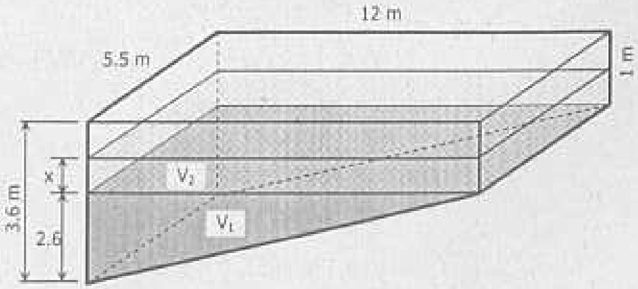
Altitude, $h = a\sqrt{\frac{2}{3}} = 10\sqrt{\frac{2}{3}}$ = **8.16 cm**

SOLUTION 14 - 46
Ans: D

(Area)$_{ABCD} = 25(25\sqrt{2})$
(Area)$_{ABCD}$ = **883.9 cm²**



SOLUTION 14 - 47
Ans: C



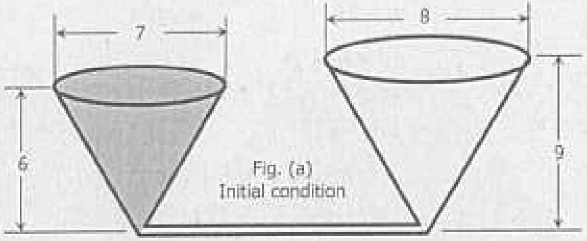
Volume of water = volume of cylindrical reservoir

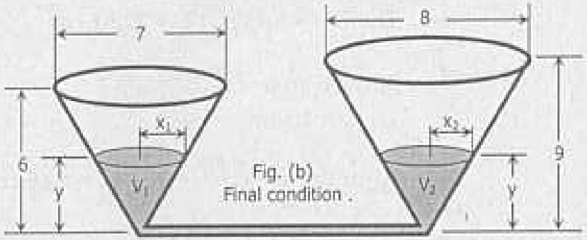
Volume of water = $\frac{\pi}{4}(3.6)^2(10) = 101.788 \text{ m}^3 = V_1 + V_2$

$V_1 = A_b \times L = ½(2.6)(12) \times 5.5 = 85.8 \text{ m}^3$
$V_2 = 101.788 - 85.8 = 15.988 \text{ m}^3$
$V_2 = 12(5.5)x = 15.988$
$x = 0.242 \text{ m}$

Depth at the deep end = 2.6 + 0.242 = **2.842 m**

SOLUTION 14 - 48
Ans: D





Volume of water (See Fig. *a*)

$V_w = \frac{1}{3}\pi(7/2)^2(6) = 24.5\pi \text{ m}^3$
$V_w = V_1 + V_2 = 24.5\pi$

From Figure *b*:

$V_1 = \frac{1}{3}\pi x_1^2 y \qquad y/x_1 = 6/(7/2);\ x_1 = 7y/12$

$V_1 = \frac{49}{432}\pi y^3$

$V_2 = \frac{1}{3}\pi x_2^2 y \qquad y/x_2 = 9/4;\ x_2 = 4y/9$

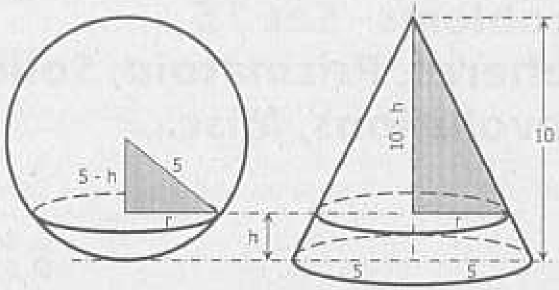
$V_2 = \frac{16}{243}\pi y^3$

$[V_1 + V_2 = 24.5\pi]$

$\frac{49}{432}\pi y^3 + \frac{16}{243}\pi y^3 = 24.5\pi$

$y^3 = 136.6657;\ y = $ **5.151 m**

SOLUTION 14 - 49
Ans: D



From the sphere:

$r^2 = 5^2 - (5 - h)^2$
$r^2 = 10h - h^2$

From the cone

$r/(10 - h) = 5/10$
$r = ½(10 - h)$
$r^2 = ¼(10 - h)^2$

$[r^2 = r^2]$

$10h - h^2 = ¼(10 - h)^2$
$40h - 4h^2 = 100 - 20h + h^2$
$5h^2 - 60h + 100 = 0; \quad h^2 - 12h + 20 = 0$
$(h - 2)(h - 10) = 0$
$h = 2$ cm **answer**
$h = 10$ cm (absurd)

SOLUTION 14 - 50
Ans: C

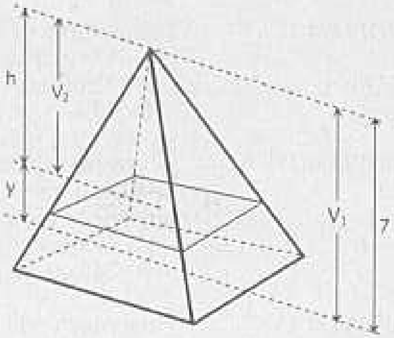
Since weight is directly proportional to volume, then
$V_2 = ½ V_1$

By similar solids relations:

$\frac{V_1}{V_2} = \left(\frac{7}{h}\right)^3$

$\frac{V_1}{1/2(V_1)} = \left(\frac{7}{h}\right)^3$

$h = 5.556$ ft.
$y = 7 - h = $ **1.444 m**



Problems - Set 15 Spheres, Prismatoid, Solids of Revolutions, Misc.

PROBLEM 15 - 1
CE Nov. 1998

What is the surface area of a sphere whose volume is 36 cu. m?

A. 52.7 m^2 C. 46.6 m^2
B. 48.7 m^2 D. 54.6 m^2

PROBLEM 15 - 2

If the surface area of the sphere is increased by 21%, its volume is increased by:

A. 13.31% C. 21%
B. 33.1% D. 30%

PROBLEM 15 - 3

The surface area of a sphere is $4\pi r^2$. Find the percentage increase in its diameter when the surface area increases by 21%.

A. 5% C. 15%
B. 10% D. 20%

PROBLEM 15 - 4

Find the percentage increase in volume of a sphere if its surface area is increased by 21%.

A. 30.2% C. 34.5%
B. 33.1% D. 30.9%

PROBLEM 15 - 5

The volume of a sphere is increased by how much if its radius is increased by 20%?

A. 32.6 % C. 44 %
B. 33% D. 72.8 %

PROBLEM 15 - 6

Given two spheres whose combined volume is known to be 819 cu. m. If their radii are in the ratio 3:4, what is the volume of the smaller sphere?

A. 576 cu. m. C. 343 cu. m.
B. 243 cu. m. D. 476 cu. m.

PROBLEM 15 - 7

How much will the surface area of a sphere be increased if its radius is increased by 5%

A. 25% C. 12.5%
B. 15.5% D. 10.25%

PROBLEM 15 - 8

The volume of a sphere is 904.78 cu. m. Find the volume of the spherical segment of height 4m.

A. 234.57 cu. m. C. 145.69 cu. m.
B. 256.58 cu. m. D. 124.58 cu. m.

PROBLEM 15 - 9

A sphere of radius r just fits into a cylindrical container of radius r and altitude $2r$. Find the empty space in the cylinder.

A. $(8/9)\pi r^3$ C. $(4/5)\pi r^3$
B. $(20/27)\pi r^3$ D. $(2/3)\pi r^3$

PROBLEM 15 - 10

If a solid steel ball is immersed in an eight cm. diameter cylinder, it displaces water to a depth of 2.25 cm. The radius of the ball is:

A. 3 cm C. 9 cm
B. 6 cm D. 12 cm

PROBLEM 15 - 11

The diameter of two spheres are in the ratio 2:3. If the sum of their volumes is 1,260 cu. m., the volume of the larger sphere is:

A. 972 cu. m. C. 856 cu. m.
B. 927 cu. m. D. 865 cu. m.

PROBLEM 15 - 12

A hemispherical bowl of radius 10 cm is filled with water to such a depth that the water surface area is equal to 75π sq. cm. The volume of water is:

A. $625/3\ cm^3$ C. $625\pi/2\ cm^3$
B. $625\pi/3\ cm^3$ D. $625\pi\ cm^3$

PROBLEM 15 - 13

A water tank is in the form of a spherical segment whose base radii are 4m and 3m and whose altitude is 6m. The capacity of the tank in gallons is:

A. 91,011 C. 95,011
B. 92,011 D. 348.72

PROBLEM 15 - 14

Find the volume of a spherical sector of altitude 3 cm. and radius 5 cm.

A. 75π cu. cm. C. 50π cu. cm.
B. 100π cu. cm. D. 25π cu. cm.

PROBLEM 15 - 15

A water tank is in the form of a spherical segment whose base radii are 4m and 3m and whose altitude is 6m. The capacity of the tank in gallons is:

A. 91,011 C. 95,011
B. 92,011 D. 348.72

PROBLEM 15 - 16

How far from the center of a sphere of a radius 10 cm should a plane be passed so that the ratio of the areas of two zones is 3:7.

A. 3 cm C. 5 cm
B. 4 cm D. 6 cm

PROBLEM 15 - 17

A 2-m diameter spherical tank contains 1396 liters of water. How many liters of water must be added for the water to reach a depth of 1.75 m?

A. 2613 C. 2542
B. 2723 D. 2472

PROBLEM 15 - 18

Find the volume of a spherical segment of radius 10 m and altitude 5 m.

A. 654.5 cu. m. C. 675.2 cu. m.
B. 659.8 cu. m. D. 680.5 cu. m.

PROBLEM 15 - 19

Find the volume of a spherical wedge of radius 10 cm. and central angle 50°.

A. 425.66 sq. m. C. 581.78 sq. m.
B. 431.25 sq. m. D. 444.56 sq. m.

PROBLEM 15 - 20

Determine the area of the zone of a sphere of radius 8 in. and altitude 12 in.

A. 192π sq. in. C. 185π sq. in.
B. 198π sq. in. D. 195π sq. in.

PROBLEM 15 - 21
CE May 1997

The corners of a cubical block touch the closed spherical shell that encloses it. The volume of the box is 2744 cc. What volume in cc, inside the shell is not occupied by the block?

A. 1356 cm^3 C. 3423 cm^3
B. 4721 cm^3 D. 7623 cm^3

PROBLEM 15 - 22
ME April 1997

A cubical container that measures 2 inches on each side is tightly packed with 8 marbles and is filled with water. All 8 marbles are in contact with the walls of the container and the adjacent marbles. All of the marbles are of the same size. What is the volume of water in the container?

A. 0.38 cu. in. C. 3.8 cu. in
B. 2.5 cu. in. D. 4.2 cu. in.

PROBLEM 15 - 23
CE Nov. 1996

The volume of the water is a spherical tank is 1470.265 cm^3. Determine the depth of water if the tank has a diameter of 30 cm.

A. 8 C. 4
B. 6 D. 10

PROBLEM 15 - 24
CE Nov. 1996

The volume of water in a spherical tank having a diameter of 4 m. is 5.236 m^3. Determine the depth of the water on the tank.

A. 1.0 C. 1.2
B. 1.4 D. 1.6

PROBLEM 15 - 25
CE May 1996,
Nov. 1999

A mixture compound from equal parts of two liquids, one white and the other black, was placed in a hemispherical bowl. The total depth of the two liquids is 6". After standing for a short time the mixture separated the white liquid settling below the black. If the thickness of the segment of the black liquid is 2", find the radius of the bowl in inches.

A. 7.53 C. 7.73
B. 7.33 D. 7.93

PROBLEM 15 - 26
CE May 1998

20.5 cubic meters of water is inside a spherical tank whose radius is 2m. Find the height of the water surface above the bottom of the tank, in m.

A. 2.7 C. 2.3
B. 2.5 D. 2.1

PROBLEM 15 - 27
ECE Nov. 1997

The volume of the sphere is 36π cu. m. The surface area of this sphere in sq. m. is:

A. 36π C. 18π
B. 24π D. 12π

PROBLEM 15 - 28

Spherical balls 1.5 cm in diameter area packed in a box measuring 6 cm by 3 cm by 3 cm. If as many balls as possible are packed in the box, how much free space remains in the box?

A. 28.41 cc C. 29.87 cc
B. 20.47 cc D. 25.73 cc

PROBLEM 15 - 29
CE May 1999

A solid has a circular base of radius r. Find the volume of the solid if every plane perpendicular to a given diameter is a square.

A. $16\,r^3/3$ C. $6\,r^3$
B. $5\,r^3$ D. $19\,r^3/3$

PROBLEM 15 - 30

A solid has circular base of diameter 20 cm. Find the volume of the solid if every cutting plane perpendicular to the base along a given diameter is an equilateral triangle.

A. 2514 cc C. 2309 cc
B. 2107 cc D. 2847 cc

PROBLEM 15 - 31

The base of a certain solid is a triangle of base b and altitude h. If all sections perpendicular to the altitude of the triangle are regular hexagons, find the volume of the solid.

A. $\frac{1}{2}\sqrt{3}\,b^2 h$ C. $\sqrt{3}\,b^2 h/3$
B. $2\sqrt{3}\,b^2 h$ D. $\sqrt{3}\,b^2 h$

PROBLEM 15 - 32

The volume generated by the circle $x^2 + y^2 + 4x - 6y - 12 = 0$ revolved about the line $2x - 3y - 12 = 0$ is:
A. 3242 cubic units C. 3452 cubic units
B. 3342 cubic units D. 3422 cubic units

PROBLEM 15 - 33

The volume generated by rotating the curve $9x^2 + 4y^2 = 36$ about the line
$4x + 3y = 20$ is:
A. 48π C. 42π
B. $58\pi^2$ D. $48\pi^2$

PROBLEM 15 - 34
CE May 1997

Find the volume generated by revolving the area bounded by the ellipse $y^2/9 + x^2/4 = 1$ about the
line $x = 3$.
A. 347.23 cu. units C. 378.43 cu. units
B. 355.31 cu. units D. 389.51 cu. units

PROBLEM 15 - 35
CE May 1996

The area in the second quadrant of the circle $x^2 + y^2 = 36$ is revolved about the line $y + 10 = 0$. What is the volume generated?
A. 2218.6 C. 2233.4
B. 2228.8 D. 2208.5

PROBLEM 15 - 36

A square area of edge "a" revolves about a line through one vertex, making an angle θ with an edge and not crossing the square. Find the volume generated.
A. $3\pi a^3 (\sin\theta + \cos\theta)$ C. $2\pi a^3 (\sin\theta + \cos\theta)$
B. $\pi a^3 (\sin\theta + \cos\theta)/2$ D. $\pi a^3 (\sin\theta + \cos\theta)$

PROBLEM 15 - 37

Given an ellipse whose semi-major axis is 6 cm. and semi-minor axis is 3 cm. What is the volume generated if it is revolved about the minor axis?
A. 36π cu. cm. C. 96π cu. cm.
B. 72π cu. cm. D. 144π cu. cm.

PROBLEM 15 - 38
CE Nov. 1999

A square hole 2" × 2" is cut through a 6-inch diameter log along its diameter and perpendicular to its axis. Find the volume of wood that was removed.
A. 27.32 cu. in. C. 21.78 cu. in.
B. 23.54 cu. in. D. 34.62 cu. in.

PROBLEM 15 - 39
CE May 2003

Find the radius of the spherical wedge whose volume is 12 cu. m. with a central angle of 1.8 radians.
A. 2.36 m C. 2.52 m
B. 2.73 m D. 2.15 m

PROBLEM 15 - 40
CE May 2003

By using Pappus Theorem, determine the volume generated by revolving the area in the first and second quadrants bounded by the ellipse $4x^2 + 25y^2 = 100$ and the x-axis, about the x-axis.
A. 85.63 C. 95.35
B. 93.41 D. 83.78

PROBLEM 15 - 41
CE Nov. 2003

Determine the volume of a spherical wedge of radius 2 m and a central angle of 1.25 radians.
A. 6.67 m^3 C. 9.85 m^3
B. 8.64 m^3 D. 5.74 m^3

PROBLEM 15 - 42

Find the volume generated by revolving the triangle whose vertices are (2, 2), (4, 8), and (6, 2) about the line
$3x - 4y - 24 = 0$.
A. 365.45 C. 543.65
B. 498.12 D. 422.23

PROBLEM 15 - 43

A light bulb is placed at a certain distance from the surface of a spherical globe of radius 20 cm. If it illuminates one-third of the total surface of the globe, how far is it from the surface?
A. 30 cm C. 60 cm
B. 35 cm D. 40 cm

PROBLEM 15 - 44

A conoid has a circular base of radius 25 cm and an altitude of 30 cm. Find its volume in cc.
A. 32,457 C. 24,486
B. 29,452 D. 18,453

PROBLEM 15 - 45

A sphere having a diameter of 30 cm. is cut into 2 segments. The altitude of the first segment is 8 cm. What is the ratio of the volume of the second segment to that of the first?
A. 3.2 C. 5.8
B. 4.7 D. 2.5

ANSWER SHEET

1. Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
2. Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
3. Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
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Solutions to Set 15 Spheres, Prismatoid, Solids of Revolutions, Misc.

SOLUTION 15 - 1
Ans: A

$V = \frac{4}{3}\pi r^3 = 36$; $r = 2.048$ m

$A_s = 4\pi r^2 = 4\pi(2.048)^2 = \mathbf{52.7\ m^2}$

SOLUTION 15 - 2
Ans: B

Since all spheres are similar, then:

$$\frac{A_{s1}}{A_{s2}} = \left(\frac{r_1}{r_2}\right)^2, \text{ where } A_{s2} = 1.21\,A_{s1}$$

$$\frac{A_{s1}}{1.21A_{s1}} = \left(\frac{r_1}{r_2}\right)^2; \quad \frac{r_1}{r_2} = 0.9091$$

$$\frac{V_1}{V_2} = \left(\frac{r_1}{r_2}\right)^3 = (0.9091)^3; \quad V_2 = 1.331\,V_1$$

Therefore, the volume is increased by **33.1%**

SOLUTION 15 - 3
Ans: B

Since all spheres are similar, then;

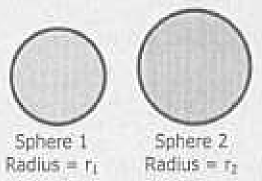
$$\frac{A_{s1}}{A_{s2}} = \left(\frac{d_1}{d_2}\right)^2, \text{ where } A_{s2} = 1.21\,A_{s1}$$

$$\frac{A_{s1}}{1.21A_{s1}} = \left(\frac{d_1}{d_2}\right)^2; \quad \frac{d_1}{d_2} = 0.909091$$

$$d_2 = 1.1d_1$$

Thus, the diameter is increased by **10%**

SOLUTION 15 - 4
Ans: B



Since all spheres are similar, then;

$$\frac{A_{s1}}{A_{s2}} = \left(\frac{d_1}{d_2}\right)^2, \text{ where } A_{s2} = 1.21\,As1$$

$$\frac{A_{s1}}{1.21A_{s1}} = \left(\frac{d_1}{d_2}\right)^2; \quad \frac{d_1}{d_2} = 0.909091$$

$$\frac{V_1}{V_2} = \left(\frac{d_1}{d_2}\right)^3 = (0.909091)^3$$

$$V_2 = 1.331\,V_1$$

Increase = $1.331\,V_1 - V_1 = 0.331\,V_1$ or 33.1%

SOLUTION 15 - 5
Ans: D

Since all spheres are similar, then;

$$\frac{V_1}{V_2} = \left(\frac{r_1}{r_2}\right)^3 ; r_2 = 1.2\, r_1$$

$$\frac{V_1}{V_2} = \left(\frac{r_1}{1.2r_1}\right)^3 = 0.5787$$

$$V_2 = 1.728V_1$$

The volume is increased by **72.8%**

SOLUTION 15 - 6
Ans: B

Let V_1 and V_2 be the volume of the bigger and smaller spheres, respectively, and whose corresponding radii be r_1 and r_2, then;

$V_1 + V_2 = 819$ → (1)
$r_2/r_1 = 3/4$ → (2)

For similar solids:

$$\frac{V_2}{V_1} = \left(\frac{r_2}{r_1}\right)^3 = \left(\frac{3}{4}\right)^3 = 27/64$$

$$V_1 = \frac{64}{27}V_2 \qquad \rightarrow (3)$$

Substitute V_1 in Eq. (3) to Eq. (1)

$$\tfrac{64}{27}V_2 + V_2 = 819$$

$$\tfrac{91}{27}V_2 = 819$$

$$\mathbf{V_2 = 243\text{ cu. m.}}$$

SOLUTION 15 - 7
Ans: D

For similar solids:

$$\frac{a}{A} = \left(\frac{r}{R}\right)^2 = \left(\frac{r}{1.05r}\right)^2$$

$$A = 1.1025\text{ a}$$

Therefore; the surface area is increased by **10.25%**

SOLUTION 15 - 8
Ans: A

Volume of sphere = $\frac{4}{3}\pi r^3 = 904.78$; $r = 6$ m

Volume of spherical segment = $\dfrac{\pi h^2}{3}(3r - h)$

Volume of spherical segment = $\dfrac{\pi(4)^2}{3}[3(6) - 4]$

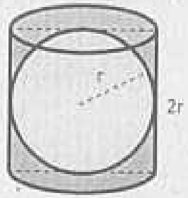
Volume of spherical segment = **234.57 cu. m.**

SOLUTION 15 - 9
Ans: D

$$V_{\text{empty space}} = V_{\text{cylinder}} - V_{\text{sphere}}$$

$$V_{\text{empty space}} = \pi r^2(2r) - \tfrac{4}{3}\pi r^3$$

$$V_{\text{empty space}} = \tfrac{2}{3}\pi r^3$$

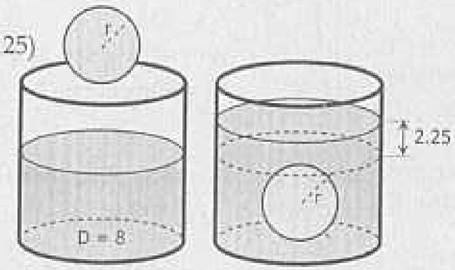


SOLUTION 15 - 10
Ans: A

$$V_{\text{sphere}} = V_{\text{displaced}}$$

$$\tfrac{4}{3}\pi r^3 = \tfrac{\pi}{4}(8)^2(2.25)$$

$$\mathbf{r = 3\text{ cm}}$$



SOLUTION 15 - 11
Ans: A

Let V, v, D, and d be the respective volume and diameter of the bigger and smaller spheres, then

$d/D = 2/3$

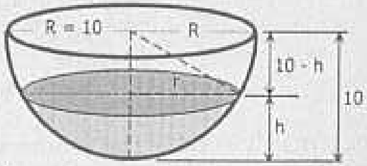
Since all spheres are similar, then

$$\frac{v}{V} = \left(\frac{d}{D}\right)^3 = \left(\frac{2}{3}\right)^3 = \frac{8}{27};\ v = \frac{8}{27}V$$

But $V + v = 1260$

$V + (8/27)V = 1260$, $\mathbf{V = 972\text{ cu. m.}}$

SOLUTION 15 - 12
Ans: B



$$\text{Volume} = \frac{\pi h^2}{3}(3R - h)$$

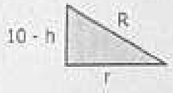
$$A_s = \pi r^2 = 75\pi;\ r^2 = 75$$

From the right triangle shown:

$$R^2 = (10 - h)^2 + r^2$$

$$(10 - h)^2 = 10^2 - 75 = 25$$

$$10 - h = 5;\ h = 5\text{ cm}$$



$$\text{Volume} = \frac{\pi(5)^2}{3}[3(10) - 5]$$

$$\text{Volume} = \mathbf{625\pi/3\text{ cm}^3}$$

SOLUTION 15 - 13
Ans: B

Volume of spherical segment of two bases:

$$V = \frac{\pi h}{6}[\,3a^2 + 3b^2 + h^2\,]$$

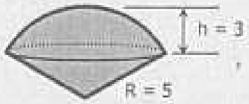
$$V = \frac{\pi(6)}{6}[\,3(3)^2 + 3(4)^2 + 6^2\,]$$

$$V = 348.72\text{ m}^3 \times \frac{1000\text{ lit/m}^3}{3.79\text{ lit/gal}} = \mathbf{92{,}011\ gals.}$$

SOLUTION 15 - 14
Ans: C

$$V = \tfrac{2}{3}\pi r^2 h = \tfrac{2}{3}\pi(5)^2(3)$$

$$V = \mathbf{50\pi\ cu.\ cm.}$$



SOLUTION 15 - 15
Ans: B

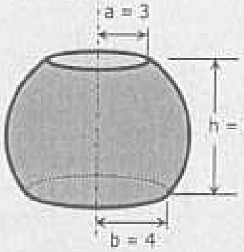
Volume of spherical segment of two bases:

$$V = \frac{\pi h}{6}[\,3a^2 + 3b^2 + h^2\,]$$

$$V = \frac{\pi(6)}{6}[\,3(3)^2 + 3(4)^2 + 6^2\,]$$

$$V = 348.72\text{ m}^3 \times \frac{1000\text{ lit/m}^3}{3.79\text{ lit/gal}}$$

$$V = \mathbf{92{,}011\ gals.}$$



SOLUTION 15 - 16
Ans: B

$$A_{zone} = 2\pi rh$$

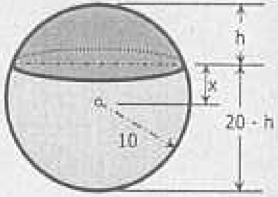
$$\frac{A_{\text{upper zone}}}{A_{\text{lower zone}}} = \frac{3}{7}$$

$$\frac{2\pi(10)h}{2\pi(10)(20-h)} = \frac{3}{7}$$

$$7h = 60 - 3h$$

$$h = 6\text{ cm}$$

$$x = r - h = 10 - 6 = \mathbf{4\ cm}$$



SOLUTION 15 - 17
Ans: A

The volume of spherical segment is:

$$V = \frac{\pi h^2}{3}(3R - h), \text{ where } R = 1\text{ m}$$

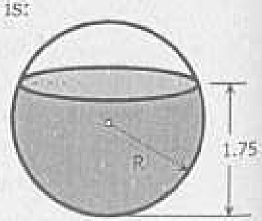
When the depth is $h = 1.75$ m

$$V = \frac{\pi(1.75)^2}{3}[3(1) - 1.75]$$

$$V = 4.009\text{ m}^3 = 4{,}009\text{ lit}$$

$$V_{added} = 4{,}009 - 1{,}396$$

$$V_{added} = \mathbf{2{,}613\ liters}$$



SOLUTION 15 - 18
Ans: A

$$\text{Volume} = \frac{\pi h^2}{3}(3R - h)$$

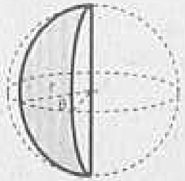
$$\text{Volume} = \frac{\pi(5)^2}{3}[3(10) - 5] = \mathbf{654.5\ cu.\ m.}$$

SOLUTION 15 - 19
Ans: C

$$\frac{V}{\theta} = \frac{(4/3)\pi r^3}{360°}$$

$$\frac{V}{50°} = \frac{(4/3)\pi(10)^3}{360°}$$

$$V = \mathbf{581.78\ cm^3}$$

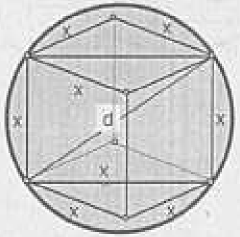


SOLUTION 15 - 20
Ans: A

$$A_Z = 2\pi rh = 2\pi(8)(12)$$

$$A_z = \mathbf{192\pi\ sq.\ in.}$$

SOLUTION 15 - 21
Ans: B



Volume required = $V_{sphere} - V_{cube}$

From the figure:

$$d = \sqrt{x^2 + (x\sqrt{2})^2} = x\sqrt{3}$$

Volume of cube = $x^3 = 2744$; $x = 14$

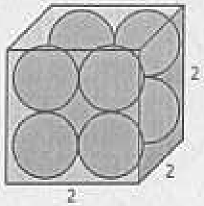
$$d = 14\sqrt{3} = 24.2487\text{ cm.}$$

Radius of sphere, $r = 12.124$ cm.

Volume required = $\frac{4}{3}\pi r^3 - 2744$

Volume required = $\frac{4}{3}\pi(12.124)^3 - 2744 = \mathbf{4721\ cm^3}$

SOLUTION 15 - 22
Ans: C



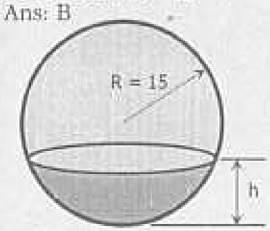
For the 8 marbles to fit the box tightly, each must have a diameter of 1" or a radius of 0.5"

The volume of water in the container is equal to the volume of the cube minus the total volume of the 8 marbles.

$$V_{water} = (2)^3 - \tfrac{4}{3}\pi(0.5)^3 \times 8$$

$$V_{water} = \mathbf{3.811\ cu.\ inches}$$

SOLUTION 15 - 23
Ans: B



The water in the tank has the shape of a spherical segment of depth h.

The volume is given by:

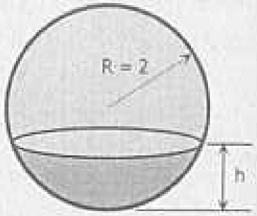
$$V = \frac{\pi h^2}{3}(3R - h)$$

$$1470.265 = \frac{\pi h^2}{3}[\,3(15) - h\,];\quad 1404 = 45h^2 - h^3$$

By trial and error using the choices, h = **6 cm**

SOLUTION 15 - 24
Ans: A

The water in the tank has the shape of a spherical segment of depth h.



The volume is given by:

$$V = \frac{\pi h^2}{3}(3R - h)$$

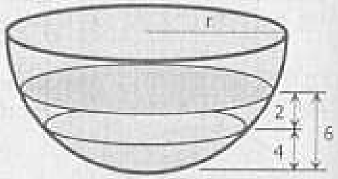
$$5.236 = \frac{\pi h^2}{3}[\,3(2) - h\,]$$

$$5 = 6h^2 - h^3$$

By trial and error using the choices, $h = 1$ m

SOLUTION 15 - 25
Ans: B

The volume of spherical segment is:



$$V = \frac{\pi h^2}{3}(3r - h)$$

$$V_{\text{total}} = \frac{\pi(6)^2}{3}(3r - 6)$$

$$V_{\text{total}} = 12\pi\,(3r - 6)$$

$V_{\text{total}} = V_{\text{black}} + V_{\text{white}}$; but $V_{\text{black}} = V_{\text{white}}$

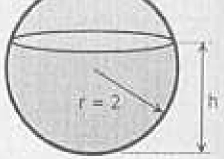
$$V_{\text{total}} = V_{\text{white}} + V_{\text{white}} = 2\,V_{\text{white}}$$

$$12\pi\,(3r - 6) = 2\,\frac{\pi(4)^2}{3}(3r - 4)$$

$54r - 108 = 48r - 64;\; 6r = 44;\; r =$ **7.33 inches**

SOLUTION 15 - 26
Ans: C

The water inside the tank will assume the shape of a spherical segment.



The volume is, $V = \frac{\pi h^2}{3}(3r - h)$

$$20.5 = \frac{\pi h^2}{3}[\,3(2) - h\,]$$

By trial and error using the choices, $h = 2.3$ m

SOLUTION 15 - 27
Ans: A

The volume of the sphere of radius r is:

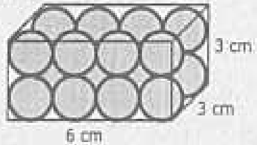
$V = \frac{4}{3}\pi r^3 = 36\pi$; $r = 3$ m

The surface area of a sphere is:

$A_s = 4\pi r^2 = 4\pi(3)^2$

$A_s = 36\pi$ **sq. m.**

SOLUTION 15 - 28
Ans: D



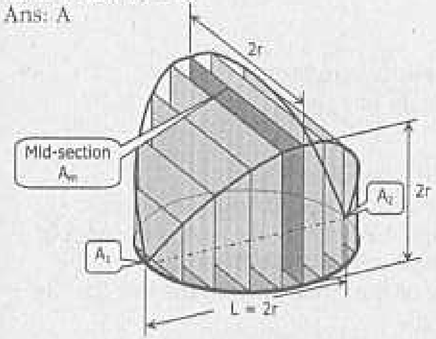
No. of balls that can tightly be fitted = 4(2)(2) = 16

Free space = $V_{\text{box}} - 16 \times V_{\text{sphere}}$

Free space = $6(3)(3) - 16\,\frac{4}{3}\,\pi(0.75)^3$

Free space = **25.73 cc**

SOLUTION 15 - 29
Ans: A



$$V = \frac{L}{6}(A_1 + 4A_m + A_2)$$

$$V = \frac{2r}{6}[0 + 4(2r)(2r) + 0]$$

$$V = \mathbf{16\,r^3/3}$$

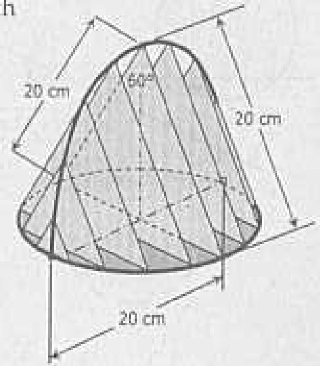
SOLUTION 15 - 30
Ans: C

The solid is a prismatoid with

$A_1 = 0,\; A_2 = 0,\; L = 20$

$A_m = \frac{1}{2}(20)(20)\sin 60°$

$A_m = 173.205\ \text{cm}^2$

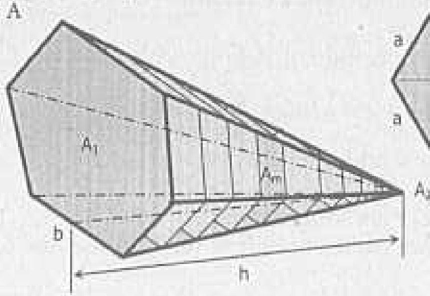


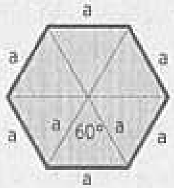
$$V = \frac{L}{6}(A_1 + 4A_m + A_2)$$

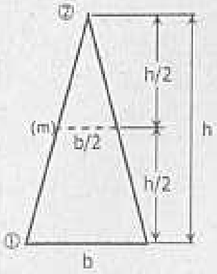
$$V = \frac{20}{6}[0 + 4(173.205) + 0]$$

$$V = \mathbf{2309.4\ cm^3}$$

SOLUTION 15 - 31
Ans: A







The area of a regular hexagon of side a is:

$$A = ½ a^2 \sin 60° \times 6 = 3\sqrt{3}\, a^2/2$$

$$V = \frac{L}{6}\left[A_1 + 4A_m + A_2\right];\ L = h;\ A_2 = 0$$

$$A_1 = 3\sqrt{3}\, b^2/2;\ A_m = 3\sqrt{3}\,(b/2)^2/2 = 3\sqrt{3}\, b^2/8$$

$$V = \frac{h}{6}\left[3\sqrt{3}b^2/2 + 4\times 3\sqrt{3}b^2/8 + 0\right]$$

$$V = ½\sqrt{3}\, b^2 h$$

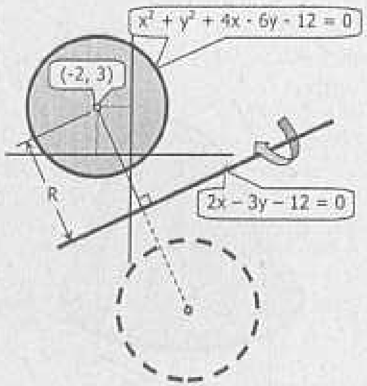
Note: The given solid is a pyramid.

$$V = \tfrac{1}{3} Ah = \tfrac{1}{3}\,[3\sqrt{3}\, b^2/2]h$$

$$V = ½\sqrt{3}\, b^2 h$$

SOLUTION 15 - 32
Ans: D

By the second proposition of Pappus, Volume = $A \times 2\pi R$, where A is the area of the circle and R is the distance from the center of the circle (h, k) to the line $2x - 3y - 12 = 0$.



Solving for the radius of the circle:

$$x^2 + y^2 + 4x - 6y = 12$$
$$x^2 + 4x + 4 + y^2 - 6y + 9 = 12 + 4 + 9 = 25$$
$$(x + 2)^2 + (y - 3)^2 = 5^2$$

$r = 5$ and center $(h, k) = (-2, 3)$.

Solving for R:
Distance from $2x - 3y - 12 = 0$ to $(-2, 3)$

$$R = \left|\frac{2(-2) - 3(3) - 12}{\sqrt{2^2 + 3^2}}\right| = 6.934$$

Volume = $[\pi(5)^2] \times 2\pi\,(6.934)$
Volume = **3422 cu. units**

SOLUTION 15 - 33
Ans: D

By the second proposition of Pappus, Volume = $A \times 2\pi R$

For the ellipse $9x^2 + 4y^2 = 36$ or $\frac{x^2}{4} + \frac{y^2}{9} = 1$

$a = 3$ and $b = 2$ and center is at the origin (0, 0).

$A = \pi ab = \pi(3)(2) = 6\pi$ square units
R = distance from center of ellipse (0, 0) to the line $4x + 3y - 20 = 0$

$$R = \left|\frac{4(0) + 3(0) - 20}{\sqrt{4^2 + 3^2}}\right| = 4 \text{ units}$$

Volume = $6\pi \times 2\pi(4)$
Volume = **48 π^2 cubic units**

SOLUTION 15 - 34
Ans: B

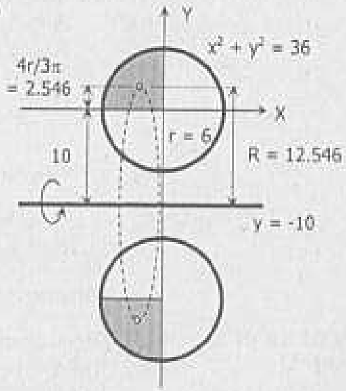
$$\frac{y^2}{3^2} + \frac{x^2}{2^2} = 1$$

$V = A \times 2\pi r$
$A = \pi a b = \pi(3)(2) = 6\pi$
$r = 3$
$V = 6\pi \times 2\pi(3)$
V = **355.31 cu. units**

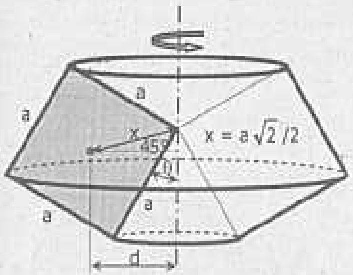
SOLUTION 15 - 35
Ans: B

The circle has a radius of 6 with center at (0, 0)

Using Pappu's Theorem:
$V = A \times 2\pi R$
$V = ¼\pi\,(6)^2 \times 2\pi\,(12.546)$
V = 2228.83 cu. units



SOLUTION 15 - 36
Ans: D



$$d = x \sin(45° + \theta)$$

$$d = (a\sqrt{2}/2) \sin(45° + \theta)$$

By the second proposition of Pappus:

$$V = A\, 2\pi d = (a^2)\, 2\pi\, (a\sqrt{2}/2) \sin(45° + \theta)$$
$$V = \pi a^3 \sqrt{2}\ (\sin 45° \cos\theta + \cos 45° \sin\theta)$$

but $\sin 45° = \cos 45° = 1/\sqrt{2}$

$$V = \pi a^3 \sqrt{2}\ [(1/\sqrt{2}) \cos\theta + (1/\sqrt{2}) \sin\theta]$$

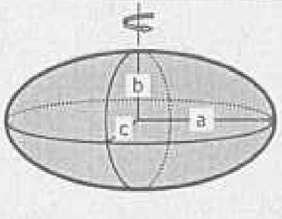
$V = \pi a^3$ **(sin θ + cos θ)**

SOLUTION 15 - 37
Ans: D

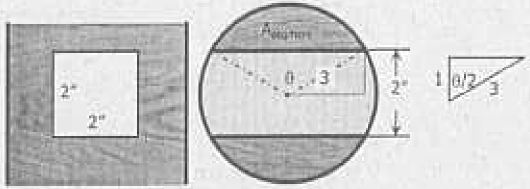
Volume = $\frac{4}{3}\pi abc$, where $c = a$

Volume = $\frac{4}{3}\pi(6)(3)(6)$

Volume = **144π cu. cm.**



SOLUTION 15 - 38
Ans: B



Volume = $A \times h$

$A = A_{circle} - 2 \times A_{segment}$

$A_{segment} = A_{sector} - A_{triangle}$

$\cos\frac{\theta}{2} = \frac{1}{3};\ \theta = 141.06°$

$$A_{segment} = \frac{\pi(3)^2(141.06°)}{360°} - ½(3)^2 \sin 141.06°$$

$A_{segment} = 8.25$

$A = \pi\,(3)^2 - 2 \times 8.25 = 11.77\ in^2$

Volume = 11.77 × 2 = **23.54 in³**

SOLUTION 15 - 39
Ans: D

The volume of a spherical wedge of central angle θ is directly proportional to the volume of sphere, which is a spherical wedge of central angle 2π.

$$\frac{V_{wedge}}{\theta} = \frac{\frac{4}{3}\pi r^3}{2\pi} = \frac{2r^3}{3}$$

$$\frac{12}{1.8} = \frac{2r^3}{3};\ r = \mathbf{2.154\ m}$$

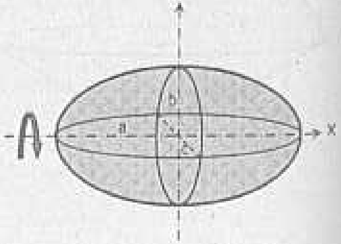
SOLUTION 15 - 40
Ans: D

Ellipse: $4x^2 + 25y^2 = 100$

$$\frac{x^2}{5^2} + \frac{y^2}{2^2} = 1$$

$a = 5$ and $b = 2$

The solid formed is a prolate spheroid where $c = b = 2$



$$\text{Volume} = \frac{4}{3}\pi abc = \frac{4}{3}\pi ab^2$$

$$\text{Volume} = \frac{4}{3}\pi(5)(2)^2 = \mathbf{83.776\ cubic\ units}$$

Note: *No need to use Pappus Theorem as suggested in the problem*

SOLUTION 15 - 41
Ans: A

$$\frac{V_{wedge}}{\theta} = \frac{\frac{4}{3}\pi r^3}{2\pi};\quad \frac{V_{wedge}}{1.25} = \frac{\frac{4}{3}\pi(2)^3}{2\pi}$$

V_{wedge} = **6.67 cu. m.**

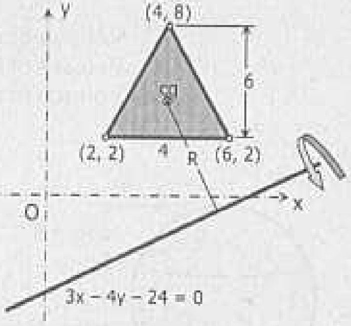
SOLUTION 15 - 42
Ans: D

Area = ½(4)(6) = 12

By inspection, cg (4, 4)

$$R = d = \left|\frac{3(4) - 4(4) - 24}{\sqrt{3^2 + (-4)^2}}\right|$$

$R = 5.6$



Volume = $A\ 2\pi R$

Volume = 12 × 2π(5.6)

Volume = **422.23**

SOLUTION 15 - 43
Ans: D

$A_{zone} = \frac{1}{3} A_{sphere}$

$2\pi Rh = \frac{1}{3} \times 4\pi R^2$

$2\pi(20)h = \frac{4}{3}\pi(20)^2$

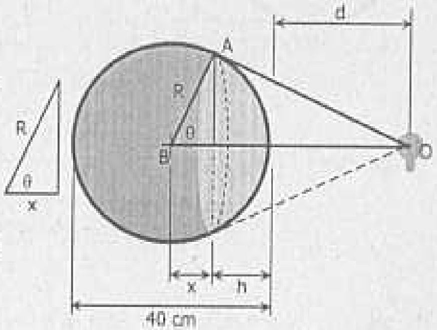
$h = 13.333$ cm

$x = 20 - 13.333$

$x = 6.667$ cm

$$\cos\theta = \frac{x}{R} = \frac{6.667}{20}$$

$\theta = 70.529°$



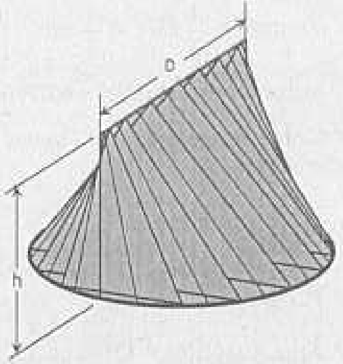
In triangle *BOA*:

$$\cos\theta = \frac{R}{OB};$$

$$\cos 70.529° = \frac{20}{OB};\ OB = 60\ cm$$

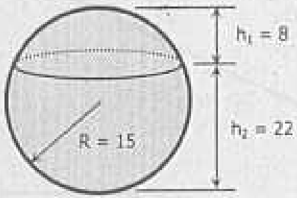
$d = OB - R = 60 - 20 =$ **40 cm**

SOLUTION 15 - 44
Ans: B



Volume of conoid = ½ Vol. of circumscribing cylinder
Volume of conoid = ½ $A_b \times h$ = ½$[\pi(25)^2 \times 30]$
Volume of conoid = **29,452.4 cc**

SOLUTION 15 - 45
Ans: B



$$\text{Volume} = \frac{\pi h^2}{3}(3R - h)$$

$$\text{Ratio} = \frac{V_2}{V_1} = \frac{\frac{\pi(22)^2}{3}[3(15) - 22]}{\frac{\pi(8)^2}{3}[3(15) - 8]}$$

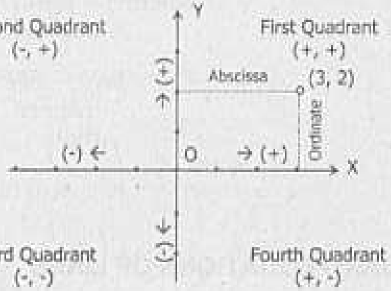
Ratio = **4.7**

Part 5

ANALYTIC GEOMETRY

Plane Analytic Geometry

CARTESIAN OR RECTANGULAR COORDINATE SYSTEM



Distance between Two Points

The distance between point $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ is:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

STRAIGHT LINE

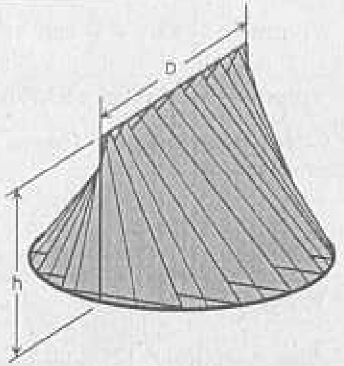
A straight line is a line that does not change in direction. Thu it has a uniform slope.

General Equation of a Line

The general equation of a straight line is:

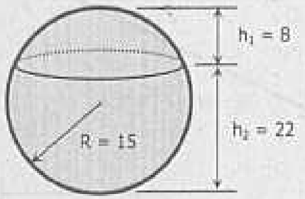
$$Ax + By + C = 0$$

SOLUTION 15 - 44
Ans: B



Volume of conoid = ½ Vol. of circumscribing cylinder
Volume of conoid = ½ $A_b \times h$ = ½[π(25)² × 30]
Volume of conoid = 29,452.4 cc

SOLUTION 15 - 45
Ans: B



$$\text{Volume} = \frac{\pi h^2}{3}(3R - h)$$

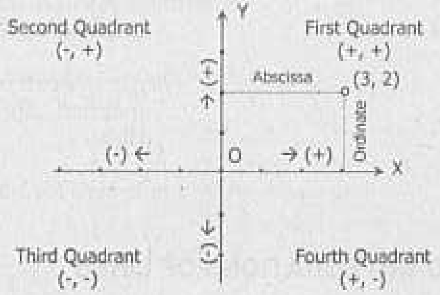
$$\text{Ratio} = \frac{V_2}{V_1} = \frac{\frac{\pi(22)^2}{3}[3(15) - 22]}{\frac{\pi(8)^2}{3}[3(15) - 8]}$$

Ratio = 4.7

Part 5
ANALYTIC GEOMETRY

Plane Analytic Geometry

CARTESIAN OR RECTANGULAR COORDINATE SYSTEM



Distance between Two Points

The distance between point $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ is:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

STRAIGHT LINE

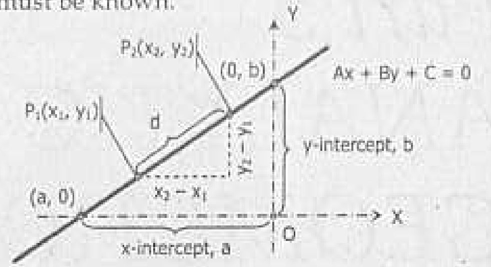
A straight line is a line that does not change in direction. Thu it has a uniform slope.

General Equation of a Line

The general equation of a straight line is:

$$Ax + By + C = 0$$

To solve a line, either *two points* or *one point and a slope* must be known.



Slope of the Line

The slope of the line passing through points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ is:

$$\text{Slope, } m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$$

where m is positive if the line is inclined upwards to the right.

m is negative if the line is inclined downwards to the right.

m is zero for horizontal lines.

STANDARD EQUATIONS OF LINES

1. Point-slope form Given a point $P_1(x_1, y_1)$ and slope its m:

$$y - y_1 = m\,(x - x_1)$$

2. Slope-intercept form Given a slope m and y-intercept:

$$y = m\,x + b$$

3. Intercept form Given x-intercept a and y-intercept b:

$$\frac{x}{a} + \frac{y}{b} = 1$$

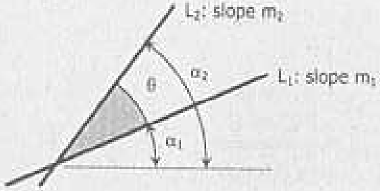
4. Two-point form Given two points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$:

$$\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1} = m$$

Note: All these four standard equations can be reduced into the *point-slope form*.

Angle between Two Lines

The angle between lines L_1 and L_2 is the angle θ that L_1 must be rotated in a counterclockwise direction to make it coincide with L_2.



$$\alpha_1 = \arctan m_1$$
$$\alpha_2 = \arctan m_2$$

$$\theta = \alpha_2 - \alpha_1$$

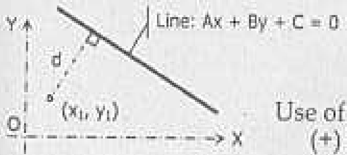
$$\tan\theta = \frac{m_2 - m_1}{1 + m_1 m_2}$$

Lines are parallel if $m_1 = m_2$

Lines are perpendicular if $m_2 = -\dfrac{1}{m_1}$

Distance from a Point to a Line

The distance (nearest) from a point $P_1(x_1, y_1)$ to a line $Ax + By + C = 0$ is:



$$d = \frac{Ax_1 + By_1 + C}{\pm\sqrt{A^2 + B^2}}$$

Use of Sign:
- (+) if B is a positive number
- (-) if B is a negative number
- (+) if the point is above the line or to the right of the line for a vertical line
- (-) if the point is below the line or to the left of the line for a vertical line.

That is;

If B is positive and the point is above the line, then use (+)(+) = (+)

If B is positive and the point is below the line, then use (+)(-) = (-)

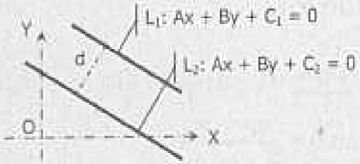
If B is negative and the point is above the line, then use (-)(+) = (-)

If B is negative and the point is below the line, then use (-)(-) = (+)

If only the distance is required then use the *absolute value.*

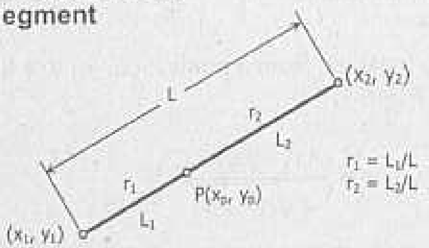
$$d=\left|\frac{Ax_1+By_1+C}{\sqrt{A^2+B^2}}\right|$$

Distance between Two Parallel Lines



$$d=\left|\frac{C_2-C_1}{\sqrt{A^2+B^2}}\right|$$

Division of Line Segment



$$x_p=\frac{x_1r_2+x_2r_1}{r_1+r_2}$$

$$y_p=\frac{y_1r_2+y_2r_1}{r_1+r_2}$$

Midpoint of a Line Segment

The midpoint $P_m(x_m, y_m)$ of a line segment through from $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ is:

$$x_m=\frac{x_1+x_2}{2} \qquad y_m=\frac{y_1+y_2}{2}$$

AREA OF POLYGON BY COORDINATES

Let (x_1, y_1), (x_2, y_2), (x_3, y_3), …, (x_{n-1}, y_{n-1}), (x_n, y_n) be the consecutive vertices of a polygon arranged in *counterclockwise* sense. The area is:

$$A=\frac{1}{2}\begin{vmatrix} x_1 & x_2 & x_3 & \cdots & x_{n-1} & x_n & x_1 \\ y_1 & y_2 & y_3 & \cdots & y_{n-1} & y_n & y_1 \end{vmatrix}$$

$$A=\frac{1}{2}\left[(x_1y_2+x_2y_3+\ldots)-(y_1x_2+y_2x_3+\ldots)\right]$$

CONIC SECTIONS

Conic sections a locus (or path) of a point that moves such that the ratio of its distance from a fixed point (called the *focus*) and a fixed line (called the *directrix*) is constant. This constant ratio is called the *eccentricity, e* of the conic.

The term *conic section* was based on the fact that these are sections formed if a plane is made to pass though a cone.

If the cutting plane is *parallel to the base* of a cone, the section formed is a **circle**. If it is *parallel to the element* (or generator) of the cone, the section formed is a **parabola**. If it is *perpendicular to the base* of the cone, the section formed is a **hyperbola**. If it is oblique to the base or element of the cone, the section formed is an **ellipse**.

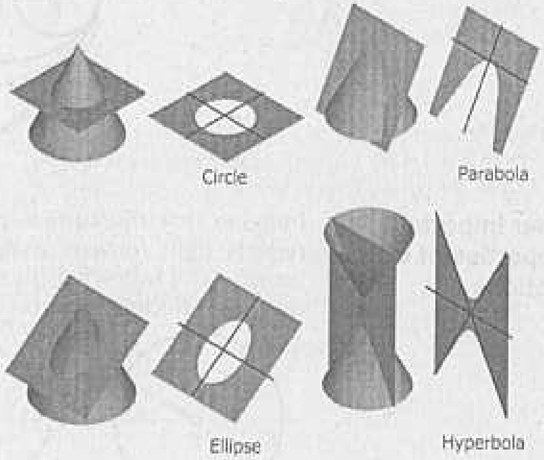


Figure: Conic Sections

General Equation of Conics

$$Ax^2+Bxy+Cy^2+Dx+Ey+F=0$$

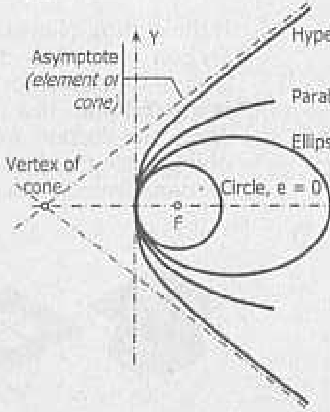
If $B \neq 0$, the axis of the conic is oblique with the coordinate axes (i.e. not parallel to X or Y axes). Thus if the axis is parallel to either X or Y-axes, the equation becomes

$$Ax^2+Cy^2+Dx+Ey+F=0$$

From the foregoing equations:

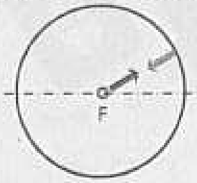
If $B^2 < 4AC$, the conic is an *ellipse*
If $B^2 = 4AC$, the conic is a *parabola*
If $B^2 > 4AC$, the conic is a *hyperbola*

Also, a conic is a *circle* if $A = C$, an *ellipse* if $A \neq C$ but have the same sign, a parabola if either $A = 0$ or $C = 0$, and a hyperbola if A and C have different signs.

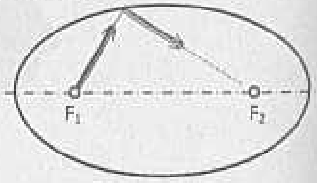


Other Important Properties of Conic Sections

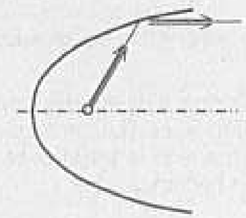
Imagine that the surface of a conic is mirror that reflects light (or any material that reflects sound beam). The following illustration shows how these rays are reflected from its source.



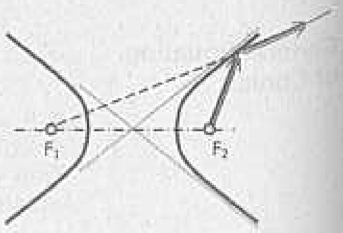
Circle



Ellipse

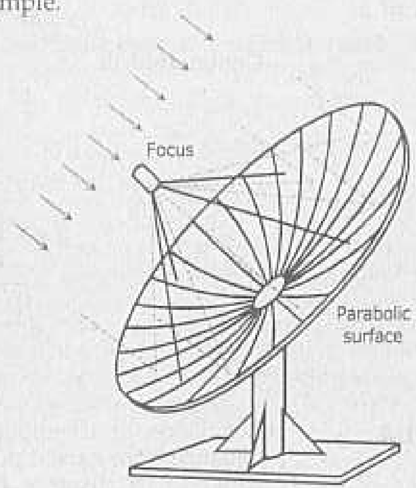


Parabola

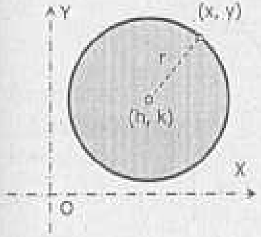


Ellipse

The *parabolic antenna* used in communication is one example.



CIRCLE



Circle is the locus of a point that moves such that it is always equidistant from a fixed point called the center. The constant distance is called the radius of the circle.

r = radius
(h, k) = center

General Equation of a Circle (A = C)

$$Ax^2 + Ay^2 + Dx + Ey + F = 0$$
$$\text{or } x^2 + y^2 + Dx + Ey + F = 0$$

To solve a circle, either one of the following two conditions must be known:

1. Three points along the circle,
   Solution: *Use the general form.*
2. Center (h, k) and the radius r,
   Solution: *Use the standard equation*

Standard Equations of a Circle

Center at (h, k)

$$(x-h)^2+(y-k)^2=r^2$$

Center at (0, 0)

$$x^2+y^2=r^2$$

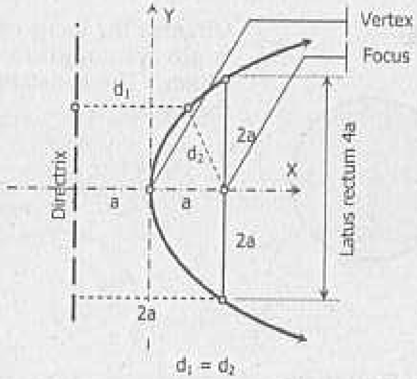
For the circle $Ax^2 + Ay^2 + Dx + Ey + F = 0$:

$$h=\frac{-D}{2A};\ k=\frac{-E}{2A}$$

$$r=\sqrt{\frac{D^2+E^2-4AF}{4A^2}}$$

PARABOLA

The locus of the point that moves such that its distance from a fixed point called the focus is always equal to its distance from a fixed line called the directrix.

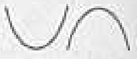


a = distance from the vertex to the focus
LR = length of latus rectum

General equation of parabola (A or C is zero)

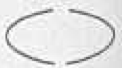
$C = 0$

$$Ax^2 + Dx + Ey + F = 0$$
$$\text{or}\quad x^2 + Dx + Ey + F = 0$$



$A = 0$

$$Cy^2 + Dx + Ey + F = 0$$
$$\text{or}\quad y^2 + Dx + Ey + F = 0$$



To solve a parabola, either one of the following conditions must be known:

1. Three points along the parabola and an axis (either vertical or horizontal)
   Solution: *Use the general form*
2. Vertex (h, k), distance from vertex to focus a, and axis.
   Solution: *Use the standard equation*
3. Vertex (h, k), and location of focus.
   Solution: *Use the standard equation*

Eccentricity

The eccentricity of a conic is the ratio of its distance from the focus (d_2) and from the directrix (d_1).
For a parabola, the eccentricity is equal to 1.

$$e = 1$$

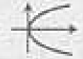
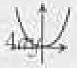
Latus Rectum, LR

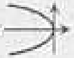
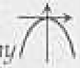
Latus rectum is a chord passing through the focus and parallel to the directrix or perpendicular to the axis.

$$LR = 4a$$

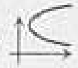
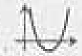
Standard Equations of Parabola

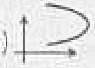
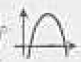
Vertex at (0, 0)

$y^2 = 4ax$  $x^2 = 4ay$ 

$y^2 = -4ax$  $x^2 = -4ay$ 

Vertex at (h, k)

$(y-k)^2 = 4a(x-h)$  $(x-h)^2 = 4a(y-k)$ 

$(y-k)^2 = -4a(x-h)$  $(x-h)^2 = -4a(y-k)$ 

For the parabola $Ax^2 + Dx + Ey + F = 0$ (axis vertical)

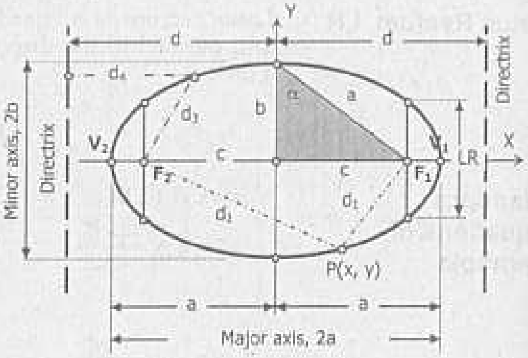
$$h = \frac{-D}{2A};\ k = \frac{D^2 - 4AF}{4AE};\ a = \frac{-E}{4A}$$

For the parabola $Cy^2 + Dx + Ey + F = 0$ (axis horizontal)

$$h = \frac{E^2 - 4CF}{4CD};\ k = \frac{-E}{2C};\ a = \frac{-D}{4C}$$

ELLIPSE

The locus of the point that moves such that the sum of its distances from two fixed points called the foci is constant. The constant sum is the length of the major axis, $2a$. It can also be defined as the locus of the point that moves such that the ratio of its distance from a fixed point, called the focus, and a fixed line, called the directrix, is constant and is less than one (1).



$d_1 + d_2 = 2a$

Elements of Ellipse

$$a^2 = b^2 + c^2$$

$$\text{Eccentricity (first eccentricity), } e = \frac{d_3}{d_4} = \frac{c}{a} < 1.0$$

$$\text{Distance from center to directrix, } d = \frac{a}{e}$$

$$\text{Length of latus rectum, } LR = \frac{2b^2}{a}$$

$$\text{Second eccentricity, } e' = \frac{c}{b}$$

$$\text{Angular eccentricity, } \alpha = \frac{c}{a}$$

$$\text{Ellipse flatness, } f = \frac{a-b}{a}$$

$$\text{Second flatness, } f' = \frac{a-b}{b}$$

General equation (A ≠ C but have the same sign)

$$Ax^2 + Cy^2 + Dx + Ey + F = 0$$

$$\text{or } x^2 + Cy^2 + Dx + Ey + F = 0$$

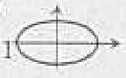
To solve a parabola, either one of the following conditions must be known:

1. Four points along the ellipse,
   Solution: *Use the general form*
2. Center (h, k), semi-major axis a, and semi-minor axis b,
   Solution: *Use the standard equation*

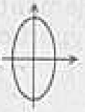
Standard Equations of Ellipse

Center at (0, 0)

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

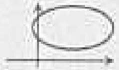


$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$

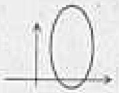


Center at (h, k)

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$



$$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$$

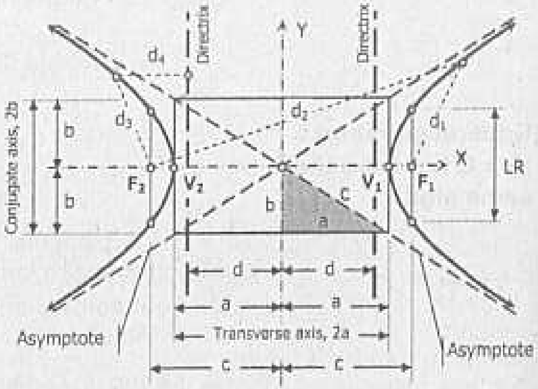


Note: $a > b$
For the ellipse $Ax^2 + Cy^2 + Dx + Ey + F = 0$

$$h = \frac{-D}{2A},\ k = \frac{-E}{2C}$$

HYPERBOLA

The locus of the point that moves such that the difference of its distance between two fixed points called the foci is constant. The constant difference is the length of the transverse axis $2a$. It may also be defined as the locus of the point that moves such that the ratio of its distance from a fixed point, called the focus, and a fixed line, called the directrix, is constant and is greater than one (1).



$$d_2 - d_1 = 2a$$

Elements of Hyperbola

$$c^2 = a^2 + b^2$$

$$\text{Eccentricity, } e = \frac{d_3}{d_4} = \frac{c}{a} > 1.0$$

$$\text{Distance from center to the directrix, } d = \frac{a}{e}$$

$$\text{Length of latus rectum, } LR = \frac{2b^2}{a}$$

Equation of Asymptote

The asymptotes of hyperbola has the following equations:

$$y - k = \pm m(x - h)$$

Where (h, k) is the center of the hyperbola and m is the slope. Use (+) for upward asymptote and (-) for downward asymptote.

$m = b/a$ if the axis is horizontal
$m = a/b$ if the axis is vertical

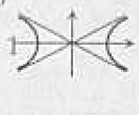
General Equation of Hyperbola (A and C have opposite sign)

$$Ax^2 - Cy^2 + Dx + Ey + F = 0$$
$$\text{or } x^2 - Cy^2 + Dx + Ey + F = 0$$

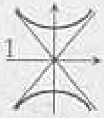
Standard Equations of Hyperbola

Center at (0, 0)

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

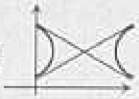


$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$

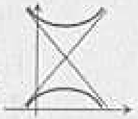


Center at (h, k)

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$



$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$



Note: "a" may be greater, equal, or less than "b".

For the hyperbola $Ax^2 - Cy^2 + Dx + Ey + F = 0$:

$$h = -\frac{D}{2A}, \quad k = -\frac{E}{2C}$$

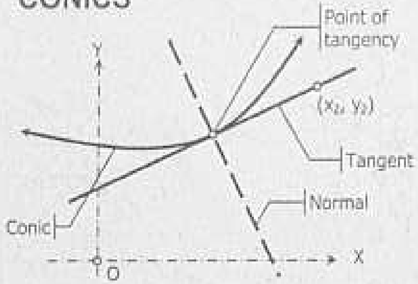
Variations of Problems in Conics

1. Given the equation of the conic, find the elements (center, eccentricity, focus, latus rectum, vertex, etc.)
   Solution: *Reduce the equation to standard form and apply the necessary formulas.*

2. Find the equation of a conic through given points. (3 points for circle, 3 points & axis for parabola, 4 points for ellipse and hyperbola)
   Solution: *Substitute the points to the general equation and solve for A, B, C, etc.*

3. Find the equation of a conic, given the elements (center & radius for circle; vertex and a for parabola; center, a, and b for ellipse and hyperbola)
   Solution: *Reduce the equation to standard form.*

TANGENTS AND NORMALS TO CONICS



Unlike other curves, the tangent to any conic will pass only through one point. To find the equation of the tangent to a conic, we may use differential calculus or make use of the following substitution for the variables in the equation:

x^2 → replace with $x\,x_1$
y^2 → replace with $y\,y_1$
x → replace with $\frac{x + x_1}{2}$
y → replace with $\frac{y + y_1}{2}$

where (x_1, y_1) is the point of tangency and (x, y) is any point on the line.

Case I

Find the equation of the tangent through a given point (x_1, y_1) on the conic.

- To find the equation of the tangent, we simply replace the variables in the conic with the above expressions and substitute for (x_1, y_1) the given point.

- We can also use differential calculus to solve for the slope $m = dy/dx$ of the curve at the given point (x_1, y_1), then use the point-slope form of the line,
$y - y_1 = m(x - x_1)$.

Case II

Find the equation of the tangent that passes through a given point (x_2, y_2) outside the conic.

- To find the equation of the tangent, we apply the necessary replacements of variables leaving x_1 and y_1 unknown. Another equation relating x_1 and y_1 can be found by substituting (x_1, y_1) to the equation of the conic. By expressing y_1 in terms of x_1 in either equation and substituting the other equation, a quadratic equation is derived in the form $Ax_1^2 + Bx_1 + C = 0$. With (x_1, y_1) known, the tangent can now be solved.

Case III

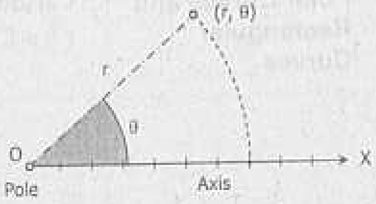
Find the equation of the tangent, given the slope m of the tangent.

- To find the equation of the tangent, we use the slope-intercept form $y = mx + b$ for the tangent line with m and b unknown. Since the line and the conic crosses, we can substitute this value of y to the value of y in the conic resulting to a quadratic equation in the form $Ax^2 + Bx + C = 0$ with A, B, and C as function of m and b. Since the tangent passes through one point only, we set $B^2 = 4AC$ and solve for b. With b known and m given, the tangent can now be solved.

- Or, for this case, the value of (x_1, y_1) can be found by differential calculus knowing that dy/dx = slope = m. After solving m, the equation of the line can be found using the point-slope form $y - y_1 = m(x - x_1)$

POLAR COORDINATE SYSTEM

In this system, the location of a point is expressed by its distance r from a fixed point called the pole and its angle θ from a fixed line O_x.

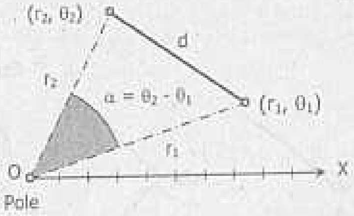


Sign Convention

- θ is positive (+) if measured counterclockwise.
- θ is negative (-) if measure clockwise.
- r is positive (+) if laid off at the terminal side of θ.
- r is negative (-) if laid off at the prolongation through O from the terminal side of θ.

Distance between Two Points

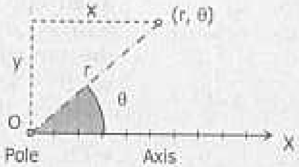
The distance between two points (r_1, θ_1) and (r_2, θ_2) can be found using cosine law.



$$d = \sqrt{r_1^2 + r_2^2 - 2r_1r_2\cos(\theta_2 - \theta_1)}$$

Relationship between Polar and Cartesian Coordinate Systems

Equations in rectangular form may be converted to polar form or vice versa. The following relationships can be found from the figure shown.



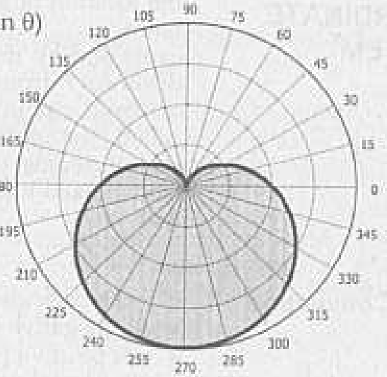
$$r^2 = x^2 + y^2$$

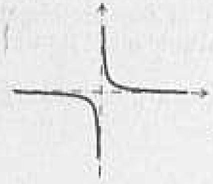
$$x = r\cos\theta \qquad y = r\sin\theta \qquad \tan\theta = \frac{y}{x}$$

Polar Curves and Rectangular Curves

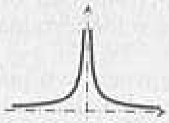
Cardioid

$r = a(1 - \sin\theta)$

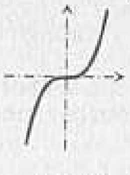




$y = 1/x$

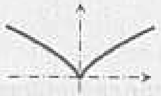


$y = |1/x|$

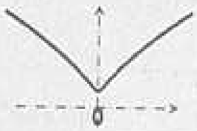


$y = a x^3$

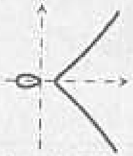
Cubic curve



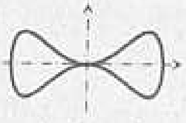
$y^3 = a x^2$



$y^2 - y = x^2$

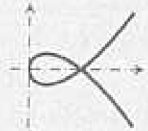


$y^2 = x(x + 3)(x - 2)$

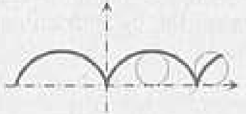


$y^2 = x^4(1 - x^2)$

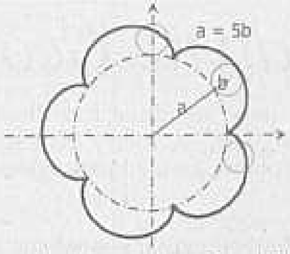
Bowtie



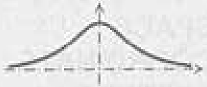
$y^2 = x(x - 1)^2$



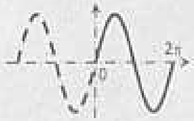
$x = a(\theta - \sin\theta)$
$y = a(1 - \cos\theta)$
a = radius of rolling circle
θ = angle of rotation
Cycloid



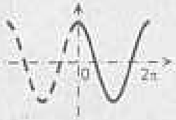
$x = (a + b)\cos\theta - b\cos[(a + b)\theta/b]$
$y = (a + b)\sin\theta - b\sin[(a + b)\theta/b]$
a = radius of fixed inner circle
b = radius of rolling outer circle
Epicycloid



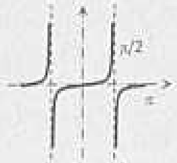
$x^2 y = 4a^2(2a - y)$
Witch of Agnesi



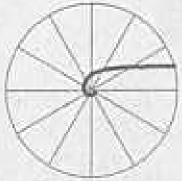
$y = \sin(x)$



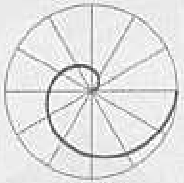
$y = \cos(x)$



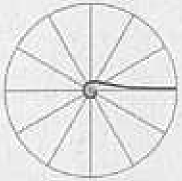
$y = \tan(x)$



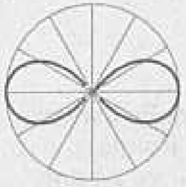
$r\theta = a$
Reciprocal Spiral



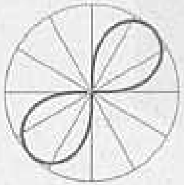
$r = a\theta$
Spiral of Archimedes



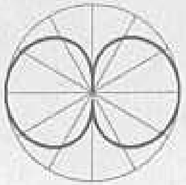
$r^2\theta = a^2$
Trumpet



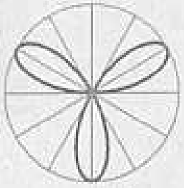
$r^2 = a^2\cos 2\theta$
Lemniscate



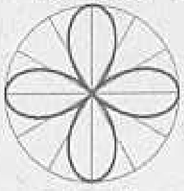
$r^2 = a^2\sin 2\theta$
Lemniscate



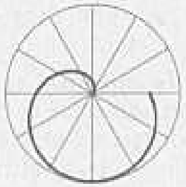
$r^2 = a^2\cos\theta$



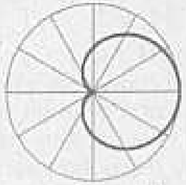
$r = a\sin 3\theta$
Three-leaved rose



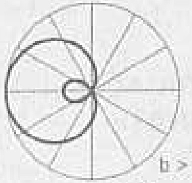
$r = a\cos 2\theta$
Four-leaved rose



$r = a\sin^3(\theta/3)$
Limacon of Pascal



$r = a(1 + \cos\theta)$
Cardiod



$r = a - b\cos\theta$
Limacon of Pascal

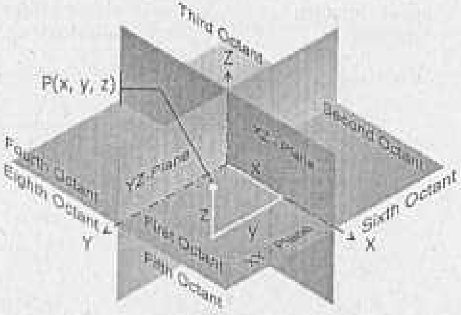
Space Analytic Geometry

SPACE COORDINATE SYSTEMS

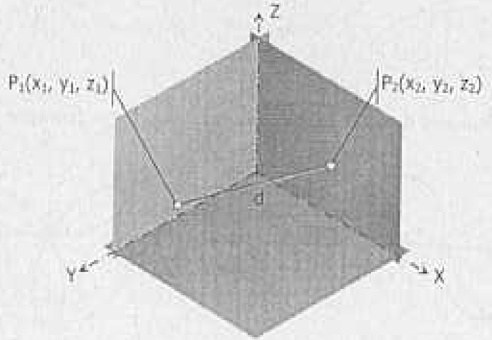
There are three coordinate systems used in space analytic geometry. The rectangular, cylindrical, and spherical coordinate systems.

RECTANGULAR COORDINATE SYSTEM IN SPACE

In rectangular coordinate system, a point $P(x, y, z)$ in space is fixed by its three distance x, y, and z from the three coordinate planes.



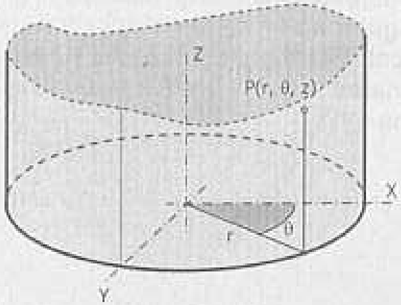
Distance between Two Points $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$



$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

CYLINDRICAL COORDINATE SYSTEM

A point P in space may be imagined as being on the surface of a cylinder perpendicular to the XY-plane. $P(r, \theta, z)$ is fixed by its distance z from the XY-plane and by the polar coordinates (r, θ) of the projection of P on the XY-plane.



Relations between Rectangular and Cylindrical Coordinates Systems

The relations between rectangular coordinates (x, y, z) and cylindrical coordinates (r, θ, z) are:

$$x = r\cos\theta$$

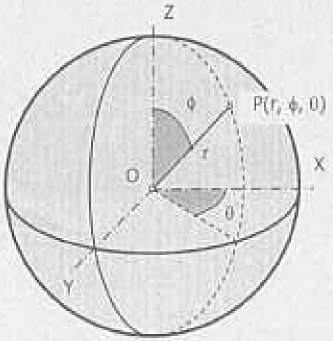
$$\theta = \arctan\frac{y}{x}$$

$$r^2 = x^2 + y^2 \quad z = z$$

$$y = r\sin\theta$$

SPHERICAL COORDINATES SYSTEM

A point P in space may be imagined as being on the surface of a sphere with center at the origin O and radius r. $P(r, \phi, \theta)$ is fixed by its distance r from O, the angle ϕ between OP and the Z-axis, and the angle θ which is the angle between the X-axis and the projection of OP on the XY-plane.



Relations between Rectangular and Spherical Coordinates Systems

The relations between rectangular coordinates (x, y, z) and spherical coordinates (r, ϕ, θ) are:

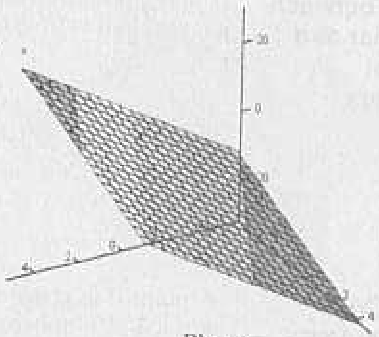
$$x = r \sin\phi \cos\theta; \quad r^2 = x^2 + y^2 + z^2$$

$$y = r \sin\phi \sin\theta$$

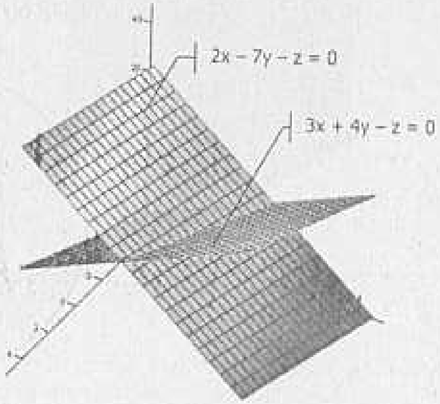
$$\phi = \arctan \frac{\sqrt{x^2 + y^2}}{z}; \quad \theta = \arctan \frac{y}{x}$$

$$z = r \cos\phi$$

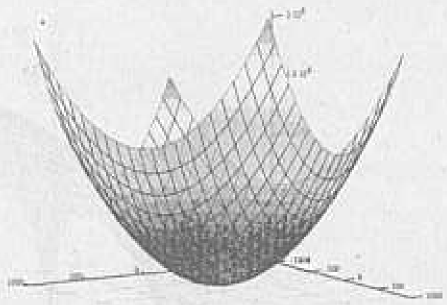
3D GRAPHS



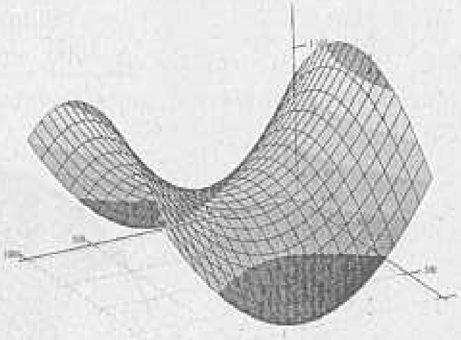
Plane
$4x - 2y - z = 0$



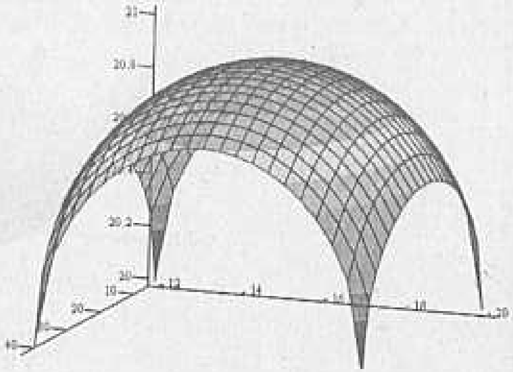
Intersecting Planes



Paraboloid
$z = x^2 + y^2$

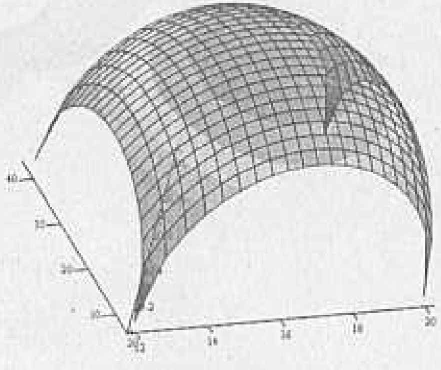


Hyperbolic Paraboloid
$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = \frac{2z}{c}$$



Ellipsoid (part only)

$$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}+\frac{(z-p)^2}{c^2}=1$$



Worms eye view of ellipsoid (part only)

Problems - Set 16
Points, Lines, Circles

PROBLEM 16 - 1

State the quadrant in which the coordinate (15, -2) lies.

A. I C. II
B. IV D. III

PROBLEM 16 - 2
ECE Nov. 1997

Of what quadrant is A, if sec A is positive and csc A is negative?

A. III C. IV
B. I D. II

PROBLEM 16 - 3
ECE April 1998

The segment from (-1, 4) to (2, -2) is extended three times its own length. The terminal point is

A. (11, -18) C. (11, -20)
B. (11, -24) D. (-11, -20)

PROBLEM 16 - 4
ECE Nov. 1997

The midpoint of the line segment between $P_1(x, y)$ and $P_2(-2, 4)$ is P_m (2, -1). Find the coordinate of P_1.

A. (6,-5) C. (6,-6)
B. (5,-6) D. (-6,6)

PROBLEM 16 - 5
ECE Nov. 1997

Find the coordinates of the point $P(2, 4)$ with respect to the translated axis with origin at (1,3).

A. (1, -1) C. (-1, -1)
B. (1, 1) D. (-1, 1)

PROBLEM 16 - 6

Find the median through (-2, -5) of the triangle whose vertices are (-6, 2), (2, -2), and (-2, -5).

A. 3 C. 5
B. 4 D. 6

PROBLEM 16 - 7

Find the centroid of a triangle whose vertices are (2, 3), (-4, 6) and (2, -6).

A. (0, 1) C. (1, 0)
B. (0, -1) D. (-1, 0)

PROBLEM 16 - 8
CE Nov. 1996

Find the area of triangle whose vertices are A (-3, -1), B (5, 3) and C(2, -8)

A. 34 C. 38
B. 36 D. 32

PROBLEM 16 - 9
CE May 1997

Find the distance between the points (4, -2) and (-5, 1)

A. 4.897 C. 7.149
B. 8.947 D. 9.487

PROBLEM 16 - 10
EE April 1994

Find the distance between $A(4, -3)$ and $B(-2, 5)$.

A. 11
B. 8
C. 9
D. 10

PROBLEM 16 - 11

If the distance between the points (8, 7) and $(3, y)$ is 13, what is the value of y?

A. 5
B. -19
C. 19 or -5
D. 5 or -19

PROBLEM 16 - 12

The distance between the points $(\sin x, \cos x)$ and $(\cos x, -\sin x)$ is:

A. 1
B. $\sqrt{2}$
C. $2 \sin x \cos x$
D. $4 \sin x \cos x$

PROBLEM 16 - 13

Find the distance from the point (2, 3) to the line $3x + 4y + 9 = 0$.

A. 5
B. 5.4
C. 5.8
D. 6.2

PROBLEM 16 - 14
CE Nov. 1998

Find the distance from the point (5, -3) to the line $7x - 4y - 28 = 0$.

A. 2.62
B. 2.36
C. 2.48
D. 2.54

PROBLEM 16 - 15
CE Nov. 1999

How far is the line $3x - 4y + 15 = 0$ from the origin?

A. 1
B. 2
C. 3
D. 4

PROBLEM 16 - 16
CE Nov. 1996

Determine the distance from (5, 10) to the line $x - y = 0$

A. 3.86
B. 3.54
C. 3.68
D. 3.72

PROBLEM 16 - 17
CE May 1994

The two points on the lines $2x + 3y + 4 = 0$ which are at distance 2 from the line $3x + 4y - 6 = 0$ are:

A. (-8, -8) and (-16, -16)
B. (-44, 64) and (-5, 2)
C. (-5.5, 1) and (-5, 2)
D. (64, -44) and (4, -4)

PROBLEM 16 - 18
ME April 1996

The intercept form for algebraic straight-line equation is:

A. $a/x + y/b = 1$
B. $y = mx + b$
C. $Ax + By + C = 0$
D. $x/a + y/b = 1$

PROBLEM 16 - 19
ME April 1998,
April 1996

Find the slope of the line defined by $y - x = 5$

A. 1
B. -1/2
C. ¼
D. $5 + x$

PROBLEM 16 - 20

The slope of the line $3x + 2y + 5 = 0$ is:

A. -2/3
B. -3/2
C. 3/2
D. 2/3

PROBLEM 16 - 21
CE May 1997

Find the slope of the line whose parametric equation is $y = 5 - 3t$ and $x = 2 + t$.

A. 3
B. -3
C. 2
D. -2

PROBLEM 16 - 22
CE Nov. 1997

Find the slope of the curve whose parametric equations are $x = -1 + t$ and $y = 2t$.

A. 2
B. 3
C. 1
D. 4

PROBLEM 16 - 23
CE May 1999

Find the angle that the line $2y - 9x - 18 = 0$ makes with the x-axis.

A. 74.77°
B.4.5°
C. 47.77°
D. 77.47°

PROBLEM 16 - 24

Which of the following is perpendicular to the line $x/3 + y/4 = 1$?

A. $x - 4y - 8 = 0$
B. $4x - 3y - 6 = 0$
C. $3x - 4y - 5 = 0$
D. $4x + 3y - 11 = 0$

PROBLEM 16 - 25

Find the equation of the bisector of the obtuse angle between the lines $2x + y = 4$ and $4x - 2y = 7$.

A. $4y = 1$
B. $8x = 15$
C. $2y = 3$
D. $8x + 4y = 6$

PROBLEM 16 - 26

The equation of the line through (1, 2) and parallel to the line $3x - 2y + 4 = 0$ is:

A. $3x - 2y + 1 = 0$
B. $3x - 2y - 1 = 0$
C. $3x + 2y + 1 = 0$
D. $3x + 2y - 1 = 0$

PROBLEM 16 - 27

If the points (-3, -5), (x , y), and (3, 4) lie on a straight line, which of the following is correct?

A. $3x + 2y - 1 = 0$
B. $2x + 3y + 1 = 0$
C. $2x + 3y - 1 = 0$
D. $3x - 2y - 1 = 0$

PROBLEM 16 - 28

One line passes through the points (1, 9) and (2, 6), another line passes through (3, 3) and (-1, 5). The acute angle between the two lines is:

A. 30°
B. 45°
C. 60°
D. 135°

PROBLEM 16 - 29

The two straight lines $4x - y + 3 = 0$ and $8x - 2y + 6 = 0$

A. intersects at the origin
B. are coincident
C. are parallel
D. are perpendicular

PROBLEM 16 - 30

A line which passes through (5, 6) and (-3, -4) has an equation of

A. $5x + 4y + 1 = 0$
B. $5x - 4y - 1 = 0$
C. $5x - 4y + 1 = 0$
D. $5x + y - 4 = 0$

PROBLEM 16 - 31
ME April 1997

Find the equation of the line with slope of 2 and y-intercept of -3.
A. $y = -3x + 2$
B. $y = 2x - 3$
C. $y = \frac{2}{3}x + 1$
D. $y = 3x - 2$

PROBLEM 16 - 32
CE May 1996

What is the equation of the line that passes through (4, 0) and is parallel to the line $x - y - 2 = 0$?
A. $y + x + 4 = 0$
B. $y - x + 4 = 0$
C. $y - x - 4 = 0$
D. $y + x - 4 = 0$

PROBLEM 16 - 33
ECE April 1998

Determine B such that $3x + 2y - 7 = 0$ is perpendicular to $2x - By + 2 = 0$.
A. 2
B. 3
C. 4
D. 5

PROBLEM 16 - 34
ME April 1998

The equation of a line that intercepts the x-axis at $x = 4$ and the y-axis at $y = -6$ is:
A. $2x - 3y = 12$
B. $3x + 2y = 12$
C. $3x - 2y = 12$
D. $2x - 37 = 12$

PROBLEM 16 - 35
CE May 1996

How far from the y-axis is the center of the curve $2x^2 + 2y^2 + 10x - 6y - 55 = 0$?
A. -3.0
B. 2.75
C. -3.25
D. 2.5

PROBLEM 16 - 36

Find the area of the circle whose center is at (2, -5) and tangent to the line $4x + 3y - 8 = 0$.
A. 6π
B. 9π
C. 3π
D. 12π

PROBLEM 16 - 37
CE May 1999

Determine the area enclosed by the curve $x^2 - 10x + 4y + y^2 = 196$
A. 15π
B. 225π
C. 12π
D. 144π

PROBLEM 16 - 38

Find the shortest distance from the point (1, 2) to a point on the circumference of the circle defined by the equation $x^2 + y^2 + 10x + 6y + 30 = 0$.
A. 5.61
B. 5.71
C. 5.81
D. 5.91

PROBLEM 16 - 39
CE Nov. 1998

Determine the length of the chord common to the circles $x^2 + y^2 = 64$ and $x^2 + y^2 - 16x = 0$.
A. 13.86
B. 12.82
C. 13.25
D. 12.28

PROBLEM 16 - 40

If (3, -2) lies on a circle with center (-1, 1), then the area of the circle is:
A. 5π
B. 25π
C. 4π
D. 3π

PROBLEM 16 - 41
CE Nov. 1994

The radius of the circle $2x^2 + 2y^2 - 3x + 4y - 1 = 0$ is:
A. $\sqrt{33} / 4$
B. $33 / 16$
C. $\sqrt{33} / 3$
D. 17

PROBLEM 16 - 42
ME April 1998

What is the radius of a circle with the following equation? $x^2 - 6x + y^2 - 4y - 12 = 0$
A. 3.46
B. 5
C. 7
D. 6

PROBLEM 16 - 43
ECE April 1998

The diameter of a circle described by $9x^2 + 9y^2 = 16$ is:
A. 16/9
B. 4/3
C. 4
D. 8/3

PROBLEM 16 - 44
CE Nov. 1999

Find the center of the circle $x^2 + y^2 - 6x + 4y - 23 = 0$.
A. (3, -2)
B. (3, 2)
C. (-3, 2)
D. (-3, -2)

PROBLEM 16 - 45

Determine the equation of the circle whose center is at (4, 5) and tangent to the circle whose equation is $x^2 + y^2 + 4x + 6y - 23 = 0$.
A. $x^2 + y^2 - 8x + 10y - 25 = 0$
B. $x^2 + y^2 + 8x - 10y + 25 = 0$
C. $x^2 + y^2 - 8x - 10y + 25 = 0$
D. $x^2 + y^2 - 8x - 10y - 25 = 0$

PROBLEM 16 - 46

The equation of the circle with center at (-2, 3) and which is tangent to the line $20x - 21y - 42 = 0$ is:
A. $x^2 + y^2 + 4x - 6y - 12 = 0$
B. $x^2 + y^2 + 4x - 6y + 12 = 0$
C. $x^2 + y^2 + 4x + 6y - 12 = 0$
D. $x^2 + y^2 - 4x - 6y - 12 = 0$

PROBLEM 16 - 47

A circle has a diameter whose ends are at (-3, 2) and (12, -6). Its Equation is:
A. $4x^2 + 4y^2 - 36x + 16y + 192 = 0$
B. $4x^2 + 4y^2 - 36x + 16y - 192 = 0$
C. $4x^2 + 4y^2 - 36x - 16y - 192 = 0$
D. $4x^2 + 4y^2 + 36x + 16y - 192 = 0$

PROBLEM 16 - 48

Find the equation of the circle with center on $x + y = 4$ and $5x + 2y + 1 = 0$ and having a radius of 3.
A. $x^2 + y^2 + 6x - 16y + 64 = 0$
B. $x^2 + y^2 + 8x - 14y + 25 = 0$
C. $x^2 + y^2 + 6x - 14y + 49 = 0$
D. $x^2 + y^2 + 6x - 14y + 36 = 0$

PROBLEM 16 - 49

If (3, -2) lies on the circle with center (-1, 1) then the equation of the circle is:

A. $x^2 + y^2 + 2x - 2y - 23 = 0$
B. $x^2 + y^2 + 4x - 2y - 21 = 0$
C. $x^2 + y^2 + 2x - y - 33 = 0$
D. $x^2 + y^2 + 4x - 2y - 27 = 0$

PROBLEM 16 - 50
ECE April 1998

Find the equation of k for which the equation $x^2 + y^2 + 4x - 2y - k = 0$ represents a point circle.

A. 5
B. -5
C. 6
D. -6

ANSWER SHEET

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Solutions to Set 16
Points, Lines, Circles

SOLUTION 16 - 1
Ans: B

Coordinate (15, -2) is in the fourth (IV) quadrant

SOLUTION 16 - 2
Ans: C

This can be determined by assumption.

First quadrant: (try $\theta = 30°$)
$\sec 30° = +1.1547$; $\csc 30° = +2$

Second quadrant: (try $\theta = 120°$)
$\sec 120° = -2$; $\csc 120° = 1.1547$

Third quadrant: (try $\theta = 210°$)
$\sec 210° = -1.1547$; $\csc 210° = -2$

Fourth quadrant: (try $\theta = 300°$)
$\sec 300° = 2$; $\csc 300° = -1.1547$

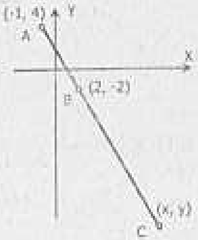
Thus, it is at the fourth quadrant:

SOLUTION 16 - 3
Ans: C

$BC = 3 \times AB$
$x - 2 = 3[2 - (-1)]$
$x = 11$

$y - (-2) = 3[-2 - 4]$
$y = -20$

Thus, the terminal point is **(11, -20)**



SOLUTION 16 - 4
Ans: C

The x-coordinate of the midpoint of a line segment is:
$x_m = (x_1 + x_2) / 2$
$2 = [x + (-2)] / 2$; $x = 6$

The y-coordinate of the midpoint of a line segment is:
$y_m = (y_1 + y_2) / 2$
$-1 = (y + 4) / 2$; $y = -6$

The coordinate of P_1 is **(6, -6)**

SOLUTION 16 - 5
Ans: B

The coordinate of a point (x, y) with respect to a translated axis (x_t, y_t) is $(x - x_t, y - y_t)$
Coordinate = (2 - 1, 4 - 3) = (1, 1)

SOLUTION 16 - 6
Ans: C

The vertices of the triangle are (-6, 2), (2, -2), and (-2, -5). The median of a triangle is the line from one vertex to the midpoint of the opposite side. Since the median required is through (-2, -5), the other end is at the midpoint of the line segment through (-6, 2) and (2, -2).

Locate the midpoint of (-6, 2) and (2, 2)

$x_m = (x_1 + x_2)/2 = (-6 + 2)/2 = -2$

$y_m = (y_1 + y_2)/2 = (2 - 2)/2 = 0$ midpoint (-2, 0)

The ends of the median is at (-2, 0) and (-2, -5), then;

$$h_m = \sqrt{[-2-(-2)]^2 + [-5-0]^2} = \textbf{5 units}$$

SOLUTION 16 - 7
Ans: A

The vertices are (2, 3), (-4, 6), and (2, -6).
The centroid of a polygon of n vertices or sides is:

$x_c = \Sigma x / n$ and $y_c = \Sigma y / n$.

$x_c = (2 - 4 + 2)/3 = 0$

$y_c = (3 + 6 - 6)/3 = 1$

Therefore, the centroid is at **(0, 1)**

SOLUTION 16 - 8
Ans: C

Using area by coordinates:

$$A = \frac{1}{2}\begin{vmatrix} x_1 & x_2 & \cdots & x_n & x_1 \\ y_1 & y_2 & & y_n & y_1 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} -3 & 5 & 2 & -3 \\ -1 & 3 & -8 & -1 \end{vmatrix}$$

$$A = \frac{1}{2}[\,(-5 + 6 + 24) - (-9 - 40 - 2)\,] = \textbf{38 sq. units}$$

SOLUTION 16 - 9
Ans: D

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(-5-4)^2 + [1-(-2)]^2}$$

$d =$ **9.487 units**

SOLUTION 16 - 10
Ans: D

The distance between two points is:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(-2-4)^2 + [5-(-3)]^2}$$

$d = 10$

SOLUTION 16 - 11
Ans: C

$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$

$13^2 = (3 - 8)^2 + (y - 7)^2$

$(y - 7)^2 = 144;\ y - 7 = \pm 12$

$y =$ **19 or -5**

SOLUTION 16 - 12
Ans: B

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d = \sqrt{(\cos x - \sin x)^2 + (-\sin x - \cos x)^2}$$

$$d = \sqrt{\cos^2 x - 2\sin x\cos x + \sin^2 x + \sin^2 x + 2\sin x\cos x + \cos^2 x}$$

$$d = \sqrt{2\sin^2 x + 2\cos^2 x} = \sqrt{2(\sin^2 x + \cos^2 x)}$$

$$d = \boldsymbol{\sqrt{2}}$$

SOLUTION 16 - 13
Ans: B

$$d = \left|\frac{Ax_1 + By_1 + C}{\sqrt{A^2 + B^2}}\right| = \left|\frac{3(2) + 4(3) + 9}{\sqrt{3^2 + 4^2}}\right| = \textbf{5.4 units}$$

SOLUTION 16 - 14
Ans: B

$$d = \left|\frac{Ax_1 + By_1 + C}{\sqrt{A^2 + B^2}}\right| = \left|\frac{7(5) - 4(-3) - 28}{\sqrt{(7)^2 + (-4)^2}}\right| = 2.357$$

SOLUTION 16 - 15
Ans: C

$$d = \left|\frac{Ax_1 + By_1 + C}{\sqrt{A^2 + B^2}}\right|, \text{ where } (x_1, y_1) = (0, 0)$$

$$d = \left|\frac{3(0) - 4(0) + 15}{\sqrt{(3)^2 + (-4)^2}}\right| = 3$$

SOLUTION 16 - 16
Ans: B

$$d = \left|\frac{Ax_1 + By_1 + C}{\sqrt{A^2 + B^2}}\right| \text{ (absolute value)}$$

where: $A = 1;\ B = -1$
$C = 0;\ x_1 = 5;$ and $y_1 = 10$

$$d = \left|\frac{1(5) + (-1)10 + 0}{\sqrt{(1)^2 + (-1)^2}}\right| = 3.5355$$

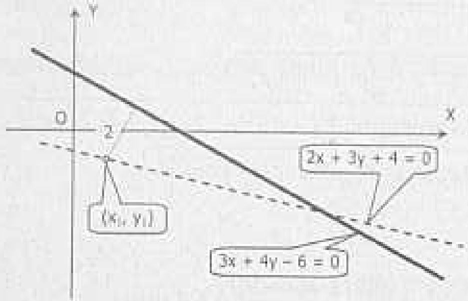
SOLUTION 16 - 17
Ans: D

$$d = \frac{Ax_1 + By_1 + C}{\pm\sqrt{A^2 + B^2}};\ A = 3;\ B = 4;\ C = -6$$

$$2 = \frac{3x_1 + 4y_1 - 6}{\pm\sqrt{3^2 + 4^2}}$$

$\pm 10 = 3x_1 + 4y_1 - 6$

$3x_1 + 4y_1 = 6 \pm 10 \rightarrow (1)$



Since (x_1, y_1) lies on
$2x + 3y + 4 = 0$, then
$2x_1 + 3y_1 = -4 \quad \rightarrow (2)$

Subtract: (1) × 2 - (2) × 3:

$$\begin{array}{rl} & 6x_1 + 8y_1 = 12 \pm 20 \\ - & 6x_1 + 9y_1 = -12 \\ \hline & -y_1 = 24 \pm 20 \end{array}$$

Using the positive sign: $y_1 = -44$ and $x_1 = 64$
(64, -44)
Using the negative sign: $y_1 = -4$ and $x_1 = 4$
(4, -4)

Thus, the points are **(64, -44) and (4, -4)**

SOLUTION 16 - 18
Ans: D

The intercept form for a straight line is: $\frac{x}{a} + \frac{y}{b} = 1$

SOLUTION 16 - 19
Ans: A

$y - x = 5$ or $y = x + 5$
Slope $m = 1$

SOLUTION 16 - 20
Ans: B

Given line: $3x + 2y + 5 = 0$

Slope-intercept form: $y = -\frac{3}{2}x - \frac{5}{2}$

Slope = -3/2

SOLUTION 16 - 21
Ans: B

$y = 5 - 3t$
$x = 2 + t;\ t = x - 2$
$y = 5 - 3(x - 2) = 5 - 3x + 6$
$y = -3x + 11$; Slope, $m = -3$

Another Solution:
Slope = dy / dt
$y = 5 - 3t;\ dt = -3\,dt$
$x = 2 + t;\ dx = dt$
Slope = $-3\,dt / dt = -3$

SOLUTION 16 - 22
Ans: A

$x = -1 + t;\ t = x + 1$
$y = 2t = 2(x + 1);\ y = 2x + 2$
Therefore, slope $m = 2$

Another Solution:
Slope = dy / dt
$y = 2t,\ dy = 2\,dt$
$x = -1 + t;\ dx = dt$

$$\text{Slope} = \frac{2\,dt}{dt} = 2$$

SOLUTION 16 - 23
Ans: D

The angle that the line makes with the x-axis is the angle of inclination of the line, which is equal to the tangent of its slope.
$2y = 9x + 18$
$y = 4.5x + 9$; slope = 4.5

$\tan\theta$ = slope = 4.5; $\theta = \arctan 4.5 = 77.47°$

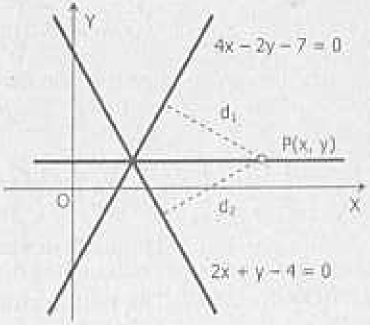
SOLUTION 16 - 24
Ans: C

For the line $x/3 + y/4 = 1$,
$y/4 = -x/3 + 1$
$y = -\frac{4}{3}x + 4$, slope $m = -\frac{4}{3}$
A line perpendicular to this line has a slope of 3/4.

Among the choices, only the line $3x - 4y - 5 = 0$ has a slope of ¾ and is therefore is perpendicular to the given line.

SOLUTION 16 - 25
Ans: A

Since the angle of inclination of the line $4x - 2y - 7$ is the same as the angle of depression of the line $2x + y - 4 = 0$, the bisector of the obtuse angle (as shown) is horizontal and passing through the intersection of the given lines.



Solving for the point of intersection:
$4x - 2y - 7 = 0 \quad \rightarrow (1)$
$2x + y - 4 = 0 \quad \rightarrow (2)$

Add: Eq. (1) + Eq.(2) × 2:
$8x - 15 = 0$
$x = 15/8$ and $y = ¼$

Thus, the equation of the bisector is $y = ¼$ or $4y = 1$

SOLUTION 16 - 26
Ans: A

The slope of the given line $3x - 2y + 4 = 0$ is $3/2$. Since the required line is parallel to this line, its slope is also $3/2$.

By point-slope form:

$$y - y_1 = m(x - x_1)$$
$$y - 2 = (3/2)(x - 1)$$
$$2y - 4 = 3x - 3;\ \mathbf{3x - 2y + 1 = 0}$$

SOLUTION 16 - 27
Ans: D

If the points $(-3, -5)$, (x, y), and $(3, 4)$ lie of a straight line, then:

$$\frac{y-(-5)}{x-(-3)} = \frac{4-(-5)}{3-(-3)} = \frac{9}{6} = \frac{3}{2}$$
$$2y + 10 = 3x + 9;\ \mathbf{3x - 2y - 1 = 0}$$

SOLUTION 16 - 28
Ans: B

The angle θ between two lines is given by

$\tan\theta = \dfrac{m_2 - m_1}{1 + m_1 m_2}$, where m_1 and m_2 is the slope of the lines and $m = (y_2 - y_1)/(x_2 - x_1)$

For the line through $(1, 9)$ and $(2, 6)$:

$$m_1 = (6 - 9)/(2 - 1) = -3$$

For the line through $(3, 3)$ and $(-1, 5)$

$$m_2 = (5 - 3)\ /\ (-1 - 3) = -1/2$$

$$\tan\theta = \frac{-1/2-(-3)}{1+(-3)(-1/2)};\ \theta = 45° \text{ and } 135°$$

Thus, the acute angle is **45°**

SOLUTION 16 - 29
Ans: B

The two straight lines given are coincident since they are equal.

SOLUTION 16 - 30
Ans: B

The slope of the line is, $m = \dfrac{-4-6}{-3-5} = \dfrac{5}{4}$

By point-slope form:

$$y - y_1 = m(x - x_1)$$
$$y - 6 = (5/4)(x - 5)$$
$$4y - 24 = 5x - 25$$
$$\text{or } \mathbf{5x - 4y - 1 = 0}$$

SOLUTION 16 - 31
Ans: B

Using the slope-intercept form:

$$y = mx + b$$
$$y = 2x - 3$$

SOLUTION 16 - 32
Ans: B

The slope of the given line $x - y - 2$ is 1. Since the required line is parallel to the given line, its slope is also 1.

Using the point-slope form:

$$y - y_1 = m(x - x_1)$$
$$y - 0 = 1(x - 4)$$
$$y - x + 4 = 0$$

SOLUTION 16 - 33
Ans: B

The slope of $3x + 2y - 7 = 0$ is:

$$2y = -3x + 7$$
$$y = (-3/2)x + 7/2$$

Slope = $-3/2$

The other line must have a slope of $-1\ /\ (-3/2) = 2/3$

$$By = 2x + 2$$
$$y = (2/B)x + 2/B$$

Slope = $2/B = 2/3$; $\mathbf{B = 3}$

SOLUTION 16 - 34
Ans: C

Using the intercept-form:

$$\frac{x}{a} + \frac{y}{b} = 1$$
$$\frac{x}{4} + \frac{y}{-6} = 1$$

Multiply both sides by 12:

$$\mathbf{3x - 2y = 12}$$

SOLUTION 16 - 35
Ans: D

The center of the curve is (h, k) where h is its the distance from the y-axis.

Reduce to standard form:

$$2x^2 + 2y^2 + 10x - 6y - 55 = 0$$
$$2x^2 + 10x + 2y^2 - 6y = 55$$
$$2[x^2 + 5x + (5/2)^2] + 2[y^2 - 3y + (3/2)^2] = 55$$
$$+ 2(5/2)^2 + 2(3/2)^2$$
$$2(x + 5/2)^2 + 2(y - 3/2)^2 = 72$$
$$(x + 5/2)^2 + (y - 3/2)^2 = 36$$

The center of the circle is $(-5/2, 3/2)$ or $(-2.5, 1.5)$
Thus the center is 2.5 units from the y-axis.

SOLUTION 16 - 36
Ans: A

Area of circle = πr^2

The radius of the circle is the distance from the point (2, -5) to the line $4x + 3y - 8 = 0$.

$$r = d = \left|\frac{Ax_1 + By_1 + C}{\sqrt{A^2 + B^2}}\right| = \left|\frac{4(2) + 3(-5) - 8}{\sqrt{4^2 + 3^2}}\right| = 3$$

Area of circle = $\pi(3)^2 = 9\pi$ **square units**

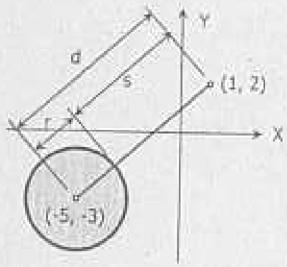
SOLUTION 16 - 37
Ans: B

Solve for the radius by reducing the equation to standard form:

$x^2 - 10x + 4y + y^2 = 196$
$x^2 - 10x + 25 + y^2 + 4y + 4 = 196 + 25 + 4$
$(x - 5)^2 + (y + 2)^2 = 225 = 15^2$
Radius = 15

Area = $\pi r^2 = \pi (15)^2 = $ **225π**

SOLUTION 16 - 38
Ans: C

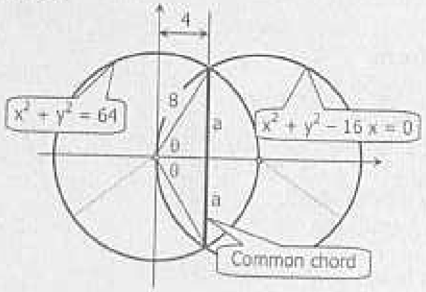


Given: $x^2 + y^2 + 10x + 6y + 30 = 0$

Reducing to standard form:
$x^2 + 10x + 25 + y^2 + 6y + 9 = -30 + 25 + 9$
$(x + 5)^2 + (y + 3)^2 = 4 = 2^2$
Center: (-5, -3); radius = 2

From the figure shown, the shortest distance is s.
$s = d - r$
$s = \sqrt{(-5-1)^2 + (-3-2)^2} - 2$
$s = $ **5.810**

SOLUTION 16 - 39
Ans: A



$x^2 + y^2 = 64$
Radius = 8, center (0, 0)

$x^2 + y^2 - 16x = 0$
$x^2 - 16x + 64 + y^2 = 64$
$(x - 8)^2 + y^2 = 64$
Radius = 8, center (8, 0)

$a^2 = 8^2 - 4^2$
$a = 6.928$

Length of common chord = $2a$
Length of common chord = **13.856**

SOLUTION 16 - 40
Ans: B

Area of circle = πr^2

The radius r of the circle is the distance from (3, -2) to (-1, 1)

$$r = \sqrt{(-1-3)^2 + [1-(-2)]^2} = 5$$

Area of circle = $\pi(5)^2 = $ **25π**

SOLUTION 16 - 41
Ans: A

Equation of circle: $2x^2 + 2y^2 - 3x + 4y - 1 = 0$

Standard form:
$2(x^2 - 3x/2 + 9/16) + 2(y^2 + 2y + 1) = 1 + 2(9/16) + 2(1)$

$2(x - 3/2)^2 + 2(y + 2)^2 = 33/8$
$(x - 3/2)^2 + (y + 2)^2 = 33/16$
$r^2 = 33/16,\ r = \sqrt{33}/4$

SOLUTION 16 - 42
Ans: B

Equation of circle: $x^2 - 6x + y^2 - 4y - 12 = 0$
Reduce to standard form:
$x^2 - 6x + y^2 - 4y = 12$
$x^2 - 6x + 9 + y^2 - 4y + 4 = 12 + 9 + 4$
$(x - 3)^2 + (y - 2)^2 = 25 = 5^2$

The radius r is 5.

SOLUTION 16 - 43
Ans: D

Reduce the equation to standard form:
$9x^2 + 9y^2 = 16$
$x^2 + y^2 = 16/9 = (4/3)^2$

Thus, the radius is 4/3 and the diameter is 8/3

SOLUTION 16 - 44
Ans: A

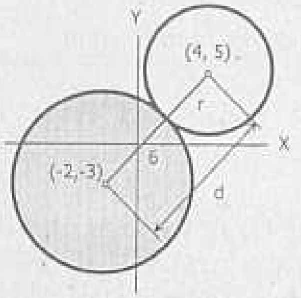
Reduce the equation to standard form:

$x^2 + y^2 - 6x + 4y - 23 = 0$
$(x^2 - 6x + 9) + (y^2 + 4y + 4) = 23 + 9 + 4$
$(x - 3)^2 + (y + 2)^2 = 36$

Center **(3, -2)**

SOLUTION 16 - 45
Ans: C

Solve for the center and radius of the given circle:
$x^2 + y^2 + 4x + 6y - 23 = 0$
$x^2 + 4x + 4 + y^2 + 6y + 9 = 23 + 4 + 9$
$(x + 2)^2 + (y + 3)^2 = 36 = 6^2$
Center (-2, -3); radius = 6



$d = \sqrt{[4-(-2)]^2 + [5-(-3)]^2} = 10$

Radius of required circle = $d - 6 = 4$

Equation of required circle:

$(x - h)^2 + (y - k)^2 = r2$

$(x - 4)^2 + (y - 5)^2 = 4^2$

$x^2 - 8x + 16 + y^2 - 10y + 25 = 16$

$\mathbf{x^2 + y^2 - 8x - 10y + 25 = 0}$

SOLUTION 16 - 46
Ans: A

Center of circle, $(h, k) = (-2, 3)$

The radius of the circle is the distance from (-2, 3) to the line $20x - 21y - 42 = 0$

$$r = d = \left| \frac{20(-2) - 21(3) - 42}{\sqrt{(20)^2 + (-21)^2}} \right| = 5$$

Equation of circle:

$(x - h)^2 + (y - k)^2 = r^2$

$[x - (-2)]^2 + (y - 3)^2 = 5^2$

$x^2 + 4x + 4 + y^2 - 6y + 9 = 25$

$\mathbf{x^2 + y^2 + 4x - 6y - 12 = 0}$

SOLUTION 16 - 47
Ans: B

The diameter of the circle is the distance from (-3, 2) and (12, -6), and its center is at the midpoint of the line segment defined by these points.

Diameter = $2r = \sqrt{[12-(-3)]^2 + [-6-2]^2} = 17;\ \ r = 17/2$

Center, $(h, k) = (x_m, y_m)$

$x_m = h = ½(x_1 + x_2)$

$x_m = ½(-3 + 12) = 9/2$

$y_m = k = ½(2 - 6) = -2$

Equation of circle:

$(x - h)^2 + (y - k)^2 = r^2$

$(x - 9/2)^2 + (y + 2)^2 = (17/2)^2$

$x^2 - 9x + 81/4 + y^2 + 4y + 4 = 289/4$

Multiply both sides by 4

$4x^2 - 36x + 81 + 4y^2 + 16y + 16 = 289$

$\mathbf{4x^2 + 4y^2 - 36x + 16y - 192 = 0}$

SOLUTION 16 - 48
Ans: C

The center of the circle is the intersection of the lines $x + y = 4$ & $5x + 2y + 1 = 0$.

From the given lines, $y = 4 - x$. Substitute this to the other line:

$5x + 2(4 - x) + 1 = 0$

$3x = -9$

$x = \underline{-3 = h}$ and $y = \underline{7 = k}$

Equation of circle: $(x - h)^2 + (y - k)^2 = r^2$

$(x + 3)^2 + (y - 7)^2 = 32$

$x^2 + 6x + 9 + y^2 - 14y + 49 = 32$

$\mathbf{x^2 + y^2 + 6x - 14y + 49 = 0}$

SOLUTION 16 - 49
Ans: A

Center of circle = (-1, 1)

Radius of circle is the distance from (-1, 1) to (3, -2)

$r = \sqrt{[3-(-1)]^2 + [-2-1]^2}$

$r = 5$

Equation of circle: $(x - h)^2 + (y - k)^2 = r^2$

$[x - (-1)]^2 + (y - 1)^2 = 5^2$

$x^2 + 2x + 1 + y^2 - 2y + 1 = 25$

$\mathbf{x^2 + y^2 + 2x - 2y - 23 = 0}$

SOLUTION 16 - 50
Ans: B

A point circle has a radius of zero.

$x^2 + y^2 + 4x - 2y - k = 0$

$x^2 + y^2 + 4x - 2y = k$

$x^2 + 4x + 4 + y^2 - 2y + 1 = k + 4 + 1$

$(x + 2)^2 + (y - 1)^2 = k + 5$

Then $k + 5 = 0$, $k = -5$

Problems - Set 17
Parabola, Ellipse, Hyperbola, Polar, Space

PROBLEM 17 - 1

The vertex of the parabola $y^2 - 2x + 6y + 3 = 0$ is at:

A. (-3, 3)
B. (3, 3)
C. (-3, 3)
D. (-3, -3)

PROBLEM 17 - 2

The length of the latus rectum of the parabola $y^2 = 4px$ is:

A. $4p$
B. $2p$
C. p
D. $-4p$

PROBLEM 17 - 3

Given the equation of the parabola: $y^2 - 8x - 4y - 20 = 0$. The length of its latus rectum is:

A. 2
B. 4
C. 6
D. 8

PROBLEM 17 - 4
CE May 1998,
May 1995

What is the length of the latus rectum of the curve $x^2 = -12y$?

A. 12
B. -3
C. 3
D. -12

PROBLEM 17 - 5

Find the equation of the directrix of the parabola $y^2 = 16x$.

A. $x = 8$
B. $x = 4$
C. $x = -8$
D. $x = -4$

PROBLEM 17 - 6
ME April 1997

The curve $y = -x^2 + x + 1$ opens:

A. upward
B. to the left
C. to the right
D. downward

PROBLEM 17 - 7
EE April 1994

The parabola $y = -x^2 + x + 1$ opens:

A. to the right
B. to the left
C. upward
D. downward

PROBLEM 17 - 8
ECE April 1998

Find the equation of the axis of symmetry of the function $y = 2x^2 - 7x + 5$.

A. $4x + 7 = 0$
B. $x - 2 = 0$
C. $4x - 7 = 0$
D. $7x + 4 = 0$

PROBLEM 17 - 9

Find the equation of the locus of the center of the circle which moves so that it is tangent to the y-axis and to the circle of radius one (1) with center at (2, 0).

A. $x^2 + y^2 - 6x + 3 = 0$
B. $x^2 - 6x + 3 = 0$
C. $2x^2 + y^2 - 6x + 3 = 0$
D. $y^2 - 6x + 3 = 0$

PROBLEM 17 - 10

Find the equation of the parabola with vertex at (4, 3) and focus at (4, -1).

A. $y^2 - 8x + 16y - 32 = 0$
B. $y^2 + 8x - 16y - 32 = 0$
C. $x^2 + 8x - 16y + 32 = 0$
D. $x^2 - 8x + 16y - 32 = 0$

PROBLEM 17 - 11
CE May 1997

Find the area bounded by the curves $x^2 + 8y + 16 = 0$, $x - 4 = 0$, the x-axis, and the y-axis.

A. 10.67 sq. units
B. 10.33 sq. units
C. 9.67 sq. units
D. 8 sq. units

PROBLEM 17 - 12
ECE April 1998

Find the area (in sq. units) bounded by the parabolas $x^2 - 2y = 0$ and $x^2 + 2y - 8 = 0$.

A. 11.7
B. 10.7
C. 9.7
D. 4.7

PROBLEM 17 - 13

The length of the latus rectum of the curve $(x - 2)^2 / 4 + (y + 4)^2 / 25 = 1$ is:

A. 1.6
B. 2.3
C. 0.80
D. 1.52

PROBLEM 17 - 14
CE Nov. 1997

Find the length of the latus rectum of the following ellipse:

$$25x^2 + 9y^2 - 300x - 144y + 1251 = 0$$

A. 3.4
B. 3.2
C. 3.6
D. 3.0

PROBLEM 17 - 15

If the length of the major and minor axes of an ellipse is 10 cm and 8 cm, respectively, what is the eccentricity of the ellipse?

A. 0.50
B. 0.60
C. 0.70
D. 0.80

PROBLEM 17 - 16

The eccentricity of the ellipse $x^2 / 4 + y^2 / 16 = 1$ is:

A. 0.725
B. 0.256
C. 0.689
D. 0.866

PROBLEM 17 - 17

An ellipse has the equation $16x^2 + 9y^2 + 32x - 128 = 0$. Its eccentricity is:

A. 0.531
B. 0.66
C. 0.824
D. 0.93

PROBLEM 17 - 18

The center of the ellipse $4x^2 + y^2 - 16x - 6y - 43 = 0$ is at:

A. (2, 3)
B. (4, -6)
C. (1, 9)
D. (-2, -5)

PROBLEM 17 - 19
CE May 1999
CE May 2003

Find the ratio of the major axis to the minor axis of the ellipse:

$$9x^2 + 4y^2 - 24y - 72x - 144 = 0$$

A. 0.67
B. 1.8
C. 1.5
D. 0.75

PROBLEM 17 - 20

The area of the ellipse $9x^2 + 25y^2 - 36x - 189 = 0$ is equal to:

A. 15π sq. units
B. 20π sq. units
C. 25π sq. units
D. 30π sq. units

PROBLEM 17 - 21
CE Nov. 1999

The area of the ellipse is given as $A = 3.1416\ a\ b$. Find the area of the ellipse $25x^2 + 16y^2 - 100x + 32y = 284$.

A. 86.2 square units
B. 62.8 square units
C. 68.2 square units
D. 82.6 square units

PROBLEM 17 - 22
ECE Nov. 1995

The semi-major axis of an ellipse is 4 and its semi-minor axis is 3. The distance from the center to the directrix is:

A. 6.532
B. 6.047
C. 0.6614
D. 6.222

PROBLEM 17 - 23
ECE Nov. 1997

Given an ellipse $x^2/36 + y^2/32 = 1$. Determine the distance between foci.

A. 2
B. 3
C. 4
D. 8

PROBLEM 17 - 24
CE May 1998

How far apart are the directrices of the curve $25x^2 + 9y^2 - 300x - 144y + 1251 = 0$?

A. 12.5
B. 14.2
C. 13.2
D. 15.2

PROBLEM 17 - 25
ECE April 1998

The major axis of the elliptical path in which the earth moves around the sun is approximately 186,000,000 miles and the eccentricity of the ellipse is 1/60. Determine the apogee of the earth.

A. 94,550,000 miles
B. 94,335.100 miles
C. 91,450,000 miles
D. 93,000,000 miles

PROBLEM 17 - 26

Find the equation of the ellipse whose center is at (-3, -1), vertex at (2, -1), and focus at (1, -1).

A. $9x^2 + 36y^2 - 54x + 50y - 116 = 0$
B. $4x^2 + 25y^2 + 54x - 50y - 122 = 0$
C. $9x^2 + 25y^2 + 50x + 50y + 109 = 0$
D. $9x^2 + 25y^2 + 54x + 50y - 119 = 0$

PROBLEM 17 - 27
ECE April 1998

Point $P(x, y)$ moves with a distance from point (0, 1) one-half of its distance from line $y = 4$, the equation of its locus is

A. $4x^2 + 3y^2 = 12$
B. $2x^2 - 4y^2 = 5$
C. $x^2 + 2y^2 = 4$
D. $2x^2 + 5y^3 = 3$

PROBLEM 17 - 28
CE Nov. 1998

The chords of the ellipse $64x^2 + 25y^2 = 1600$ having equal slopes of 1/5 are bisected by its diameter. Determine the equation of the diameter of the ellipse.

A. $5x - 64y = 0$
B. $64x - 5y = 0$
C. $5x + 64y = 0$
D. $64x + 5y = 0$

PROBLEM 17 - 29

Find the equation of the upward asymptote of the hyperbola whose equation is $(x - 2)^2 / 9 - (y + 4)^2 / 16$.

A. $3x + 4y - 20 = 0$
B. $4x - 3y - 20 = 0$
C. $4x + 3y - 20 = 0$
D. $3x - 4y - 20 = 0$

PROBLEM 17 - 30

The semi-conjugate axis of the hyperbola $x^2 / 9 - y^2 / 4 = 1$ is:

A. 2
B. -2
C. 3
D. -3

PROBLEM 17 - 31
ECE Nov. 1996

What is the equation of the asymptote of the hyperbola

$$\frac{x^2}{9} - \frac{y^2}{4} = 1\ ?$$

A. $2x - 3y = 0$
B. $3x - 2y = 0$
C. $2x - y = 0$
D. $2x + y = 0$

PROBLEM 17 - 32

The graph $y = (x - 1) / (x + 2)$ is not defined at:

A. 0
B. 2
C. -2
D. 1

PROBLEM 17 - 33
ME Oct. 1997

The equation $x^2 + Bx + y^2 + Cy + D = 0$ is:

A. hyperbola
B. parabola
C. ellipse
D. circle

PROBLEM 17 - 34
ME Oct. 1997

The general second degree equation has the form $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$ and describes an ellipse if:

A. $B^2 - 4AC = 0$
B. $B^2 - 4AC > 0$
C. $B^2 - 4AC = 1$
D. $B^2 - 4AC < 0$

PROBLEM 17 - 35

Find the equation of the tangent to the circle $x^2 + y^2 - 34 = 0$ through point (3, 5).

A. $3x + 5y - 34 = 0$
B. $3x - 5y - 34 = 0$
C. $3x + 5y + 34 = 0$
D. $3x - 5y + 34 = 0$

PROBLEM 17 - 36

Find the equation of the tangent to the curve $x^2 + y^2 + 4x + 16y - 32 = 0$ through (4, 0).

A. $3x - 4y + 12 = 0$
B. $3x - 4y - 12 = 0$
C. $3x + 4y + 12 = 0$
D. $3x + 4y - 12 = 0$

PROBLEM 17 - 37

Find the equation of the normal to the curve $y^2 + 2x + 3y = 0$ though point (-5,2)

A. $7x + 2y + 39 = 0$
B. $7x - 2y + 39 = 0$
C. $2x - 7y - 39 = 0$
D. $2x + 7y - 39 = 0$

PROBLEM 17 - 38
ME April 1996

Determine the equation of the line tangent to the graph $y = 2x^2 + 1$, at the point (1, 3).

A. $y = 4x + 1$
B. $y = 4x - 1$
C. $y = 2x - 1$
D. $y = 2x + 1$

PROBLEM 17 - 39
CE May 1997

Find the equation of the tangent to the curve $x^2 + y^2 = 41$ through (5,4)

A. $5x + 4y = 41$ C. $4x + 5y = 41$
B. $4x - 5y = 41$ D. $5x - 4y = 41$

PROBLEM 17 - 40
CE Nov. 1996

Find the equation of a line normal to the curve $x^2 = 16y$ at (4, 1).

A. $2x - y - 9 = 0$ C. $2x + y - 9 = 0$
B. $2x - y + 9 = 0$ D. $2x + y + 9 = 0$

PROBLEM 17 - 41
CE May 1996

What is the equation of the tangent to the curve $9x^2 + 25y^2 - 225 = 0$ at (0, 3)?

A. $y + 3 = 0$ C. $x - 3 = 0$
B. $x + 3 = 0$ D. $y - 3 = 0$

PROBLEM 17 - 42
CE May 1995

What is the equation of the normal to the curve $x^2 + y^2 = 25$ at (4, 3)?

A. $3x - 4y = 0$ C. $5x - 3y = 0$
B. $5x + 3y = 0$ D. $3x + 4y = 0$

PROBLEM 17 - 43

The polar form of the equation $3x + 4y - 2 = 0$ is:

A. $3r \sin\theta + 4r \cos\theta = 2$ C. $3r\cos\theta + 4r\sin\theta = 2$
B. $3r\cos\theta + 4r\sin\theta = -2$ D. $3r\sin\theta + 4r\tan\theta = -2$

PROBLEM 17 - 44

The polar form of the equation $3x^2 + 2y^2 = 8$ is:

A. $r^2 = 8$ C. $r = 8$
B. $r = \dfrac{8}{\cos^2\theta + 2}$ D. $r^2 = \dfrac{8}{\cos^2\theta + 2}$

PROBLEM 17 - 45

The distance between points (5, 30°) and (-8, -50°) is:

A. 9.84 C. 6.13
B. 10.14 D. 12.14

PROBLEM 17 - 46
ECE Nov. 1997

Convert $\theta = \pi/3$ to Cartesian equation.

A. $x = \sqrt{3}\,x$ C. $3y = \sqrt{3}\,x$
B. $y = x$ D. $y = \sqrt{3}\,x$

PROBLEM 17 - 47

The point of intersection of the planes $x + 5y - 2z = 9$, $3x - 2y + z = 3$, and $x + y + z = 2$ is:

A. (2, 1, -1) C. (-1, 1, -1)
B. (2, 0, -1) D. (-1, 2, 1)

PROBLEM 17 - 48

A warehouse roof needs a rectangular skylight with vertices (3, 0, 0), (3, 3, 0), (0, 3, 4), and (0, 0, 4). If the units are in meter, the area of the skylight is:

A. 12 sq. m. C. 15 sq. m.
B. 20 sq. m. D. 9 sq. m.

PROBLEM 17 - 49

The distance between points in space whose coordinates are (3, 4, 5) and (4, 6, 7) is:

A. 1 C. 3
B. 2 D. 4

PROBLEM 17 - 50
ME April 1997

What is the radius of the sphere with center at origin and which passes through the point (8, 1, 6)?

A. 10 C. $\sqrt{101}$
B. 9 D. 10.5

PROBLEM 17 - 51
CE Nov. 2002
CE May 2003
CE Nov. 2003

Points *C* (5, 7, *z*) and *D* (4, 1, 6) are 7.28 cm apart. Find the value of *z*.

A. 3 cm C. 2 cm
B. 4 cm D. 1 cm

PROBLEM 17 - 52

What is the total length of the curve $r = 4\sin\phi$?

A. 8π C. 2π
B. π D. 4π

PROBLEM 17 - 53
CE Nov. 2002

A triangle have vertices at *A*(-3, -2), *B*(2, 6), and *C*(4, 2). What is the abscissa of the centroid of the triangle?

A. 3/4 C. 3/2
B. 5/4 D. 1

PROBLEM 17 - 54
CE May 2002

What is the distance between the vertices of the following ellipse: $64x^2 + 25y^2 + 16x - 16y - 648 = 0$?

A. 6.324 C. 10.21
B. 12.54 D. 5.105

PROBLEM 17 - 55

Determine the equation of the curve such that the sum of the distances of any point of the curve from two points whose coordinates are (-3, 0) and (3, 0) is always equal to 8.

A. $4x^2 + 49y^2 - 343 = 0$ C. $7x^2 + 16y^2 - 112 = 0$
B. $7x^2 + 16y^2 - 112 = 0$ D. $7x^2 + 16y^2 - 112 = 0$

PROBLEM 17 - 56

Find the volume of the tetrahedron bounded by the coordinate planes and the plane $8x + 12y + 4z - 24 = 0$.

A. 5 C. 6
B. 9 D. 12

PROBLEM 17 - 57

The distances from the focus to the vertices of an ellipse are 4 and 6 units. Determine the ellipse flatness.

A. 0.0202 C. 0.0312
B. 0.206 D. 0.0187

PROBLEM 17 - 58

If the length of the latus rectum of an ellipse is three-fourth of the length of the minor axis, determine its eccentricity.

A. 0.775 C. 0.661
B. 0.332 D. 0.553

PROBLEM 17 - 59 Transform $r = \dfrac{3}{3 + 2\cos\theta}$ into Cartesian coordinates.

A. $5x^2 - 9y^2 + 12x + 9 = 0$
B. $5x^2 + 9y^2 - 12x - 9 = 0$
C. $5x^2 + 9y^2 + 12x + 9 = 0$
D. $5x^2 + 9y^2 + 12x - 9 = 0$

PROBLEM 17 - 60 Find the polar coordinates for the point whose rectangular coordinate of (-6, -8).

A. (10, -233.23°)
B. (10, 233.23°)
C. (10, 126.87°)
D. (10, -53.13°)

ANSWER SHEET

1.	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	21.	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	41.	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
2.	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	22.	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	42.	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
3.	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	23.	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	43.	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
4.	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	24.	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	44.	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
5.	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	25.	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	45.	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
6.	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	26.	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	46.	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
7.	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	27.	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	47.	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
8.	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	28.	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	48.	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
9.	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	29.	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	49.	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
10.	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	30.	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	50.	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
11.	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	31.	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	51.	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
12.	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	32.	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	52.	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
13.	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	33.	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	53.	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
14.	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	34.	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	54.	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
15.	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	35.	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	55.	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
16.	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	36.	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	56.	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
17.	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	37.	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	57.	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
18.	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	38.	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	58.	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
19.	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	39.	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	59.	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
20.	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	40.	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	60.	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ

Solutions to Set 17 Parabola, Ellipse, Hyperbola, Polar, Space

SOLUTION 17 - 1
Ans: D

Reduce the equation to standard form:

$$y^2 - 2x = 2x - 3$$

Add 9 both sides:

$$y^2 + 6y + 9 = 2x - 3 + 9$$
$$(y + 3)^2 = 2x + 6$$
$$(y + 3)^2 = 2(x + 3)$$

Vertex is at **(-3, -3)**

SOLUTION 17 - 2
Ans: A

The length of the latus rectum of the parabola $y^2 = 4px$ is $4p$.

SOLUTION 17 - 3
Ans: D

The length of the latus rectum (LR) of a parabola is $4a$.

Reduce the given equation to standard form:

$$y^2 - 8x - 4y - 20 = 0$$
$$y^2 - 4y = 8x + 20 \text{ add "4" both side}$$
$$y^2 - 4y + 4 = 8x + 20 + 4$$
$$(y - 2)^2 = 8(x + 3)$$

From the standard equation, $LR = 4a = 8$

SOLUTION 17 - 4
Ans: A

The curve $x^2 = -12y$ is a parabola with a standard equation of $x^2 = -4ay$.
The length of the latus rectum of the parabola is $4a$.

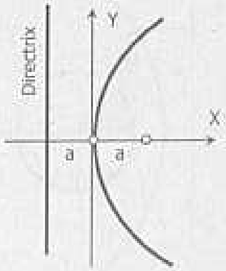
Thus, for the parabola $x^2 = -12y$, the length of the latus rectum, $LR = 12$

SOLUTION 17 - 5
Ans: D

Equation of parabola: $y^2 = 16x$

$$4a = 16;\ a = 4$$

From the figure shown, the equation of directrix is $x = -4$



SOLUTION 17 - 6
Ans: D

The curve $y = -x^2 + x + 1$ is a parabola. To determine the behavior of the curve, reduce it the standard form.

$$x^2 - x = -y + 1$$
$$x^2 - x + \frac{1}{4} = -y + 1 + \frac{1}{4}$$
$$(x - ½)^2 = -(y - 5/4)$$

The curve is symmetrical with respect to the y-axis and since the right side of the equation is negative, the curve opens downward.

SOLUTION 17 - 7
Ans: D

Reducing the standard form:

$$x^2 - x = -y + 1$$
$$x^2 - x + ¼ = -y + 1 + ¼$$
$$(x - ½)^2 = -(y - 5/4)$$

Therefore the parabola opens downward.

SOLUTION 17 - 8
Ans: C

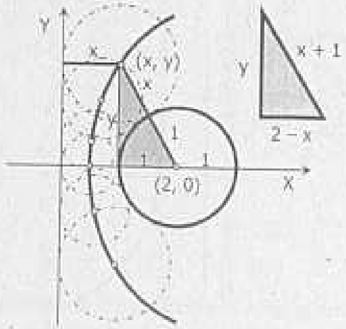
The given equation is a parabola. The axis of symmetry is the axis of the parabola, which is the line that passes through the vertex and the focus.

Reducing the equation to standard form:

$$y = 2x^2 - 7x + 5$$
$$y - 5 = 2[\ x^2 - (7/2)x\ ]$$
$$y - 5 + (49/16)2 = 2\ [\ x^2 - (7/2)x + 49/16\ ]$$
$$y + 9/8 = 2(x - 7/4)^2$$
$$(x - 7/4)^2 = ½(y + 9/8)$$

The parabola has a vertical axis with the vertex at (7/4, -9/8). Thus the axis (also the axis of symmetry) is the line $x = 7/4$ or $4x - 7 = 0$.

SOLUTION 17 - 9
Ans: D

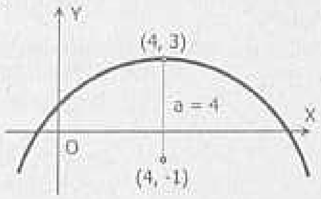


From the right triangle shown:

$$(x + 1)^2 = y^2 + (2 - x)^2$$
$$x^2 + 2x + 1 = y^2 + 4 - 4x + x^2$$
$$y^2 - 6x + 3 = 0$$

SOLUTION 17 - 10
Ans: D

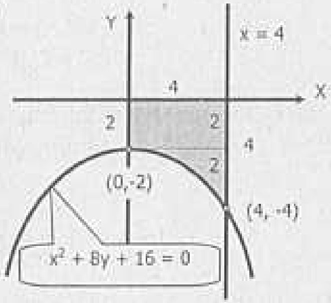
The standard equation of the parabola shown is:



$$(x - h)^2 = -4a(y - k)$$
$$(x - 4)^2 = -4(4)(y - 3)$$
$$x^2 - 8x + 16 = -16y + 48$$
$$x^2 - 8x + 16y - 32 = 0$$

SOLUTION 17 - 11
Ans: A

$x^2 + 8y + 16 = 0$ (parabola)
$x^2 = -8(y + 2)$
Vertex at (0, -2)

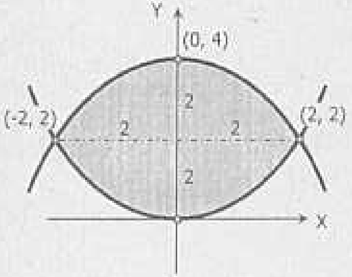


Point of intersection (PI):
When $x = 4$
$4^2 = -8(y + 2);\ y = -4$
PI @ (4, -4)
Area = $(4)(2) + \frac{1}{3}(4)(2)$
Area = 10.67 sq. units

SOLUTION 17 - 12
Ans: B

The standard forms of the parabolas are: $x^2 = 2y$ and $x^2 = -2(y - 4)$.

The plot of the curves is as shown:



Solving for the points of intersection:
$x^2 = x^2$
$2y = -2(y - 4)$
$y = 2;\ x = \pm 2$
PI @ (2, -2) and (2, 2)

From the figure shown, the area is:

$$A = (2/3)(4)(2) + (2/3)(4)(2) = \mathbf{10.67\ sq.\ units.}$$

SOLUTION 17 - 13
Ans: A

Given conic $\frac{(x-2)^2}{4} + \frac{(y+4)^2}{25} = 1$ (an ellipse)

Length of latus rectum, $LR = \frac{2b^2}{a}$, where $a = 5$ and $b = 2$

$$LR = \frac{2(2)^2}{5} = 8/5 = 1.6$$

SOLUTION 17 - 14
Ans: C

Length of latus rectum, $LR = 2b^2/a$

Reduce the given equation to standard for:
$25x^2 - 300x + 9y^2 - 144y = -1251$
$25(x^2 - 12x + 36) + 9(y^2 - 16y + 64) = -1251 + 25(36) + 9(64)$

$25(x - 6)^2 + 9(y - 8)^2 = 225$ Divide both sides by 225

$$\frac{(x-6)^2}{9} + \frac{(y-8)^2}{25} = 1$$

$a = 5; b = 3$
$LR = 2(3)^2/5 = \mathbf{3.6}$

SOLUTION 17 - 15
Ans: B

Major axis = $2a = 10;\ a = 5$
Minor axis = $2b = 8;\ \ b = 4.$

$a^2 = b^2 + c^2$
$5^2 = 4^2 + c^2;\ c = 3$
Eccentricity, $e = c / a = 3/5 = \mathbf{0.6}$

SOLUTION 17 - 16
Ans: D

Eccentricity of ellipse, $e = c / a$

For the ellipse $\frac{x^2}{4} + \frac{y^2}{16} = 1;\ a^2 = 16,\ a = 4;\ b^2 = 4,\ b = 2$
$a^2 = b^2 + c^2$
$16 = 4 + c^2,\ c = \sqrt{12}$
Eccentricity, $e = \sqrt{12}/4 = \mathbf{0.866}$

SOLUTION 17 - 17
Ans: B

The eccentricity of an ellipse is $e = c/a$.
Solving for a and b by reducing the equation to standard form:
$16x^2 + 9y^2 + 32x - 128 = 0$
$16(x^2 + 2 + 1) + 9y^2 = 128 + 16$
$16(x + 1)^2 + 9y^2 = 144$

$$\frac{(x+1)^2}{9} + \frac{y^2}{16} = 1; \quad a = 4 \text{ and } b = 3.$$

$[a^2 = b^2 + c^2] \qquad 4^2 = 3^2 + c^2;\ c = \sqrt{7}$
Eccentricity, $e = \sqrt{7}/4 = \mathbf{0.66}$

SOLUTION 17 - 18
Ans: A

$4x^2 + y^2 - 16x - 6y - 43 = 0$

Reduce to standard form:
$4x^2 - 16x + y^2 - 6y = 43$
$4(x^2 - 4x + 4) + (y^2 - 6y + 9) = 43 + 4(4) + 9$
$4(x - 2)^2 + (y - 3)^2 = 68;$ Center is at **(2, 3)**

SOLUTION 17 - 19
Ans: C

Reduce the equation to standard form:

$9x^2 - 72x + 4y^2 - 24y = 144$
$9(x^2 - 8x + 16) + 4(y^2 - 6y + 9) = 144 + 9(16) + 4(9)$
$9(x - 4)^2 + 4(y - 3)^2 = 324$ Divide both sides by 324

$$\frac{(x-4)^2}{36} + \frac{(y-3)^2}{81} = 1;\ a = 9 \text{ and } b = 6$$

$$\text{Ratio} = \frac{\text{Major axis}}{\text{Minor axis}} = \frac{2a}{2b} = \frac{2(9)}{2(6)} = \mathbf{1.5}$$

SOLUTION 17 - 20
Ans: A

Equation of ellipse: $9x^2 + 25y^2 - 36x - 189 = 0$

Reduce to standard form by completing square:
$9x^2 - 36x + 25y^2 = 189$
$9(x^2 - 4x + 4) + 25y^2 = 189 + 9(4)$
$9(x - 2)^2 + 25y^2 = 225$ Divide both sides by 225

$$\frac{(x-2)^2}{25} + \frac{y^2}{9} = 1$$

$a^2 = 25,\ a = 5;\ b^2 = 9,\ b = 3.$

Area of ellipse = $\pi ab = \pi(5)(3) = \mathbf{15\pi}$ **sq. units**

SOLUTION 17 - 21
Ans: B

Reduce the equation to standard form:

$25x^2 + 16y^2 - 100x + 32y = 284$
$25(x^2 - 4x + 4) + 16(y^2 + 2y + 1) = 284 + 25(4) + 16(1)$
$25(x - 2)^2 + 16(y + 1)^2 = 400$ Divide both sides by 400

$$\frac{(x-2)^2}{16} + \frac{(y+1)^2}{25} = 400;\ \text{thus } a = 5 \text{ and } b = 4$$

Area = 3.1416(5)(4) = **62.8 sq. units**

SOLUTION 17 - 22
Ans: B

Distance from the center to the directix, $d = \frac{a}{e}$

$a = 4;\ \ b = 3$

$a^2 = b^2 + c^2$
$4^2 = 3^2 + c^2;\ c = \sqrt{7}$

$$e = \frac{c}{a} = \frac{\sqrt{7}}{4} = 0.6614$$

$$d = \frac{a}{e} = \frac{4}{0.6614} = \mathbf{6.047}$$

SOLUTION 17 - 23
Ans: C

The distance from the center to each of the foci is c. The distance between foci is $2c$.

For the ellipse $x^2/36 + y^2/32 = 1$, $a^2 = 36$ and $b^2 = 32$. The value of c can be solved from the relationship:

$$a^2 = b^2 + c^2$$
$$36 = 32 + c^2;\ c = 2$$

Thus, the distance between foci is $2c$ = **4 units**

SOLUTION 17 - 24
Ans: A

The distance of each directrix to the center of the ellipse is $d = a\ /\ e$, where a is the semi-major axis and e is the eccentricity.

Solve for a and b by reducing the general equation to standard form:

$$25x^2 + 9y^2 - 300x - 144y + 1251 = 0$$
$$25(x^2 - 12x + 36) + 9(y^2 - 16y + 64) = -1251 + 36(25) + 64(9)$$
$$25(x - 6)^2 + 9(y - 8)^2 = 225$$
$$\frac{(x-6)^2}{9} + \frac{(y-8)^2}{25} = 1$$

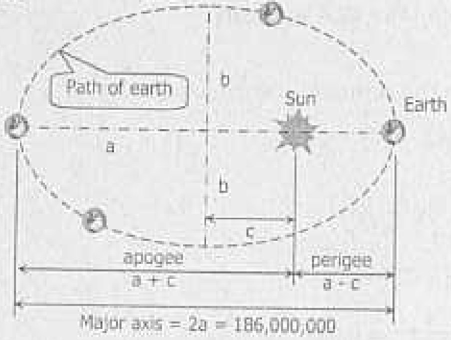
$a = 5$ and $b = 3$

$$a^2 = b^2 + c^2$$
$$5^2 = 3^2 + c^2;\ c = 4$$

$$e = c/a = 4\ /\ 5 = 0.80$$
$$d = a/e = 5/0.8 = 6.25$$

The distance between the directrices is $2d = 2(6.25)$ = **12.5 units**

SOLUTION 17 - 25
Ans: A

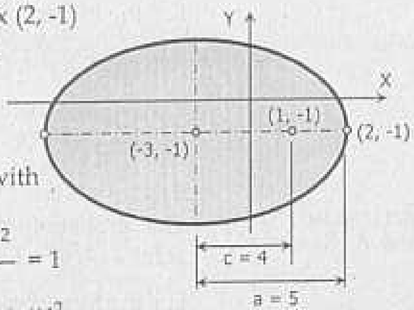


$$2a = 186,000,000$$
$$a = 93,000,000$$
$$e = a/c$$
$$1/60 = 93,000,000\ /\ c$$
$$c = 1,550,000$$

Apogee = $a + c$
Apogee = **94,550,000 miles**

SOLUTION 17 - 26
Ans: D

From the figure shown:
Center (-3, -1), vertex (2, -1)
one of the foci (1, -1)



$$a^2 = b^2 + c^2$$
$$5^2 = b^2 + 4^2;\ b = 3$$

Equation of ellipse with axis horizontal:

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$
$$\frac{[x-(-3)]^2}{5^2} + \frac{[y-(-1)]^2}{3^2} = 1$$
$$\frac{x^2+6x+9}{225} + \frac{y^2+2y+1}{9} = 1$$

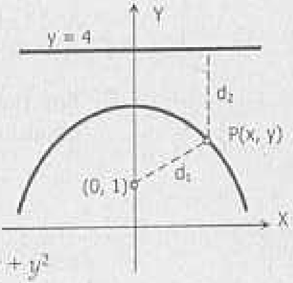
Multiply both sides by 225 and simplify

$$\mathbf{9x^2 + 25y^2 + 54x + 50y - 119 = 0}$$

SOLUTION 17 - 27
Ans: A

Distance of $P(x, y)$ to (0, 1):

$$d_1 = \sqrt{(x-0)^2 + (y-1)^2}$$
$$d_1 = \sqrt{x^2 + y^2 - 2y + 1}$$



Distance of $P(x, y)$ to $y = 4$:

$$d_2 = 4 - y$$
$$d_1 = ½\, d_2$$
$$d_1^2 = (1/4)\, d_2^2\ ;\ 4d_1^2 = d_2^2$$
$$4(x^2 + y^2 - 2y + 1) = 16 - 8y + y^2$$

$\mathbf{4x^2 + 3y^2 = 12}$ (an ellipse)

SOLUTION 17 - 28
Ans: D

$$64\,x^2 + 25\,y^2 = 1600$$

Differentiate with respect to x:
$128\,x + 50\,y(dy\ /\ dx) = 0$
dy/dx = slope of the chord = 1/5

$$128\,x + 50\,y(1/5) = 0$$
$128\,x + 10\,y = 0$ or $\mathbf{64\,x + 5\,y = 0}$

SOLUTION 17 - 29
Ans: B

The equation of the asymptote of a hyperbola is $(y - k) = m(x - h)$, where (h, k) is the center of the hyperbola and $m = b/a$ since the axis is horizontal.

For the hyperbola $\frac{(x-2)^2}{9} - \frac{(y+4)^2}{16} = 1$

$a = 3, b = 4, (h, k) = (2, -4)$

The equation of the asymptote is:

$y - (-4) = (4/3)(x - 2)$

$3y + 12 = 4x - 8$ or $\mathbf{4x - 3y - 20 = 0}$

SOLUTION 17 - 30
Ans: A

The semi-conjugate axis of the hyperbola $x^2/a^2 - y^2/b^2 = 1$ is b.

For the hyperbola $x^2/9 - y^2/4 = 1$, $b^2 = 4$ or $\mathbf{b = 2}$.

SOLUTION 17 - 31
Ans: A

The hyperbola $\frac{x^2}{9} - \frac{y^2}{4} = 1$ has its center at (0, 0) with the transverse axis along the x-axis. The equation of the asymptote can be derived from the point-slope form:

$$y - k = m(x - h)$$

where (h, k) is the center of the hyperbola which is at (0, 0), m is the slope of the line which is $+b/a$ (upward) and $-b/a$ (downward) for a hyperbola with axis horizontal.

For the given hyperbola: $a = 3$ and $b = 2$. Thus the equation is:

$y - 0 = \frac{2}{3}(x - 0)$; $\mathbf{2x - 3y = 0}$

SOLUTION 17 - 32
Ans: C

The graph is not defined at $x = -2$ since the divisor is zero for this value.

SOLUTION 17 - 33
Ans: D

The equation $x^2 + Bx + y^2 + Cy + D = 0$ represents a circle.

SOLUTION 17 - 34
Ans: D

The general second degree equation $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$ and describes an ellipse if $B^2 < 4AC$ or $B^2 - 4AC < 0$.

SOLUTION 17 - 35
Ans: A

Since the circle $x^2 + y^2 - 34 = 0$ passes through the point (3, 5), then $(x_1, y_1) = (3, 5)$.
Replace x^2 with $x\,x_1$ and y^2 with $y\,y_1$:

$x\,x_1 + y\,y_1 - 34 = 0$

$x(3) + y(5) - 34 = 0$

The equation of the tangent is: $\mathbf{3x + 5y - 34 = 0}$

SOLUTION 17 - 36
Ans: D

Equation of the curve: $x^2 + y^2 + 4x + 16y - 32 = 0$

Check if the point (4,0) lies on the curve:

$4^2 + 0^2 + 4(4) + 16(0) - 32 = 0; 0 = 0$ (O.K.)

Slope of the curve at any point, m = y'

$x^2 + y^2 + 4x + 16y - 32 = 0$

$2x + 2yy' + 4 + 16y' = 0$

@ (4, 0)

$2(4) + 2(0)y' + 4 + 16y' = 0$; $y' = -12/16 = -3/4 = m$

Equation of line: $y - y_1 = m\,(x - x_1)$

$y - 0 = (-3/4)(x - 4)$

$\mathbf{3x + 4y - 12 = 0}$

SOLUTION 17 - 37
Ans: B

By inspection, (-5, 2) lies on the curve $y^2 + 2x + 3y = 0$.

The slope of the tangent at any point is dy/dx or y'.

$2y\,y' + 2 + 3y' = 0$

$2(2)y' + 2 + 3y' = 0$

$y' = -2/7$ = slope of the tangent.

The slope of the normal is $-\frac{1}{-2/7} = \underline{7/2}$

Using the point-slope form: $(y - y_1) = m(x - x_1)$

$y - 2 = (7/2)[x - (-5)]$

$2y - 4 = 7x + 35$

$\mathbf{7x - 2y + 39 = 0}$

SOLUTION 17 - 38
Ans: B

Since (1,3) lies on the curve $y = 2x^2 + 1$, then the slope of the tangent is:

$m = \frac{dy}{dx} = 4x = 4(1) = 4$

By point slope form:

$y - y_1 = m(x - x_1)$

$y - 3 = 4(x - 1)$; $\mathbf{y = 4x - 1}$ (equation of the tangent)

Another solution:

$y = 2x^2 + 1$

$(y + y_1)/2 = 2\,x\,x_1 + 1$

$(y + 3)/2 = 2x(1) + 1$

$y + 3 = 4x + 2$

$\mathbf{y = 4x - 1}$

SOLUTION 17 - 39
Ans: A

$x^2 + y^2 = 41$

Replace x^2 with $x\,x_1$ and y^2 with $y\,y_1$:

$x\,x_1 + y\,y_1 = 41$

Since (5, 4) lies on the curve then $x_1 = 5$ and $y_1 = 4$

$x(5) + y(4) = 41$
$5x + 4y = 41$

SOLUTION 17 - 40
Ans: C

Using the point-slope form:
$y - y_1 = m(x - x_1)$ → (1)
where $x_1 = 4$ and $y_1 = 1$

The slope of the normal $m = \frac{1}{m_T}$, where m_T = slope of the tangent to the curve $x^2 = 16y$ at (4,1)

Solve for m_T:
$m_T = dy/dx = y'$
$16y = x^2$
$16y' = 2x = 2(4) = 8;\ y' = ½ = m_T$

Then $m = -1/(1/2) = -2$

Substitute $m = -2$ to Eq. (1):
$y - 1 = -2(x - 4) = -2x + 8$

The equation of the normal is: **$2x + y - 9 = 0$**

SOLUTION 17 - 41
Ans: D

Since (0, 3) lies on the curve, then $x_1 = 0$ and $y_1 = 3$.

$9\,x\,x_1 + 25\,y\,y_1 - 225 = 0$
$9x(0) + 25\,y(3) - 225 = 0$
$75y - 225 = 0;\ y - 3 = 0$

SOLUTION 17 - 42
Ans: A

Solving for the slope of the tangent to the curve:

$x^2 + y^2 = 25$
$x\,x_1 + y\,y_1 = 25$

Since (4, 3) lies on the curve)
$x(4) + y(3) = 25$
$4x + 3y = 25$, slope $m = -4/3$

Thus, the slope of the normal is $-1/(-4/3) = ¾$.

The equation of the normal through (4, 3) with slope of ¾ is:

$y - 3 = ¾(x - 4)$
$4y - 12 = 3x - 12$
$3x - 4y = 0$

SOLUTION 17 - 43
Ans: C

In polar form, $x = r\cos\theta$, $y = r\sin\theta$

The polar form of $3x + 4y - 2 = 0$ is
$3r\cos\theta + 4r\sin\theta - 2 = 0$

SOLUTION 17 - 44
Ans: D

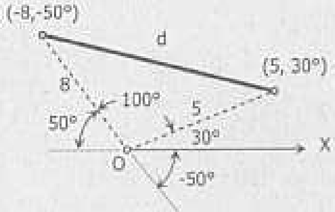
Replace x with $r\cos\theta$ and y with $r\sin\theta$.

$3(r\cos\theta)^2 + 2(r\sin\theta)^2 = 8$
$r^2[3\cos^2\theta + 2\sin^2\theta] = 8$

But $\sin^2\theta = 1 - \cos^2\theta$:
$r^2[3\cos^2\theta + 2(1 - \cos^2\theta)] = 8$
$r^2[\cos^2\theta + 2] = 8$

$$r^2 = \frac{8}{\cos^2\theta + 2}$$

SOLUTION 17 - 45
Ans: B



Given points in polar form: (5, 30°) and (-8, -50°)

By Cosine Law:
$d^2 = (8)^2 + (5)^2 - 2(8)(5)\cos 100°$
$d^2 = 102.892$
$d = 10.14$ units

SOLUTION 17 - 46
Ans: D

To convert polar equation to Cartesian form, the following relationships are applicable:
$r^2 = x^2 + y^2$
$y = r\sin\theta;\ x = r\cos\theta;\ \tan\theta = y/x$

$\theta = \pi/3 = 60°$
$\tan\theta = y/x = \tan 60° = \sqrt{3}$
$y = \sqrt{3}\,x$

SOLUTION 17 - 47
Ans: A

Equation of planes:
$x + 5y - 2z = 9$ → (1)
$3x - 2y + z = 3$ → (2)
$x + y + z = 2$ → (3)

Solve x, y, and z by Cramer's Rule:

$$D = \left|\begin{array}{ccc|cc} 1 & 5 & -2 & 1 & 5 \\ 3 & -2 & 1 & 3 & -2 \\ 1 & 1 & 1 & 1 & 1 \end{array}\right|$$

$$D = (-2 + 5 - 6) - (4 + 1 + 15) = -23$$

$$N_x = \left|\begin{array}{ccc|cc} 9 & 5 & -2 & 9 & 5 \\ 3 & -2 & 1 & 3 & -2 \\ 2 & 1 & 1 & 2 & 1 \end{array}\right|$$

$$N_x = (-18 + 10 - 6) - (8 + 9 + 15) = -46$$

$$x = N_x/D = -46/(-23) = 2$$

$$N_y = \left|\begin{array}{ccc|cc} 1 & 9 & -2 & 1 & 9 \\ 3 & 3 & 1 & 3 & 3 \\ 1 & 2 & 1 & 1 & 2 \end{array}\right|$$

$$N_y = (3 + 9 - 12) - (-6 + 2 + 27) = -23$$

$$y = N_y/D = -23/(-23) = 1$$

$$N_z = \left|\begin{array}{ccc|cc} 1 & 5 & 9 & 1 & 5 \\ 3 & -2 & 3 & 3 & -2 \\ 1 & 1 & 2 & 1 & 1 \end{array}\right|$$

$$N_z = (-4 + 15 + 27) - (-18 + 3 + 30) = 23$$

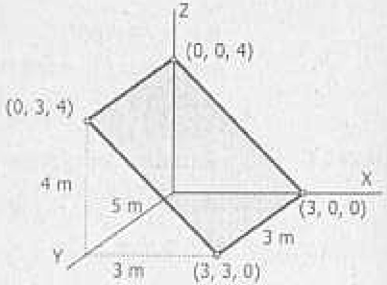
$$z = N_z/D = 23/(-23) = -1$$

Therefore the point of intersection is at (2, 1, -1)

Another way to get the correct answer is to substitute each point in the choices to the given equation of the planes. The one that satisfies all planes is the correct answer.

SOLUTION 17 - 48
Ans: C

Area = (5)(3)
Area = **15 m^2**



SOLUTION 17 - 49
Ans: C

The distance between (x_1, y_1, z_1) and (x_2, y_2, z_2) is:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Thus, the distance between (3, 4, 5) and (4, 6, 7) is:

$$d = \sqrt{(4-3)^2 + (6-4)^2 + (7-5)^2} = \mathbf{3}$$

SOLUTION 17 - 50
Ans: C

Since the center of the sphere is at the origin, its radius is equal to the distance from the origin to the point (8, 1, 6).

$$r = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

$$r = \sqrt{(8-0)^2 + (1-0)^2 + (6-0)^2} = \sqrt{101}$$

SOLUTION 17 - 51
Ans: C

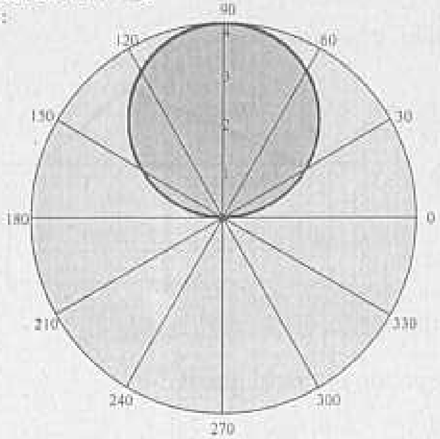
The distance d between $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$ is given as:

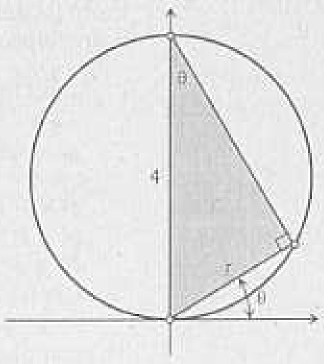
$$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2$$

Then,

$$7.28^2 = (4 - 5)^2 + (1 - 7)^2 + (6 - z)^2$$
$$(6 - z)^2 = 16$$
$$6 - z = \pm 4$$
$$z = 6 \pm 4 = \mathbf{10 \text{ and } \underline{2}}$$

SOLUTION 17 - 52
Ans:





r = 4 sin θ

The curves $r = a \sin\theta$ and $r = a \cos\theta$ are circles with diameter a.

For the given curve, $a = 4$ = diameter.
Total length (perimeter) = $\pi D = \pi(4) = \mathbf{4\pi}$

SOLUTION 17 - 53
Ans: D

Let $(\overline{x}, \overline{y})$ be the coordinate of the centroid. Then,

$$\text{Abscissa, } \overline{x} = \frac{x_A + x_B + x_C}{3}$$

$$\text{Abscissa, } \overline{x} = \frac{-3+2+4}{3} = 1$$

$$\text{Ordinate, } \overline{y} = \frac{y_A + y_B + y_C}{3}$$

$$\text{Ordinate, } \overline{y} = \frac{-2+6+2}{3} = 2$$

SOLUTION 17 - 54
Ans: C

Distance between the vertices = $2a$

$64x^2 + 25y^2 + 16x - 16y - 648 = 0$
$64x^2 + 16x + 25y^2 - 16y = 648$
$64(x^2 + x/4 + 1/64) + 25(y^2 - 16y/25 + 64/625) = 648 + 64(1/64)$
$64(x + 1/8)^2 + 25(y - 8/25)^2 = 651.56$ divide by 651.56

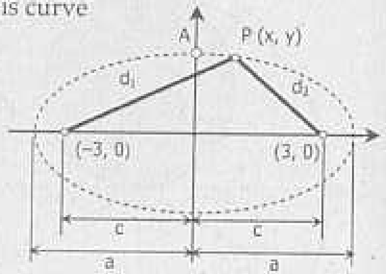
$$\frac{(x+1/8)^2}{10.1806} + \frac{(y-8/25)^2}{26.0624} = 1$$

$a^2 = 26.0624$
$a = 5.105$

Distance between the vertices = $2a$ = **10.21**

SOLUTION 17 - 55
Ans: B

From its definition, this curve is an ellipse.



From the figure:
$c = 3$
$d_1 + d_2 = 8 = 2a$
$a = 4$
$a^2 = b^2 + c^2$
$b^2 = 4^2 - 3^2 = 7$

Center of ellipse = (0, 0)

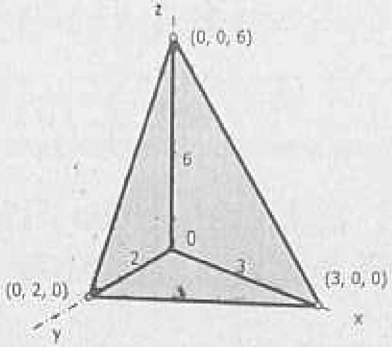
The standard equation of this ellipse is:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\frac{x^2}{4^2} + \frac{y^2}{7} = 1 \quad \text{multiplying both sides by 112}$$

$7x^2 + 16y^2 = 112$ or **$7x^2 + 16y^2 - 112 = 0$**

SOLUTION 17 - 56
Ans: C



For the plane $8x + 12y + 4z - 24 = 0$

When $x = 0$ & $y = 0$; $z = 6$
When $x = 0$ & $z = 0$; $y = 2$
When $y = 0$ & $z = 0$; $x = 3$

Volume = $\frac{1}{3} A_b h$
Volume = $\frac{1}{3}$ [½(2)(3)](6)
Volume = **6 cu. units**

SOLUTION 17 - 57
Ans: A

$2a = 6 + 4$; $a = 5$
$a - c = 4$
$5 - c = 4$; $c = 1$
$a^2 = b^2 + c^2$
$5^2 = b^2 + 1^2$
$b = 4.899$

$$\text{Ellipse flatness, } f = \frac{a-b}{a}$$

$$\text{Ellipse flatness, } f = \frac{5-4.899}{5}$$

Ellipse flatness, f = **0.0202**

SOLUTION 17 - 58
Ans: C

$LR = \frac{2b^2}{a}$ Minor axis = $2b$

$a^2 = b^2 + c^2$ Eccentricity = $\frac{c}{a}$

LR = ¾ Minor axis

$\frac{2b^2}{a} = ¾(2b)$; $b = ¾\,a$

$[\,a^2 = b^2 + c^2\,]$
$a^2 = (¾\,a)^2 + c^2$
$c = 0.6614$ a

Eccentricity = $\frac{c}{a}$ = **0.6614**

SOLUTION 17 - 59
Ans: D

$$r = \frac{3}{3 + 2\cos\theta}$$

$$3r + 2r\cos\theta = 3$$

note: $r = \sqrt{x^2 + y^2}$
$r\cos\theta = x$

$$3\sqrt{x^2 + y^2} + 2x = 3$$

$$3\sqrt{x^2 + y^2} = 3 - 2x \qquad \text{Square both sides}$$

$$9(x^2 + y^2) = 9 - 12x + 2x^2$$

$$5x^2 + 9y^2 + 12x - 9 = 0$$

SOLUTION 17 - 60
Ans: B

Point (-6, 8), second quadrant

$$r = \sqrt{(-6)^2 + (-8)^2} = 10$$

$$\tan\theta = |y/x| = |8/-6|$$

$$\theta = 53.13°$$

Since the point is in the third quadrant,

$$\theta = 180 + 53.23°$$

$$\theta = 233.23°$$

In polar coordinate the point is **(10, 233.23°)**

Part 6
RECENT BOARD EXAMS

Problems - Set 18
Recent Board Exams

PROBLEM 18 - 1
ME APRIL 2004

Solve the system:

$$xy = 24$$
$$y - 2x + 2 = 0$$

A. {(-3, 4)}
B. {(4, -8), (-3, 6)}
C. {(4, 6), (-3, -8)}
D. {-3, 4), (-8, 6)}

PROBLEM 18 - 2
ME APRIL 2004

Solve the equation: $\sqrt{x^2} - \sqrt{3x} = 2x - 6$

A. 12
B. 4
C. 3
D. 5

PROBLEM 18 - 3
ME APRIL 2004

$\sqrt{8} + 3\sqrt{18} - 7\sqrt{2} =$

A. $3\sqrt{2}$
B. $5 - 3\sqrt{2}$
C. $4\sqrt{2}$
D. $2\sqrt{2}$

PROBLEM 18 - 4
ME APRIL 2004

If $-9 < x < -4$ and $-12 < y < -6$, then

A. $108 < xy < 24$
B. $108 > xy > 24$
C. $108 < xy < 120$
D. $-21 < x + y < -10$

PROBLEM 18 - 5
CE MAY 2004

What is the value of E in the following equation?

$$\frac{2x^4 + 3x^3 + 7x^2 + 10x + 10}{(x-1)(x^2+3)^2} = \frac{A}{x-1} + \frac{Bx + C}{x^2 + 3} + \frac{Dx + E}{(x^2+3)^2}$$

A. 2
B. 3
C. 0
D. -1

PROBLEM 18 - 6
ME APRIL 2004

If $3^x = 54$, then 3^{x-2} is equal to:

A. 6
B. 5
C. 8
D. 9

PROBLEM 18 - 7
ME OCT 2003

Which of the following is the value of $x^2 + 1/x^2$ when $x + 1/x = 5$?

A. 28
B. 24
C. 23
D. 25

PROBLEM 18 - 8
ME APRIL 2004

Evaluate the following logarithm: $\log_2 \frac{\sqrt{2}}{8}$.

A. 2.5
B. 5.2
C. -2.5
D. -5.2

PROBLEM 18 - 9
ME APRIL 2004

Given $\log_{10} 2 = 0.3010$, find $\log_{10} 32$.

A. 1.863
B. 0.256
C. 1.365
D. 1.505

PROBLEM 18 - 10
ECE APRIL 2004

Given: log $(2x - 3) = ½$. What is the value of x if the base of the logarithm is 9?

A. 1
B. 2
C. 3
D. 4

PROBLEM 18 - 11
ME APRIL 2004

If $\log_8 N = 2/3$, then N is equal to:

A. 4
B. 2
C. 3
D. 5

PROBLEM 18 - 12
ECE APRIL 2004

What is the value of x if log (base x) 1296 = 4?

A. 5
B. 4
C. 3
D. 6

PROBLEM 18 - 13
ME APRIL 2004

Solve for x if $2 \log x - \log (30 - 2x) = -1$.

A. 12
B. 10
C. -30
D. 15

PROBLEM 18 - 14
ME APRIL 2004

If $\log_b 2 = 0.69$ and $\log_b 3 = 1.10$, find $\log_b 6$.

A. 1.7836
B. 1.5698
C. 1.1236
D. 2.1475

PROBLEM 18 - 15
CE MAY 2004

If $\log_x 6 = 1.2925$, what is the value of $\log_x 11$?

A. 1.65
B. 1.73
C. 1.42
D. 1.51

PROBLEM 18 - 16
ECE APRIL 2004

What is the product (AB) of the following complex numbers? Express your answer in polar form.

$A = 3 + j4$ $B = 8 + j6$

A. $50\angle 90°$
B. $50\angle 30°$
C. $50\angle 45°$
D. $50\angle 60°$

PROBLEM 18 - 17
ME APRIL 2004

Add the following complex numbers: $3 + 4i$ and $2 - 5i$.

A. $12 + 2i$
B. $24 - 5i$
C. $6 - i$
D. $5 - i$

PROBLEM 18 - 18
ME APRIL 2004

The function is defined as $f(x) = \frac{1}{1+x}$. For what value of x is $f(f(x))$ undefined?

A. $\{-1, -1/2\}$
B. $\{-1, -2\}$
C. $\{-1, 0\}$
D. $\{0\}$

PROBLEM 18 - 19
ME APRIL 2004

If $f(x) = 2x^2 + 2$, find the value of $f(x + 4)$

A. $2x^2 + 16x + 24$
B. $2x^2 + 16x$
C. $2x^2 + 6x + 4$
D. $2x^2 + 16x + 34$

PROBLEM 18 - 20
ME APRIL 2004

If $f(x) = (3\sqrt{x} - 4)^2$, then how much does $f(x)$ increase as x goes from 2 to 3?

A. 1.372
B. 1.732
C. 1.273
D. 1.723

PROBLEM 18 - 21
ME APRIL 2004

If $f(x) = x^3 - x - 1$, what is the set of all c if $f(c) = f(-c)$?

A. all real numbers
B. $\{-1, 0, 1\}$
C. $\{0\}$
D. $\{0, 1\}$

PROBLEM 18 - 22
ME APRIL 2004

If $f(x) = 3x - 1$, then $f^{-1}(x)$ is equal to:

A. $x - \frac{1}{3}$
B. $3x + 1$
C. $\frac{1}{3}x + \frac{1}{3}$
D. $\frac{1}{3}x - 1$

PROBLEM 18 - 23
ME APRIL 2004

If $f(x) = x - 2x^2 + 2$, what is $f(a - 2)$?

A. $9a - 2a^2 - 8$
B. $9a - 2a^2 + 8$
C. $2a - 9a^2 - 8$
D. $2a + 9a^2 - 8$

PROBLEM 18 - 24
ME APRIL 2004

If $f(x) = 3x + 2$ and $g(f(x)) = x$, then $g(x)$ is equal to:

A. $x - 2/3$
B. $4x + 9$
C. $1/3(x - 2)$
D. $3x - 2$

PROBLEM 18 - 25
ECE APRIL 2004

Solve for x in the following equation:

$$x + 4x + 7x + 10x + \ldots + 64x = 1430$$

A. 3
B. 4
C. 5
D. 2

PROBLEM 18 - 26
ECE APRIL 2004

Four positive integers form an arithmetic progression. If the product of the first and the last term is 70 and the second and third term is 88, what is the first term?

A. 3
B. 14
C. 5
D. 8

PROBLEM 18 - 27
CE MAY 2004

What is the value of x if 1, ¼, $1/x$, 1/10,... form a harmonic progression?

A. 6
B. 7
C. 8
D. 5

PROBLEM 18 - 28
ME APRIL 2004

Find the sum of the infinite progression: $2^{-1}, 2^{-3}, 2^{-5}, \ldots$

A. 3/4
B. 1/2
C. 2/3
D. 1/3

PROBLEM 18 - 29
ME APRIL 2004

The sides of a right triangle are in arithmetic progression whose common difference is 6. Find the hypotenuse.

A. 6
B. 30
C. 18
D. 24

PROBLEM 18 - 30
ME APRIL 2004

If 3, 5, 8.333, and 13.889 are the first four terms of a sequence, then, which of the following could define the sequence?

A. $A_0 = 3; A_n = A_{n-1} + \frac{40}{9}$
B. $A_0 = 3; A_{n+1} = A_{n+2}$
C. $A_0 = 3; A_n = \frac{5}{3} A_{n-1}$
D. $A_0 = 3; A_{n+1} = 2A_{n+2}$

PROBLEM 18 - 31
ME APRIL 2004

Two numbers differ by 40 and their arithmetic mean exceeds their geometric mean by 2. What is the smaller number?

A. 81
B. 96
C. 64
D. 45

PROBLEM 18 - 32
ECE APRIL 2004

If $kx^3 - (k + 3)x^2 + 13$ is divided by $x - 4$ the remainder is 157. What is the value of k?

A. 5
B. 4
C. 3
D. 7

PROBLEM 18 - 33
ECE APRIL 2004

The average rate of production of PCB is one (1) unit for every 2 hours work by two workers. How many PCB's can be produced in one month by 80 workers working 200 hours during the month?

A. 2000
B. 5000
C. 3000
D. 4000

PROBLEM 18 - 34
ECE APRIL 2004

A can do a job 50% faster than B and 20% faster than C. Working altogether, they can finish the job in 4 days. How many days will it take A to finish the job if he work alone?

A. 16
B. 18
C. 10
D. 12

PROBLEM 18 - 35
ME APRIL 2004

If John can paint a room in 30 minutes and Tom can paint it in 1 hour, how many minutes will it take them to paint the room if they work together?

A. 15
B. 20
C. 12
D. 10

PROBLEM 18 - 36
CE MAY 2004

A salesman started walking from office A at 9:30 am at the rate of 2.5 kph. He arrived office B 12 seconds late. Had he started at A at 9:00 am and walked at 1.5 kph, he would have arrived at B one minute before the required time. At what time was he supposed to be at B?

A. 10:13 am
B. 10:16 am
C. 10:22 am
D. 10:18 am

PROBLEM 18 - 37
ME APRIL 2004

A man walks a certain distance at the rate of 5 kph and returns at a rate of 4 kph. if the total time that it takes him is 3 hours and 36 minutes, what is the total distance that he walked?

A. 8
B. 9
C. 18
D. 16

PROBLEM 18 - 38
ME APRIL 2004

John can shovel a driveway in 50 minutes. If Mary can shovel the driveway in 20 minutes, how long will it take them, to the nearest minute, to shovel the driveway if they work together?

A. 12
B. 13
C. 14
D. 16

PROBLEM 18 - 39
ME OCT 2003

Carpenter Pedro can make a bookshelf in 5 working days alone. Carpenter Juan can do the same bookshelf in 10 days working alone. How long will it take them to finish the same bookshelf if they work together?

A. 4 - 1/3 days
B. 3 - 2/3 days
C. 3 - 1/3 days
D. 2 - 2/3 days

PROBLEM 18 - 40
ECE APRIL 2004

At exactly what time after 2 o'clock will the hour hand and the minute hand of a continuously driven clock extend in the opposite direction for the first time?

A. 2:43:6.8
B. 2:43:58.1
C. 2:43:12.2
D. 2:43:38.2

PROBLEM 18 - 41
ECE APRIL 2004

A speed boat can make a trip of 100 km in one hour and 30 minutes if it travels upstream. If it travels downstream, it will take one hour and 15 minutes to travel the same distance. What is the speed of the boat in clam water?

A. 80 kph C. 66.67 kph
B. 123.33 kph D. 73.33 kph

PROBLEM 18 - 42
ME APRIL 2004

A train can pass an average of 3 stations every 10 minutes. at this rate, how many stations will it pass in one hour?

A. 30 C. 18
B. 26 D. 15

PROBLEM 18 - 43
ME OCT 2003

Going against the wind, a domestic plane can travel 5/8 of the distance it can travel in one hour if it is going with the wind. If the plane can fly 300 kilometers an hour in calm air, what is the velocity of the wind?

A. 69.23 kph C. 63.92 kph
B. 62.93 kph D. 96.23 kph

PROBLEM 18 - 44
ME APRIL 2004

John is four times as old as Harry. In six years, John will be twice as old as Harry. What is the age of Harry now?

A. 2 C. 4
B. 3 D. 5

PROBLEM 18 - 45
ECE APRIL 2004

What is the value of the following determinant?

$$\begin{vmatrix} 1 & 4 & 6 & 1 \\ 2 & 3 & 8 & 2 \\ 1 & 4 & 6 & 1 \\ 2 & 1 & 5 & 2 \end{vmatrix}$$

A. 1 C. 4
B. 3 D. 0

PROBLEM 18 - 46
ME APRIL 2004

Find the number of ways two balls, four dolls, and six toy guns can be given to 12 children, if each child gets a toy.

A. 12,606 C. 13,860
B. 10,620 D. 11,640

PROBLEM 18 - 47
ME OCT 2003

If three coins are tossed, how many possible ways are there for at least one coin showing tails?

A. 5 C. 8
B. 6 D. 7

PROBLEM 18 - 48
ME APRIL 2004

What is the indicated root $\sqrt[3]{-125}$?

A. -25 C. -5
B. 5 D. 25

PROBLEM 18 - 49
ECE APRIL 2004

Given that w varies as directly as the product of x and y and inversely as the square of z and that $w = 4$ when $x = 2$, $y = 6$, and $z = 3$. What is the value of w when $x = 1$, $y = 4$, and $z = 2$?

A. 3 C. 5
B. 4 D. 2

PROBLEM 18 - 50
ECE APRIL 2004

If 16 is four more than $3x$, then $x^2 + 5$ is equal to:

A. 18 C. 8
B. 21 D. 10

PROBLEM 18 - 51
ECE APRIL 2004

A professional organization is composed of x ECEs and $2x$ CEs. If 6 ECEs are replaced by 6 CEs, 1/6 of the members will be ECEs. What is the value of x?

A. 24 C. 12
B. 36 D. 18

PROBLEM 18 - 52
ME APRIL 2004

An hour-long test has 60 problems. If a student completes 30 problems in 20 minutes, how many seconds does he have on average for completing each of the remaining problems?

A. 60 C. 120
B. 90 D. 80

PROBLEM 18 - 53
ME APRIL 2004

The sum of two numbers is 10 and the sum of the squares of the numbers is 52. Find the product of the two numbers.

A. 26 C. 32
B. 24 D. 16

PROBLEM 18 - 54
ME APRIL 2004

After 8:00 pm, a ride in a cab cost ₱25.00 plus ₱3.00 for every fifth of a kilometer traveled. If a passenger travel x kilometers, what is the cost of the trip in pesos, as a function of x?

A. $28x$ C. $25 + 15x$
B. $25 + 3x$ D. $25 + 0.6x$

PROBLEM 18 - 55
ME APRIL 2004

On a scaled map, a distance of 10 cm represents 5 km. If a street is 750 meters long, what is the length on the map, in centimeters?

A. 15
B. 1.5
C. 150
D. 0.15

PROBLEM 18 - 56
ME APRIL 2004

Between 1950 and 1960, the population of Singapore increased by 3.5 million. If the amount of increase between 1960 and 1970 was 1.75 million more than the increase from 1950 to 1960, compute for the total amount of increase in the population of Singapore between 1950 to 1970.

A. 5.75 million
B. 4.25 million
C. 5.25 million
D. 8.75 million

PROBLEM 18 - 57
ME APRIL 2004

In a typical month ½ of the UFO sightings in the California State are attributable to airplanes and 1/3 of the remaining sightings are attributable to weather balloons. If there were 108 sightings during one typical month, how many would be attributable to weather balloons?

A. 36
B. 54
C. 24
D. 18

PROBLEM 18 - 58
ME APRIL 2004

If x is an integer, which of the following must be an odd integer?

A. $4(x - 1)$
B. $4x - 1$
C. $3x^2 - 2$
D. $x^2 - 3$

PROBLEM 18 - 59
ME OCT 2003

Ernie's average in 6 subjects is 83. If his lowest grade is disregarded, the average of his remaining subject is 84. What is his lowest grade?

A. 76
B. 77
C. 79
D. 78

PROBLEM 18 - 60
ME OCT 2003

A product has a current selling price of P325. If its selling price is expected to decline at the rate of 10% per annum because of obsolescence, what will be its selling price four years hence?

A. P302.75
B. P202.75
C. P213.23
D. P156.00

PROBLEM 18 - 61
ME OCT 2003

Instead of multiplying the number by 17, Karla divided it by 17. If the answer he obtained was 1, what should have been the correct answer?

A. 285
B. 276
C. 289
D. 295

PROBLEM 18 - 62
ME OCT 2003

Kim was sent to the store to get 11 boxes of sardines. Kim could carry only two boxes at a time. How many trips would Kim have to make?

A. 4
B. 6
C. 7
D. 5

PROBLEM 18 - 63
ME APRIL 2004

Simplify cos (30° - A) - cos (30° + A) as a function of angle A only.

A. sin A
B. tan A
C. cos A
D. sec A

PROBLEM 18 - 64
ME APRIL 2004

In triangle ABC, $\frac{\sin A}{\sin B} = \frac{7}{10}$ and $\frac{\sin B}{\sin C} = \frac{5}{2}$. If angle A, B, and C are opposite side a, b, and c, respectively, and the triangle had a perimeter of 16, what is the value of a?

A. 12.33
B. 8
C. 4.67
D. 5.33

PROBLEM 18 - 65
ME APRIL 2004

At one side of a road is a pole 25 ft high fixed on top of a wall a5 ft high. On the other side of the road, at a point on the ground directly opposite, the flagstaff and the wall subtend equal angles. Find the width of the road.

A. 30 feet
B. 20 feet
C. 45 feet
D. 50 feet

PROBLEM 18 - 66
ME OCT 2003

If sec $2A = 1 / \sin 13A$, determine the value of A.

A. 3°
B. 6°
C. 7°
D. 5°

PROBLEM 18 - 67
ECE APRIL 2004

The sides of a triangle are 8, 15, and 17 units. If each side is doubled, by how many square units will the area of the triangle be increased?

A. 120
B. 60
C. 180
D. 240

PROBLEM 18 - 68
CE MAY 2004

Find the area of the spherical triangle ABC having the following parts:

Angle A = 140°
Angle B = 75°
Angle C = 86°
Radius of sphere = 4 m

A. 32.78 m^2
B. 41.41 m^2
C. 33.79 m^2
D. 34.56 m^2

PROBLEM 18 - 69
ECE APRIL 2004

What is the area of a rhombus whose diagonals are 12 and 24?

A. 144 C. 108
B. 164 D. 132

PROBLEM 18 - 70
ECE APRIL 2004

A wire with length of 52 cm is cut into two unequal lengths. Each par is bent to form a square. If the sum of the area of the two squares is 97 sq. cm., what is the area of the smaller square?

A. 16 C. 64
B. 81 D. 49

PROBLEM 18 - 71
ME APRIL 2004

The perimeter of a circular sector, whose central angle is 60° is 14 feet. Find the radius of the circle.

A. 3.68 feet C. 6.32 feet
B. 4.59 feet D. 8.74 feet

PROBLEM 18 - 72
ME OCT 2003

An equilateral triangle has an altitude of $5\sqrt{3}$ cm long. Find the area of the triangle.

A. $25\sqrt{3}$ C. $15\sqrt{3}$
B. $3\sqrt{5}$ D. $10\sqrt{3}$

PROBLEM 18 - 73
ME OCT 2003

A circle with radius r has a circumference equal to the perimeter of a square whose side is S. Which of the following is correct?

A. $r = S/2\pi$ C. $r = 2\pi/S$
B. $r = S$ D. $r = 2S/\pi$

PROBLEM 18 - 74
ME OCT 2003

How long is each side of a regular hexagon with perimeter 108 centimeters.

A. 18 C. 32
B. 24 D. 12

PROBLEM 18 - 75
ME OCT 2003

How many sides have a polygon if the sum of the measures of the angles is 900°?

A. 9 C. 8
B. 6 D. 7

PROBLEM 18 - 76
ME OCT 2003

A rectangular room is to contain 125 square meters of flooring. If its length is to be five times its width, what should its dimensions be?

A. 5 m × 75 m C. 3 m × 15 m
B. 25 m × 125 m D. 5 m × 25 m

PROBLEM 18 - 77
ME OCT 2003

If the total number of diagonals of an N-gon is 77, then what is the value of N?

A. 14 C. 12
B. 13 D. 15

PROBLEM 18 - 78
ME OCT 2003

A circle has a circumference that is numerically equal to its area. If a certain square has the same area as the circle, what would be the length of the side?

A. $3\sqrt{\pi}$ C. $\pi\sqrt{2}$
B. $\pi\sqrt{3}$ D. $2\sqrt{\pi}$

PROBLEM 18 - 79
ME APRIL 2004

A rectangular solid has dimensions 3, 4, and 5. What is its diagonal?

A. 7.071 C. 6.325
B. 7.253 D. 9.125

PROBLEM 18 - 80
ECE APRIL 2004

Find the volume of the pyramid formed in the first octant by the plane $6x + 10y + 5z - 30 = 0$.

A. 15 C. 12
B. 13 D. 14

PROBLEM 18 - 81
CE MAY 2004

The upper and lower bases of the frustum of a rectangular pyramid are 3 m by 4 m and 6 m by 8 m, respectively. If the volume of the solid is 140 m^3, how far apart are the bases?

A. 6.5 m C. 5 m
B. 4.5 m D. 3.5 m

PROBLEM 18 - 82
ECE APRIL 2004

By how many percent will the volume of the cube increase if its edge is increased by 20%

A. 44 C. 1.728
B. 72.8 D. 7.28

PROBLEM 18 - 83
CE MAY 2004

A closed conical vessel with base radius of 1 m and altitude 2.5 m has its axis vertical. In upright position (vertex uppermost), the depth of water in it is 50 cm. If the vessels was inverted (vertex lowermost), how deep is the water in it?

A. 196.8 cm C. 204.6 cm
B. 187.5 cm D. 174.8 cm

PROBLEM 18 - 84
ME OCT 2003

When an irregular-shaped object is placed in a cylindrical vessel of radius 8 cm, containing water, the water surface rises 6 cm. What is the volume of the object if it is completely submerged in water?

A. 384π cc C. 632π cc
B. 525π cc D. 245π cc

PROBLEM 18 - 85
CE MAY 2004

A right prism has a base in the shape of regular octagon inscribed in a square 10 cm by 10 cm. If its altitude of 15 cm, find its volume in cc.

A. 1,242.6 C. 1,359.7
B. 1,163.4 D. 1,421.6

PROBLEM 18 - 86
ME OCT 2003

If two points in the number line have coordinates 1 and 7, find the coordinate of a point on the line which is twice as far from 1 as from 7.

A. 3 C. 6
B. 4 D. 5

PROBLEM 18 - 87
CE MAY 2004

Find the equation of the line with slope of 2/3 and which passes through point of intersection of the lines $4x - 2y + 1 = 0$ and $x - 2y + 4 = 0$.

A. $4x - 6y + 9 = 0$ C. $6x - 4y + 11 = 0$
B. $6x - 4y + 9 = 0$ D. $4x - 6y + 11 = 0$

PROBLEM 18 - 88
ME OCT 2003

Find x if the point $(x, 2)$ is equidistant from (3, 2) and (7, 2).

A. 5 C. 8
B. 4 D. 6

PROBLEM 18 - 89
ME OCT 2003

A rectangle with sides parallel to the axes has vertices at (-3, 2), (2, 5), and (-3, 5). What is the coordinate of the fourth vertex?

A. (-5, 2) C. (5, -2)
B. (2, 2) D. (-3, -2)

PROBLEM 18 - 90
ME OCT 2003

The triangle defined by the points $A(6, 1)$, $B(2, 4)$, and $C(-2, 1)$ is what?

A. right C. isosceles
B. obtuse D. scalene

PROBLEM 18 - 91
CE MAY 2004

Given the curve $36x^2 + 9y^2 - 36 = 0$. What is the length of its latus rectum?

A. 0.6 C. 1.5
B. 0.75 D. 1

PROBLEM 18 - 92
ME APRIL 2004

Which of the following points (1, 0), (-1, 0), (4, 4), and (9, 7) belong to the graph of the equation $y = x^2 - x$?

A. (-1, 0) C. (4, 4) and (1, 0)
B. (9, 7) D. (1, 0)

PROBLEM 18 - 93
ECE APRIL 2004

A circle is described by the equation $x^2 + y^2 - 16x = 0$. What is the length of the chord that is 4 units from the center of the circle?

A. 12.563 C. 8.523
B. 13.856 D. 9.632

PROBLEM 18 - 94
ECE APRIL 2004

What is the equation of the line that passes through (-3, 5) and is parallel to the line $4x - 2y + 2 = 0$?

A. $4y - 2x + 22 = 0$ C. $4x - 2y + 11 = 0$
B. $2x + y + 10 = 0$ D. $2x - y + 11 = 0$

PROBLEM 18 - 95
ECE APRIL 2004

A circle with its center in the first quadrant is tangent to both x and y-axes. If the radius is 4, what is the equation of the circle?

A. $(x + 4)^2 + (y + 4)^2 = 16$ C. $(x - 4)^2 + (y + 4)^2 = 16$
B. $(x - 4)^2 + (y - 4)^2 = 16$ D. $(x + 2)^2 + (y - 2)^2 = 16$

PROBLEM 18 - 96
ECE APRIL 2004

What is the distance between the line $x + 2y + 8 = 0$ and the point (5, -2)?

A. 4.025 C. 4.205
B. 4.502 D. 4.052

PROBLEM 18 - 97
CE MAY 2004

Find the area enclosed by the following curve:

$$x^2 + y^2 - 10x - 10y + 25 = 0$$

A. 85.47 square units C. 78.55 square units
B. 95.61 square units D. 68.53 square units

PROBLEM 18 - 98
ME APRIL 2004

What is the diameter of a circle with the following equation: $x^2 + y^2 - 6x + 4y - 12 = 0$

A. 10 C. 16
B. 5 D. 25

PROBLEM 18 - 99
ECE APRIL 2004

What conic section is described by the equation $x^2 + y^2 - 4x + 2y - 20 = 0$?

A. circle C. hyperbola
B. parabola D. ellipse

PROBLEM 18 - 100
ME APRIL 2004

The directrix of the parabola is $y - 5 = 0$ and its focus is at (4, -3). Find the length of its latus rectum.

A. 16 C. 2
B. 12 D. 4

PROBLEM 18 - 101
ME APRIL 2004

Find the area bounded by the parabola $x^2 = 16(y - 1)$ and its latus rectum.

A. 56.33 C. 36.67
B. 46.67 D. 42.67

PROBLEM 18 - 102
ME APRIL 2004

$4x^2 - y^2 = 16$ is the equation of a:

A. circle
B. parabola
C. hyperbola
D. ellipse

PROBLEM 18 - 103
ME APRIL 2004

Find the area bounded by the parabola $4y = x^2 - 2x + 1$ and its latus rectum.

A. 8/3
B. 4/3
C. 2/3
D. 10/3

PROBLEM 18 - 104
ME OCT 2003

Point (3, 4) is the center of a circle that is tangent to the *X*-axis. What is the point of tangency?

A. (4, 0)
B. (0, 4)
C. (3, 0)
D. (0, 3)

PROBLEM 18 - 105
ME OCT 2003

An arch 18 m high has a form of a parabola with a vertical axis. The length of a horizontal beam placed across the arch 8 m from the top is 64 m. Find the width at the bottom.

A. 81
B. 96
C. 64
D. 74

PROBLEM 18 - 106
ME OCT 2003

Find k such that the line $y = 4x + 3$ is tangent to the curve $y = x^2 + k$.

A. 7
B. 6
C. 5
D. 4

PROBLEM 18 - 107
CE MAY 2004

A triangle have vertices at (0, 0), (6, 30°), and (9, 70°). What is the perimeter of the triangle?

A. 29.45 units
B. 26.74 units
C. 20.85 units
D. 23.74 units

PROBLEM 18 - 108
ECE APRIL 2004

The square root of a variance is referred to as_______

A. dispersion
B. central tendency
C. median
D. standard deviation

PROBLEM 18 - 109
ME APRIL 2004

To find the angles of a triangle, given only the length of the sides, one would use:

A. law of sines
B. law of tangents
C. orthogonal functions
D. law of cosines

PROBLEM 18 - 110
ME APRIL 2004

A curve generated by a point which moves in uniform circular motion about an axis while traveling with a constant speed parallel to the axis.

A. epicyclod
B. cyloid
C. spiral of Archimedes
D. helix

ANSWER SHEET

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ANSWER SHEET (Continuation)

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Solutions to Set 18 Recent Board Exams

SOLUTION 18 - 1

$xy = 24 \quad \rightarrow (1)$

$y = 2x - 2 \quad \rightarrow (2)$

Substitute (2) to (1):

$x(2x - 2) = 24$

$2x^2 - 2x - 24 = 0$

$x^2 - x - 12 = 0$

$(x - 4)(x + 3) = 0$; $x = 4$ and $x = -3$

when $x = 4$, $y = 6$; $(4, 6)$

when $x = -3$, $y = -8$; $(-3, -8)$

SOLUTION 18 - 2

$\sqrt{x^2} - \sqrt{3x} = 2x - 6$

$x - \sqrt{3x} = 2x - 6$

$\sqrt{3x} = 6 - x$ squaring both sides

$3x = 36 - 12x + x^2$

$x^2 - 15x + 36 = 0$

$(x - 12)(x - 3) = 0$

$x = 12$ and $\underline{3}$

Note: This can be solved by trial and error using the choices.

SOLUTION 18 - 3

$\sqrt{8} + 3\sqrt{18} - 7\sqrt{2} = \sqrt{4(2)} + 3\sqrt{9(2)} - 7\sqrt{2}$

$\sqrt{8} + 3\sqrt{18} - 7\sqrt{2} = 2\sqrt{2} + 3 \times 3\sqrt{2} - 7\sqrt{2} = 4\sqrt{2}$

Note: This can be solved by direct calculation using calculator.

SOLUTION 18 - 4

Note: if $a > b$, $c > d$, and $a, b, c, d > 0$, then $ac > bd$

If $-9 < x$ and $-12 < y$, then $(-9)(-12) < (x)(y)$, or $108 < xy$

If $x < -4$ and $y < -6$, then $(x)(y) < (-4)(-6)$ or $xy < 24$

Thus, $108 < xy < 24$

SOLUTION 18 - 5

$$\frac{2x^4 + 3x^3 + 7x^2 + 10x + 10}{(x-1)(x^2+3)^2} = \frac{A}{x-1} + \frac{Bx + C}{x^2 + 3} + \frac{Dx + E}{(x^2+3)^2}$$

$2x^4 + 3x^3 + 7x^2 + 10x + 10 = A(x^2 + 3)^2$
$\quad + (Bx + C)(x - 1)(x^2 + 3) + (Dx + E)(x - 1)$

Set $x = 1$:
$\quad 2(1)^4 + 3(1)^3 + 7(1)^2 + 10(1) + 10 = A(1^2 + 3)^2$
$\quad A = 2$

$2x^4 + 3x^3 + 7x^2 + 10x + 10 = (2x^4 + 12x^2 + 18)$
$\quad + (Bx^4 + 3Bx^2 - Bx^3 - 3Bx + Cx^3 + 3Cx - Cx^2 - 3C)$
$\quad + (Dx^2 - Dx + Ex - E)$

$2x^4 + 3x^3 + 7x^2 + 10x + 10 = (2 + B)x^4 + (C - B)x^3$
$\quad + (12 + 3B - C + D)x^2 + (3C - 3B - D + E)x$
$\quad + (18 - 3C - E)$

This equation is an IDENTITY. Each term at the left side of the equation must be equal to each term at the right side.

$2x^4 = (2 + B)x^4$
$2 = 2 + B; B = 0$

$3x^3 = (C - B)x^3$
$3 = C - B = C - 0; C = 3$

Constant:
$10 = 18 - 3C - E$
$10 = 18 - 3(3) - E; \mathbf{E = -1}$

$7x^2 = (12 + 3B - C + D)x^2$
$7 = 12 + 3B - C + D$
$7 = 12 + 3(0) - 3 + D; \; D = -2$

Thus; $A = 2, B = 0, C = 3, D = -2, E = -1$

$$\frac{2x^4 + 3x^3 + 7x^2 + 10x + 10}{(x-1)(x^2+3)^2} = \frac{2}{x-1} + \frac{3}{x^2+3} - \frac{2x+1}{(x^2+3)^2}$$

SOLUTION 18 - 6 $\quad 3^x = 54; \; 3^{x-2} = 3^x/3^2 = 54/9 = 6$

SOLUTION 18 - 7 $\quad x + 1/x = 5$ square both sides
$x^2 + 2(x)(1/x) + (1/x)^2 = 5^2$
$x^2 + 2 + 1/x^2 = 25$
$x^2 + 1/x^2 = 23$

SOLUTION 18 - 8
$$\log_2 \frac{\sqrt{2}}{8} = \frac{\log(\sqrt{2}/8)}{\log 2} = -2.5$$

SOLUTION 18 - 9 $\quad$ Using the calculator, $\log_{10} 32 = 1.505$

SOLUTION 18 - 10 $\quad \log_9 (2x - 3) = ½$
$9^{1/2} = 2x - 3; \; 3 = 2x - 3$
$x = 3$

SOLUTION 18 - 11 $\quad \log_8 N = 2/3$
$N = 8^{2/3} = 4$

SOLUTION 18 - 12 $\quad \log_x 1296 = 4; \quad x^4 = 1296$
$x = 6$

SOLUTION 18 - 13 $\quad 2 \log x - \log (30 - 2x) = 1$
$\log x^2 - \log (30 - 2x) = -1$
$\log \frac{x^2}{30 - 2x} = 1; \quad \frac{x^2}{30 - 2x} = 10^1$
$x^2 = 300 - 20x$
$x^2 + 20x - 300 = 0$
$(x + 30)(x - 10) = 0; \; x = \mathbf{10}$ and -30

Note: -30 is absurd. This can be solved by trial and error using the choices.

SOLUTION 18 - 14 $\quad \log_b 2 = 0.69$
$b^{0.69} = 2, \; b = 2.7307$
$$\log_b 6 = \frac{\log 6}{\log b} = \frac{\log 6}{\log 2.7307} = 1.7836$$

SOLUTION 18 - 15 $\quad \log_x 6 = 1.2925$
$x^{1.2925} = 6; \; x = 4$
$$\log_x 11 = \log_4 11 = \frac{\log 11}{\log 4} = \mathbf{1.7297}$$

SOLUTION 18 - 16 $\quad AB = (3 + j4)(8 + j6) = 24 + 50j + 24j^2 \qquad$ Note: $j^2 = -1$
$AB = 50j$
$\quad$ Thus, $a = 0, \; b = 50$
$\quad r = \sqrt{a^2 + b^2} = 50$
$\quad \tan\theta = b/a = 50/0; \; \theta = 90°$
Polar form $= r\angle\theta = \mathbf{50\angle 90°}$

SOLUTION 18 - 17 $\quad (3 + 4i) + (2 - 5i) = (3 + 2) + (4i - 5i) = \mathbf{5 - i}$

SOLUTION 18 - 18

$$f(x) = \frac{1}{1+x}$$

$$f(-1) = \frac{1}{1+(-1)} = \text{undefined}$$

$$f(-1/2) = \frac{1}{1+(-1/2)} = 2;\ f(2) = 1/3$$

Thus, $f(f(-1/2)) = 1/3$

$$f(-2) = \frac{1}{1+(-2)} = -1;\ f(-1) = \text{undefined}$$

Thus, $f(f(-2)) = \text{undefined}$

$f(0) = 1;\ f(1) = ½$

Thus, $f(f(x))$ is undefined at $\{-1, -2\}$

SOLUTION 18 - 19

$f(x) = 2x^2 + 2$
$f(x + 4) = 2(x + 4)^2 + 2 = 2(x^2 + 8x + 16) + 2$
$f(x + 4) = 2x^2 + 16x + 34$

SOLUTION 18 - 20

Increase $= f(3) - f(2) = (3\sqrt{3} - 4)^2 - (3\sqrt{2} - 4)^2$
Increase = **1.372**

SOLUTION 18 - 21

$f(x) = x^3 - x - 1$
$f(1) = 1^3 - 1 - 1 = -1;\quad f(-1) = (-1)^3 + 1 - 1 = -1$ (OK)
$f(0) = -1$ (OK)
$f(2) = 2^3 - 2 - 1 = 5;\ f(-2) = -8 + 2 - 1 = -7$ (Not OK)

Therefore, the set of all c is **$\{-1, 0, 1\}$**

SOLUTION 18 - 22

$f(x) = 3x - 1$, then $f^{-1}(x) = \frac{1}{3}(x + 1) = \frac{1}{3}x + \frac{1}{3}$

SOLUTION 18 - 23

$f(x) = x - 2x^2 + 2$
$f(a - 2) = (a - 2) - 2(a - 2)^2 + 2 = a - 2 - 2(a^2 - 4a + 4) + 2$
$f(a - 2) = a - 2 - 2a^2 + 8a - 8 + 2$
$f(a - 2) =$ **$9a - 2a^2 - 8$**

SOLUTION 18 - 24

$f(x) = 3x + 2$
Let $g(x) = ax + b$
$g(f(x)) = x$
$g(3x + 2) = a(3x + 2) + b = x$
$3ax + 2a + b = x$
$(3a)x + (2a + b) = x$
Then; $3a = 1;\ a = 1/3$
$2a + b = 0$

$2(1/3) + b = 0;\ b = -2/3$

$g(x) = ax + b = x/3 - 2/3 =$ **$1/3(x - 2)$**

SOLUTION 18 - 25

The terms form an A.P. with $a_1 = x$, $d = 3x$ and $a_n = 64x$
$a_n = a_1 + (n - 1)d$
$64x = x + (n - 1)3x$
$64 = 1 + 3n - 3;\ n = 22$

$$\text{Sum} = \frac{n}{2}(a_1 + a_n) = \frac{22}{2}(x + 64x) = 1430$$

$\frac{22}{2} \times 65x = 1430;$ **$x = 2$**

SOLUTION 18 - 26

Let x be the first term and y be the common difference.
$a_1 = x;\ a_2 = x + y;\ a_3 = x + 2y;\ a_4 = x + 3y$

$x(x + 3y) = 70$
$x^2 + 3xy = 70 \quad \rightarrow$ Eq. (1)

$(x + y)(x + 2y) = 88$
$x^2 + 3xy + 2y^2 = 88 \quad \rightarrow$ Eq. (2)

Replace $x^2 + 3xy$ in Eq. (2) by 70 from Eq. (1)
$70 + 2y^2 = 88$
$2y^2 = 18;\quad y^2 = 9;\ y = 3$

In Eq. (1):
$x^2 + 3x(3) = 70$
$x^2 + 9x - 70 = 0$
$(x - 5)(x + 14) = 0$
$x = 5$ and -14

SOLUTION 18 - 27

Harmonic progression: 1, ¼, $1/x$, $1/10$, or $\frac{1}{1}, \frac{1}{4}, \frac{1}{x}, \frac{1}{10}$

Their reciprocals 1, 4, x, 10 form an arithmetic progression. By inspection, the common difference of A.P. is 3. Thus $x = 7$

SOLUTION 18 - 28

The terms $2^{-1}, 2^{-3}, 2^{-5}, \ldots$ is a G.P. with $a_1 = 2^{-1}$ and $r = 2^{-2}$ or ¼.

$$\text{Sum of I.G.P.} = \frac{a_1}{1-r} = \frac{2^{-1}}{1-1/4} = \frac{1/2}{3/4}$$

Sum of I.G.P. = **2/3**

SOLUTION 18 - 29

Let the sides by x, $x + 6$, and $x + 12$.
Then, $(x + 12)^2 = x^2 + (x + 6)^2$

$x^2 + 24x + 144 = x^2 + x^2 + 12x + 36$
$x^2 - 12x - 108 = 0$
$x = \mathbf{18}$ and -6
Hypotenuse = 18 + 12 = **30**

SOLUTION 18 - 30 $A_0 = 3;\ A_n = \frac{5}{3} A_{n-1}$

SOLUTION 18 - 31 Let x and y be the numbers. ($y > x$)
$y - x = 40$ or $y = 40 + x$
Arithmetic mean, $A_m = ½(x + y)$
Geometric mean, $G_m = \sqrt{xy}$
$A_m = G_m + 2$
$½(x + y) = \sqrt{xy} + 2$
$x + y - 4 = 2\sqrt{xy}$ → square both sides
$x^2 + y^2 + 16 + 2xy - 8x - 8y = 4xy$
$x^2 + y^2 + 16 - 2xy - 8x - 8y = 0$
$x^2 + (40 + x)^2 + 16 - 2x(40 + x) - 8x - 8(40 + x) = 0$
$x^2 + 1600 + 80x + x^2 + 16 - 80x - 2x^2 - 8x - 320 - 8x = 0$
$1296 - 16x = 0;\ x = \mathbf{81}$
Another solution:
$A_m = G_m + 2$
$½(x + y) = \sqrt{xy} + 2$
$½(x + 40 + x) = \sqrt{x(40+x)} + 2$
$40 + 2x = 2\sqrt{x(40+x)} + 4$
Solve x by trial and error using the choices, $x = \mathbf{81}$.

SOLUTION 18 - 32 $f(x) = kx^3 - (k + 3)x^2 + 13$
$f(4) = k(4)^3 - (k + 3)(4)^2 + 13 = 157$
$64k - 16k - 48 + 13 = 157;\ k = \mathbf{4}$

SOLUTION 18 - 33 Number of man-hours required per unit = 2(2) = 4
Number of PCB's = 80(200)/4 = **4000 PCB's**

SOLUTION 18 - 34 Let A, B, and C be the number of days each one of them can finish the job working alone.
$1/A = 1.5\,(1/B);\qquad B = 1.5A$
$1/A = 1.2(1/C);\qquad C = 1.2A$

$$\frac{1}{A}(4) + \frac{1}{B}(4) + \frac{1}{C}(4) = 1$$

$$\frac{1}{A}(4) + \frac{1}{1.5A}(4) + \frac{1}{1.2A}(4) = 1$$

$$\frac{10}{A} = 1;\ A = \mathbf{10\ days}$$

SOLUTION 18 - 35 Rate of John = 1/30
Rate of Tom = 1/60
$(1/30)t + (1/60)t = 1$
$t = \mathbf{20}$ **minutes**

SOLUTION 18 - 36 Let S = the distance from A to B
t = required time of arrival.

Leaving at 9:30 am and walking at 2.5 kph
Time of travel = $t - 9.5 + 12/3600 = t - 9.49667$
S = speed × time of travel
$S = 2.5(t - 9.49667)$ → Eq. (1)

Leaving at 9:00 am and walking at 1.5 kph:
Time of travel = $t - 9 - 1/60 = t - 9.01667$
$S = 1.5(t - 9.01667)$ → Eq. (2)

$[S = S]$ $2.5(t - 9.49667) = 1.5(t - 9.01667)$
$2.5t - 23.7417 = 1.5t - 13.525$
$t = 10.2167$ hr = **10:13 am**

SOLUTION 18 - 37 Time, $t = S/v$

$$\frac{S}{5} + \frac{S}{4} = 3 + \frac{36}{60};\ S = 8\text{ km}$$

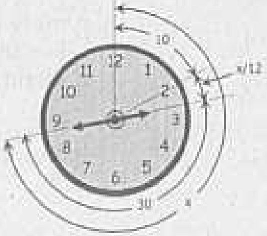
Total distance that walked = 8 + 8 = **16 km**

SOLUTION 18 - 38 $\frac{1}{50}t + \frac{1}{20}t = 1;\ t = \mathbf{14.28\ min}$

SOLUTION 18 - 39 $\frac{1}{5}t + \frac{1}{10}t = 1;\ t = 3\frac{1}{3}$ days

SOLUTION 18 - 40 From the figure shown:
$x = 10 + x/12 + 30$
$x = 43.636$ minutes
$x = 43$ min 38.2 sec

Time: 2:43:38.2



SOLUTION 18 - 41

Let x be the speed of the boat in clam water and y be the speed of the current in kph:

Traveling upstream; speed, $v = x - y$
$[S = vt]$ $100 = (x - y)(1.5)$
$x - y = 66.67 \quad \rightarrow$ Eq. (1)

Traveling downstream, speed, $v = x + y$
$[S = vt]$ $100 = (x + y)1.25$
$x + y = 80 \quad \rightarrow$ Eq. (2)

Add: Eq. (1) + Eq. (2):
$(x - y) + (x + y) = 66.67 + 80$
$2x = 146.67$; $x =$ **73.33 kph**

SOLUTION 18 - 42

Rate = 3/10 = 0.3 stations per minute
No. of stations = rate × time
No. of stations = 0.3(60) = **18 stations**

SOLUTION 18 - 43

Let x be the speed of the wind:
Velocity with the wind = $300 + x$
Velocity against the wind = $300 - x$
$S_{\text{against the wind}} = (5/8)\, S_{\text{with the wind}}$
$(300 - x)(1) = (5/8)(300 + x)(1)$
$2400 - 8x = 1500 + 5x$
$13x = 900$; $x =$ **69.23 kph**

SOLUTION 18 - 44

Let J and H be the present ages of John and Harry, respectively.

At present: $J = 4H \quad \rightarrow (1)$

In 6 years: $J + 6 = 2(H + 6)$
$4H + 6 = 2H + 12$
$H =$ **3**

SOLUTION 18 - 45

The value of a determinant is zerois there are two or more identical rows or columns.
For the given determinant, column 1 and column 4 are identical. Therefore, its value is zero.

SOLUTION 18 - 46

$$\text{Number of ways} = \frac{12!}{2!4!6!} = \mathbf{13{,}860}$$

SOLUTION 18 - 47

Note: "At least one" tail means either one tail, two tails, or three tail.

Number of possibilities:
THH, HTH, HHT, TTH, THT, HTT, TTT = 7 ways

Or, $N = C(3, 1) + C(3, 2) + C(3, 3) = 3 + 3 + 1 = 7$ ways

SOLUTION 18 - 48

$\sqrt[3]{-125} = -5$

SOLUTION 18 - 49

$w \propto xy/z^2$ or $w = kxy/z^2$
$4 = k(2)(6)/3^2$; $k = 3$
$w = 3(1)(4)/2^2 =$ **3**

SOLUTION 18 - 50

$16 = 3x + 4$; $x = 4$
$x^2 + 5 = 4^2 + 5 =$ **21**

SOLUTION 18 - 51

Total members = $x + 2x = 3x$
After replacement, the number of ECEs is $x - 6$.

$$\text{Fraction of ECE} = \frac{x-6}{3x} = \frac{1}{6}$$

$6x - 36 = 3x$
$x =$ **12**

SOLUTION 18 - 52

Remaining problems = 60 - 30 = 30
Remaining time = 60 - 20 = 40 minutes = 2400 seconds
Average time for each remaining problem = 2400/30
Average time for each remaining problem = **80 sec**

SOLUTION 18 - 53

Let x and y be the numbers.
$x + y = 10 \quad \rightarrow (1)$
$x^2 + y^2 = 52 \quad \rightarrow (2)$

Squaring both sides of Eq. (1):
$(x + y)^2 = 10^2$
$x^2 + 2xy + y^2 = 100$
$x^2 + y^2 + 2xy = 100$ but $x^2 + y^2 = 52$
$52 + 2xy = 100$
Product = xy = **24**

SOLUTION 18 - 54

Note: There are 5 fifth km in a kilometer

Cost = $25 + 3(5x) =$ **25 + 15x**

SOLUTION 18 - 55

$$\frac{x}{750} = \frac{10}{5000};\ x = \mathbf{1.5\ cm}$$

SOLUTION 18 - 56

Increase between 1950 and 1960 = 3.5 M
Increase between 1960 and 1970 = 3.5 + 1.75 = 5.25 M
Total increase between 1950 and 1970 = **8.75 million**

SOLUTION 18 - 57

Attributable to airplanes = ½(108) = 54
Attributable to weather balloons = 1/3(108 – 54) = **18**

SOLUTION 18 - 58

Among the choices, **$4x - 1$** always gives an odd integer because $4x$ is always an EVEN integer.

SOLUTION 18 - 59

Total/6 = 83; Total = 498
(Total – Lowest)/5 = 84
498 – Lowest = 420; Lowest = **78**

SOLUTION 18 - 60

Selling price after 1 year = 325(0.90)
Selling price after 2 years = 325(0.90)(0.90)
Selling price after 2 years = $325(0.90)^2$
Thus, selling price four years hence = $325(0.90)^4$
= **P213.23**

SOLUTION 18 - 61

Let x be the number:
$x/17 = 1$, $x = 17$
Correct answer = 17(17) = **289**

SOLUTION 18 - 62

Number of trips = **6**

SOLUTION 18 - 63

Using the relationship:
$\cos(x + y) = \cos x \cos y - \sin x \sin y$

$$[\cos 30^\circ \cos A + \sin 30^\circ \sin A] - [\cos 30^\circ \cos A - \sin 30^\circ \sin A]$$
$$= \sin 30^\circ \sin A + \sin 30^\circ \sin A$$
$$= 0.5 \sin A + 0.5 \sin A = \sin A$$

Note: This problem can be solved by assuming any value of angle A (say A = 1°). The choice that will give the same value as the given expression is the correct answer.

SOLUTION 18 - 64

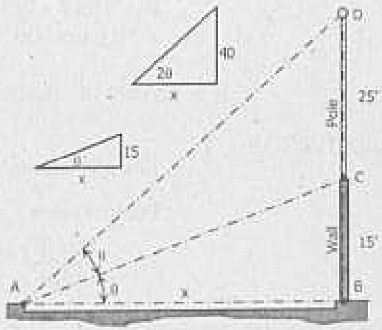
Sine law: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

$$\frac{\sin A}{\sin B} = \frac{a}{b} = \frac{7}{10};\ a = 0.7b$$

$$\frac{\sin B}{\sin C} = \frac{b}{c} = \frac{5}{2};\ c = 0.4b$$

Perimeter: $a + b + c = 16$
$0.7b + b + 0.4b = 16$; $b = 7.619$
$a = 0.7(7.619) = 5.33$

SOLUTION 18 - 65



$\tan\theta = 15/x$
$\tan 2\theta = 40/x$

$$\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}$$
$$\frac{40}{x} = \frac{2(15/x)}{1 - (15/x)^2}$$
$20 - 4500/x^2 = 15$
$4500/x^2 = 5$
$x^2 = 900$
x = **30 feet**

SOLUTION 18 - 66

$\sec 2A = 1/\sin 13A$; $1/\cos 2A = 1/\sin 13A$

By trial and error using the choices:
A. $A = 3^\circ$: $1/\cos 6^\circ = 1.0055$; $1/\sin 39^\circ = 1.5890$ ×
B. $A = 6^\circ$: $1/\cos 12^\circ = 1.0223$; $1/\sin 78^\circ = 1.0223$ ✓
C. $A = 7^\circ$: $1/\cos 14^\circ = 1.0306$; $1/\sin 91^\circ = 1.0002$ ×
D. $A = 5^\circ$: $1/\cos 10^\circ = 1.0154$; $1/\sin 65^\circ = 1.1034$ ×

SOLUTION 18 - 67

Original side: $a = 8$, $b = 15$, $c = 17$.
$s = ½(8 + 15 + 17) = 20$
$$A_o = \sqrt{20(20-8)(20-15)(20-17)} = 60 \text{ sq. units}$$

If each side is double, the area is (double)² or quadrupled.
$A_{new} = 4(60) = 240$ sq. units

Increase = 240 – 60 = **180 square units**

SOLUTION 18 - 68

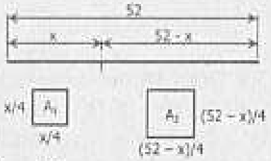
$$\text{Area} = \frac{\pi R^2 E}{180^\circ}$$
$E = A + B + C - 180^\circ = 140^\circ + 75^\circ + 86^\circ - 180^\circ = 121^\circ$
$R = 4$ m

$$\text{Area} = \frac{\pi(4)^2(121^\circ)}{180^\circ} = \mathbf{33.79\ m^2}$$

SOLUTION 18 - 69 Area of rhombus = $\frac{1}{2} d_1 d_2 = \frac{1}{2}(12)(24) =$ **144 sq. units**

SOLUTION 18 - 70

$A_1 + A_2 = 97$
$(x/4)^2 + [(52 - x)/4]^2 = 97$
$x^2/16 + (2704 - 104x + x^2)/16 = 97$
$x^2 + 2704 - 104x + x^2 = 1552$
$2x^2 - 104x + 1152 = 0$
$x^2 - 52x + 576 = 0$
$(x - 16)(x - 36) = 0$
$x = \underline{16}$ and 36



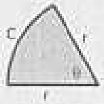
Area of smaller square = $(16/4)^2 =$ **16**

SOLUTION 18 - 71

Length of arc, $C = \dfrac{\pi r\theta}{180°}$

Perimeter = $C + 2r$

$14 = \dfrac{\pi r(60°)}{180°} + 2r$; r = **4.59 feet**



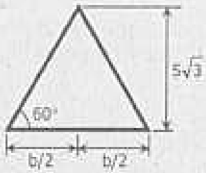
SOLUTION 18 - 72

$\tan 60° = \dfrac{5\sqrt{3}}{b/2}$

$b = 10$

Area= $\frac{1}{2}(10)(5\sqrt{3})$

Area = $25\sqrt{3}$ cm²



SOLUTION 18 - 73 Circumference of circle = perimeter of square
$2\pi r = 4S$
$r = 2S/\pi$

SOLUTION 18 - 74 A regular hexagon has 6 equal sides.
6 × side = 108
side = **18 cm**

SOLUTION 18 - 75 Sum of interior angles = $180(n - 2)$
$900 = 180(n - 2)$; n = **7 sides**

SOLUTION 18 - 76 Length × width = 125
Length = 5 × width
(5 × width) × width = 125
width = 5, length = 25
Dimensions = **5 m × 25 m**

SOLUTION 18 - 77

Number of diagonals = $\dfrac{n}{2}(n - 3)$

$77 = \dfrac{n}{2}(n - 3)$; $n^2 - 3n - 154 = 0$; $n = \mathbf{14}$

SOLUTION 18 - 78

For the circle:
Circumference = Area
$2\pi r = \pi r^2$; $r = 2$
Area = $\pi(2)^2 = 4\pi$

For the square:
Area = side² = 4π; side = $2\sqrt{\pi}$

SOLUTION 18 - 79

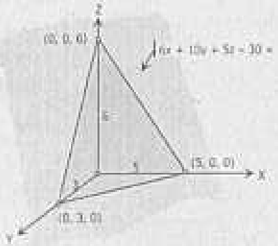
Space diagonal = $\sqrt{3^2 + 4^2 + 5^2} = 7.071$

SOLUTION 18 - 80

Solving for the intercepts:
Set $x = 0$ & $y = 0$
$6(0) + 10(0) + 5z - 30 = 0$
$z = 6$ (0, 0, 6)

Set $x = 0$ and $z = 0$
$y = 3$ (0, 3, 0)

Set $y = 0$ and $z = 0$
$x = 5$ (5, 0, 0)



From the figure:
$V = \frac{1}{3}\left[\frac{1}{2}(3)(5)\right](6) =$ **15 cubic units**

SOLUTION 18 - 81

$V = \dfrac{h}{3}[A_1 + A_2 + \sqrt{A_1 A_2}]$

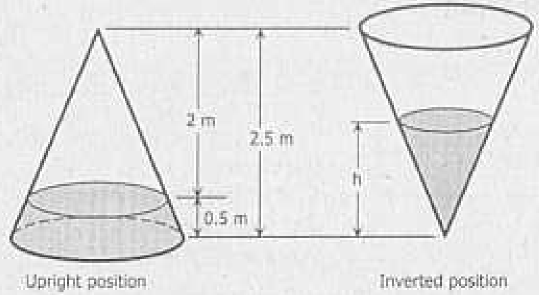
$140 = \dfrac{h}{3}[(3{\times}4) + (6{\times}8) + \sqrt{(3 \times 4)(6 \times 8)}]$; h = **5 m**

SOLUTION 18 - 82 Let V and x be the original volume and edge of the cube. When the edge is increased by 20% (edge = $1.2x$)

$\dfrac{V_n}{V} = \left(\dfrac{x_n}{x}\right)^3$; $\dfrac{V_n}{V} = \left(\dfrac{1.2x}{x}\right)^3 = 1.728$

$V_n = 1.728V$
Increase = $1.728V - V = 0.728V$
Percent increase = $(0.728V/V) \times 10\%$ = **72.8%**

SOLUTION 18 - 83



Let V = volume of the vessel; V_a = volume of air
V_w = volume of water = $V - V_a$

In upright position:
By similar solids (vessel and air):

$$\frac{V}{V_a} = \left(\frac{2.5}{2}\right)^3 ;\ V_a = 0.512\ V$$

$$V_w = V - V_a = 0.488V$$

In inverted position:
By similar solids (vessel and water):

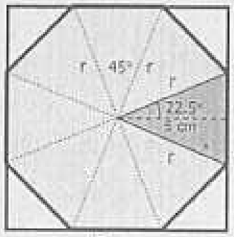
$$\frac{V}{V_w} = \left(\frac{2.5}{h}\right)^3 ;\ \frac{V}{0.488V} = \left(\frac{2.5}{h}\right)^3$$

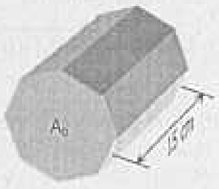
$h = 1.968$ m = **196.8 cm**

SOLUTION 18 - 84

Volume of object = Volume rise
Volume of object = $\pi r^2 h = \pi(8)^2(6) =$ **384π cc**

SOLUTION 18 - 85





Volume = $A_b \times h$

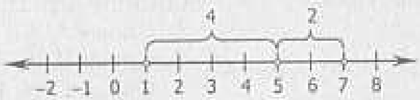
$$\cos 22.5° = \frac{5}{r}$$

$r = 5.41196$ cm
$A_b = ½\, r^2 \sin 45° \times 8$
$A_b = 4(5.41196)^2 \sin 45°$
$A_b = 82.8427$ cm²

Volume = 82.8427(15)
Volume = **1242.64 cm³**

SOLUTION 18 - 86

From the diagram shown the point has a coordinate of 5



SOLUTION 18 - 87

Solving for the point of intersection, PI:
$4x - 2y + 1 = 0$ → (1)
$x - 2y + 4 = 0$ → (2)

Subtract: Eq. (1) - Eq. (2):

$$\begin{array}{rl} & 4x - 2y + 1 = 0 \\ - & \underline{x\ - 2y + 4 = 0} \\ & 3x - 3 = 0 \end{array}$$

$x = 1; y = 5/2$ PI is at (1, 5/2)

By point-slope form:
$y - y_1 = m(x - x_1)$
$y - 5/2 = 2/3(x - 1)$
$6y - 15 = 4x - 4;$ $\mathbf{4x - 6y + 11 = 0}$

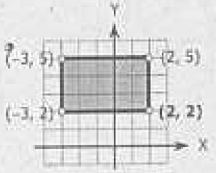
SOLUTION 18 - 88

$$d_1 = d_2 :\ \sqrt{(3-x)^2 + (2-2)^2} = \sqrt{(7-x)^2 + (2-2)^2}$$

$9 - 6x + x^2 = 49 - 14x + x^2$
$8x = 40; x = 5$

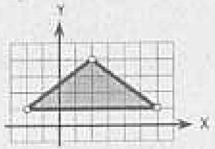
SOLUTION 18 - 89

From the figure shown,
the fourth vertex is at (2, 2)



SOLUTION 18 - 90

From the figure shown, the triangle is an *isosceles* triangle



SOLUTION 18 - 91

The given curve is an ellipse with standard equation of

$$\frac{x^2}{1} + \frac{y^2}{4} = 1.$$

$a^2 = 4, a = 2;$ $b^2 = 1, b = 1$

$$\text{Length of latus rectum} = \frac{2b^2}{a} = \frac{2(1)^2}{2} = \mathbf{1}$$

SOLUTION 18 - 92

Substitute each choice to the equation $y = x^2 - x$.

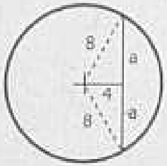
Choice A, (-1, 0): $y = (-1)^2 - (-1) = 2 \neq 0$

Choice B, (9, 7): $y = 9^2 - 9 = 72 \neq 7$

Choice C, (4, 4): $y = 4^2 - 4 = 12 \neq 4$

Choice D, (1, 0): $y = 1^2 - 1 = 0$ **OK**

SOLUTION 18 - 93



Solving for the radius of th circle:

$$x^2 + y^2 - 16x = 0$$
$$x^2 - 16x + 64 + y^2 = \underline{64}$$
$$(x-8)^2 + y^2 = 8^2 \qquad \text{Radius} = 8$$

$$a^2 = 8^2 - 4^2;\ a = 6.9282$$

Length of chord $= 2a =$ **13.856 units**

SOLUTION 18 - 94

The slope of the line $4x - 2y + 2 = 0$ is +2. Since the required line is parallel to the given line, their slopes are equal.

By point slope form:

$$y - y_1 = m(x - x_1)$$
$$y - 5 = 2[x - (-3)] = 2x + 6$$
$$\mathbf{2x - y + 11 = 0}$$

SOLUTION 18 - 95

The given circle has center $(h, k) = (4, 4)$ and radius of 4. Thus the standard equation is:

$$(x - h)^2 + (y - k)^2 = r^2$$
$$\mathbf{(x - 4)^2 + (y - 4)^2 = 16}$$

SOLUTION 18 - 96

$$d = \left| \frac{Ax_1 + By_1 + C}{\sqrt{A^2 + B^2}} \right|$$

where $A = 1, B = 2, C = 8$
$x_1 = 5, y_1 = -2$

$$d = \left| \frac{1(5) + 2(-2) + 8}{\sqrt{(1)^2 + (2)^2}} \right| = \mathbf{4.025}$$

SOLUTION 18 - 97

$$x^2 + y^2 - 10x - 10y + 25 = 0 \qquad \rightarrow \text{a circle}$$
$$(x^2 - 10x + 25) + (y^2 - 10y + 25) = -25 + 25 + 25$$
$$(x - 5)^2 + (y - 5)^2 = 25;\ \text{Radius} = 5$$

Area $= \pi R^2 = \pi(5)^2 =$ **78.55 square units**

SOLUTION 18 - 98

$$x^2 + y^2 - 6x + 4y - 12 = 0$$
$$(x^2 - 6x + 9) + (y^2 + 4y + 4) = 12 + 9 + 4$$
$$(x - 3)^2 + (y + 2)^2 = 25$$

Radius = 5, diameter = **10**

SOLUTION 18 - 99

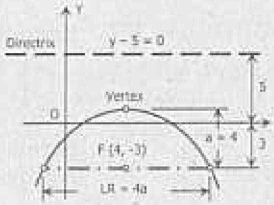
Since the coefficients of x^2 and y^2 are the same, the conic is a circle.

SOLUTION 18 - 100

From the figure shown:

$a = ½(5 + 3) = 4$

LR = $4a$ = **16**



SOLUTION 18 - 101

For the parabola $x^2 = 16(y - 1)$, $a = 4$ and $LR = 4a = 16$

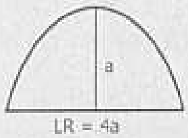
Area $= \frac{2}{3}(LR)a = \frac{2}{3}(16)(4) =$ **42.667 sq. units**

SOLUTION 18 - 102

Hyperbola

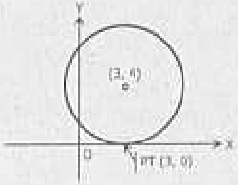
SOLUTION 18 - 103

$$4y = x^2 - 2x + 1$$
$$(x - 1)^2 = 4y; \qquad a = 1, LR = 4a = 4$$
$$\text{Area} = \tfrac{2}{3}(LR)(a) = \tfrac{2}{3}(4)(1) = 8/3$$



SOLUTION 18 - 104

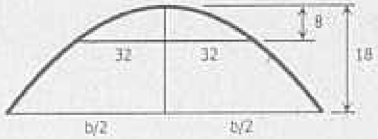
From the figure shown, the point of tangency PT is at (3, 0)



SOLUTION 18 - 105

By squared property of parabola:

$$\frac{8}{32^2} = \frac{18}{(b/2)^2}$$
$$\frac{b^2}{4} = \frac{18(32^2)}{8}$$
$$b = 96$$



SOLUTION 18 - 106 At the point of tangency, y of the line equals y of the curve.

$[y = y]$ $4x + 3 = x^2 + k$

$x^2 - 4x + (k - 3) = 0$

This quadratic equation must have only one root.

$B^2 = 4AC$

$(-4)^2 = 4(1)(k - 3)$

$4 = k - 3;\ k = 7$

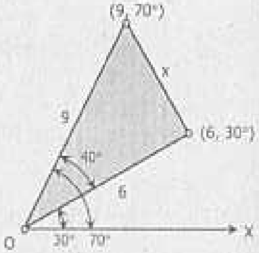
SOLUTION 18 - 107 From the figure shown:

$x^2 = 9^2 + 6^2 - 2(9)(6)\cos 40°$

$x = 5.8538$

Perimeter = 9 + 6 + 5.8538

Perimeter = **20.853 units**



SOLUTION 18 - 108 The square root of a variance is the **standard deviation**

SOLUTION 18 - 109 To find the angles of a triangle, given only the length of the sides, one would use the *law of cosines*.

SOLUTION 18 - 110 **Cycloid**

Problems - Set 19
Recent Board Exams 2

PROBLEM 19 - 1
ECE APRIL 2005

Which of the following mathematically represents a two-digit number with x as the unit's digit and y as its ten's digit.

A. $10y + 10x$ C. $10y + x$
B. $10y - x$ D. $10x + y$

PROBLEM 19 - 2
ECE APRIL 2005

What is the equation form of the statement; the amount by which 100 exceeds four times a given number?

A. $4x(100)$ C. $100 - 4x$
B. $100 + 4x$ D. $4x - 100$

PROBLEM 19 - 3
ECE APRIL 2005

Solve for x if $8^x = 2^{y+2}$ and $16^{3x-y} = 4^y$.

A. 4 C. 3
B. 2 D. 1

PROBLEM 19 - 4
ECE APRIL 2005

From the equation $7x^2 + (2k - 1)x - 3k + 2 = 0$, determine the value of k so that the sum and product of the roots are equal.

A. 4 C. 2
B. 3 D. 1

PROBLEM 19 - 5
ECE APRIL 2005

The arithmetic mean of six numbers is 17. If two numbers are added to the progression, the new arithmetic mean is 21. What are the two numbers if their difference is 4?

A. 28 and 32 C. 26 and 30
B. 30 and 34 D. 34 and 38

PROBLEM 19 - 6
ECE APRIL 2005

What is the sum of all the odd integers between 10 and 500?

A. 65,955 C. 87,950
B. 62,475 D. 124,950

PROBLEM 19 - 7
ECE APRIL 2005

How many terms of the progression 3, 5, 7, ... should there be so that their sum will be 2600?

A. 54 C. 50
B. 52 D. 55

PROBLEM 19 - 8
ECE APRIL 2005

If the first term of geometric progression is 27 and the fourth term is -1, the third term is?

A. 3
B. 2
C. -2
D. -3

SITUATION
CE MAY 2005

The fourth term of a geometric progression is 6 and the 10th term is 384.

PROBLEM 19 - 9
①

What is the common ratio of the G.P.?

A. 1.5
B. 3
C. 2.5
D. 2

PROBLEM 19 - 10
②

What is the first term?

A. 0.75
B. 1.5
C. 3
D. 0.5

PROBLEM 19 - 11
③

What is the seventh term?

A. 24
B. 32
C. 48
D. 96

PROBLEM 19 - 12
ECE APRIL 2005

Candle *A* and candle *B* of equal length are to be lighted at the same time and burning until candle *A* is twice as long as candle *B*. Candle *A* is designed to fully burn in 8 hours while candle *B* for 4 hours. How long will they be lighted?

A. 2 hrs and 40 min
B. 3 hrs and 30 min
C. 3 hrs & 40 min
D. 2 hrs & 30 min

PROBLEM 19 - 13
ECE APRIL 2005

At approximately what time between 6 and 7 o'clock will the minute and hour hands coincide?

A. 27 min and 41 sec after 6 o'clock
B. 32 min and 72 sec after 6 o'clock
C. 25 min and 38 sec after 6 o'clock
D. 32 min and 43 sec after 6 o'clock

PROBLEM 19 - 14
ECE APRIL 2005

In how many minutes after 2 p.m. will the hands of the clock extend in opposite directions for the first time?

A. 40.522
B. 43.636
C. 45.575
D. 41.725

PROBLEM 19 - 15
ECE APRIL 2005

How many ounces of pure nickel must be added to 150 ounces of alloy 70% pure to make an alloy 85% pure?

A. 150
B. 225
C. 125
D. 175

PROBLEM 19 - 16
ECE APRIL 2005

If there are nine distinct items taken three at a time, how many permutations will there be?

A. 252
B. 720
C. 504
D. 336

PROBLEM 19 - 17
ECE APRIL 2005

A bag contains 3 white and 5 red balls. If two balls are drawn in succession without returning the first ball drawn, what is the probability that the balls drawn are both red?

A. 0.299
B. 0.237
C. 0.357
D. 0.107

PROBLEM 19 - 18
ECE APRIL 2005

A janitor with a bunch of nine keys is to open a door but only one key can open. What is the probability that he will succeed in three trials?

A. 0.375
B. 0.425
C. 0.333
D. 0.255

PROBLEM 19 - 19
ECE APRIL 2005

Evaluate the expression: $(1 + i^2)^{10}$, where i is an imaginary number.

A. 0
B. -1
C. i
D. 1

SITUATION
CE MAY 2005

Given the following matrix:

$$A = \begin{Bmatrix} 2 & 5 & 1 \\ 3 & 1 & 0 \\ 6 & 7 & 4 \end{Bmatrix} \text{ and } B = \begin{Bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{Bmatrix}$$

PROBLEM 19 - 20
①

Determine the minor corresponding to the 5 entry in matrix *A*.

A. 15
B. 8
C. 12
D. 4

PROBLEM 19 - 21
②

Determine the cofactor corresponding to the 3 entry in matrix *A*.

A. 13
B. -13
C. 4
D. -4

PROBLEM 19 - 22
③

What is the product $A \times B$?

A. $\begin{bmatrix} 1 & 2 & 5 \\ 0 & 3 & 1 \\ 4 & 6 & 7 \end{bmatrix}$
C. $\begin{bmatrix} 5 & 2 & 1 \\ 1 & 3 & 0 \\ 7 & 6 & 4 \end{bmatrix}$
B. $\begin{bmatrix} 2 & 1 & 5 \\ 3 & 0 & 1 \\ 6 & 4 & 7 \end{bmatrix}$
D. $\begin{bmatrix} 2 & 5 & 1 \\ 3 & 1 & 0 \\ 6 & 7 & 4 \end{bmatrix}$

PROBLEM 19 - 23
ECE APRIL 2005

If $\sin A = 4/5$ and $\sin B = 7/25$, what is $\sin (A + B)$ if A is in the 3rd quadrant and B is in the 2nd quadrant?

A. 3/5
C. -3/5
B. 2/5
D. 4/5

PROBLEM 19 - 24
ECE APRIL 2005

If the sides of a right triangle are 3, 4, 5 m, the area of the inscribed circle is _______.

A. 2π square m
C. $3/4\ \pi$ square m
B. $3\pi/2$ square m
D. π square m

SITUATION
CE MAY 2005

A flagpole is stands on top of a pedestal. From a point O at the same elevation as the base of the pedestal, the vertical angle of the top of the flagpole is 60° and the vertical angle of the bottom of the flagpole is 52°. Point O is 14.8 meters away from the base of the pedestal.

PROBLEM 19 - 25
①

How high is the pedestal in meters?

A. 18.94
C. 15.24
B. 25.36
D. 14.28

PROBLEM 19 - 26
②

How long is the flagpole in meters?

A. 3.68
C. 8.12
B. 5.88
D. 6.69

PROBLEM 19 - 27
③

What is the distance from O to the top of the flagpole?

A. 33.2 m
C. 15.4 m
B. 29.6 m
D. 22.8 m

SITUATION
CE MAY 2005

One of the diagonals of a rhombus measure 12 inches. If the area of the rhombus is 132 square inches, determine the following:

PROBLEM 19 - 28
①

The length of the other diagonal in inches.

A. 24
C. 22
B. 8
D. 14

PROBLEM 19 - 29
②

The measure of the side of the rhombus in inches.

A. 13.85
C. 8.52
B. 10.96
D. 12.53

PROBLEM 19 - 30
③

The measure of the acute angle between the sides of the rhombus in degrees.

A. 28.61
C. 57.22
B. 45.26
D. 32.78

SITUATION
CE MAY 2005

A triangular lot ABC have side $AB = 400$ and angle $B = 50°$. The lot is to be segregated by a dividing line DE parallel to BC and 150 m long. The area of segment $BCDE$ is 50,977.4 m².

PROBLEM 19 - 31
①

Calculate the area of lot ABC, in square meter.

A. 62,365
C. 57,254
B. 59,319
D. 76.325

PROBLEM 19 - 32
②

Calculate the area of lot ADE, in square meter.

A. 8,342
C. 6,569
B. 14,475
D. 11,546

PROBLEM 19 - 33
③

Calculate the value of angle C, in degrees.

A. 57
C. 63
B. 42
D. 68

PROBLEM 19 - 34
ECE APRIL 2005

A railroad curve is to be laid in a circular path. What should be the radius if the track is to change direction by 30 degrees at a distance of 300 m?

A. 352 m
C. 287 m
B. 573 m
D. 452 m

SITUATION
CE MAY 2005

A swimming pool is shaped from two intersecting circles 9 m in radius with their centers 9 m apart.

PROBLEM 19 - 35
①

What is the area common to the two circles, in square meter?

A. 85.2
C. 128.7
B. 63.7
D. 99.5

PROBLEM 19 - 36
②

What is the total water surface area, in square meter?

A. 409.4
C. 387.3
B. 524.3
D. 427.5

PROBLEM 19 - 37
③

What is the perimeter of the pool, in meters?

A. 63.5
C. 82.4
B. 75.4
D. 96.3

PROBLEM 19 - 38
ECE APRIL 2005

Spheres of the same size are piled in the form of a pyramid with an equilateral triangle as its base. Compute the total number of spheres in the pile if each side contains 4 spheres.

A. 20 C. 16
B. 64 D. 28

PROBLEM 19 - 39
ECE APRIL 2005

What is the volume (in cm^3) of a right hexagonal prism 15 cm high and with one of its sides equal to 6 cm?

A. 985 C. 818
B. 929 D. 1185

PROBLEM 19 - 40
ECE APRIL 2005

Eight balls are tightly packed in a cubical container that measures 8 cm on each side. The balls are arranged with 4 balls per layer and in contact with the walls of the container and the adjacent balls. If the container is filled with water, what is the volume of the water (in cubic cm)?

A. 355 C. 268
B. 335 D. 244

SITUATION
CE MAY 2005

A closed conical vessel has a base radius of 2 m and is 6 m high. When in upright position, the depth of water in the vessel is 3 m.

PROBLEM 19 - 41
①

What is the volume of water in cubic meter?

A. 22 C. 28
B. 25 D. 32

PROBLEM 19 - 42
②

If the vessel is held in inverted position, how deep is the water, in meters?

A. 8.56 C. 4.12
B. 5.74 D. 6.87

PROBLEM 19 - 43
③

What is the weight of water in quintals. Unit weight of water is 9,800 N/m^3.

A. 263.4 C. 219.7
B. 195.4 D. 247.2

PROBLEM 19 - 44
ECE APRIL 2005

The total volume of two spheres is 100π cubic cm. The ratio of their areas is 4:9. What is the volume of the smaller sphere in cubic cm?

A. 75.85 C. 71.79
B. 314.16 D. 242.36

PROBLEM 19 - 45
ECE APRIL 2005

What is the distance between the points (3, 7) and (-4, -7)?

A. 18.65 C. 5.25
B. 6.54 D. 15.65

PROBLEM 19 - 46
ECE APRIL 2005

What is the area bounded by the curve defined by the equation $x^2 - 8y = 0$ and its latus rectum?

A. 5.33 C. 10.67
B. 7.33 D. 3.66

SITUATION
CE MAY 2005

Given the ellipse $9x^2 + 16y^2 - 144 = 0$.

PROBLEM 19 - 47
①

Determine the length of arc in the first quadrant.

A. 5.55 C. 7.58
B. 6.32 D. 9.97

PROBLEM 19 - 48
②

Determine the equation of its diameter bisecting all chords having equal slope of 3?

A. $4x + 25y$ C. $6x + 32y = 0$
B. $9x + 16y = 0$ D. $9x + 32y = 0$

PROBLEM 19 - 49
③

What is the volume generated if the area on the first and second quadrants is revolved about the X-axis?

A. 201.1 C. 175.4
B. 150.8 D. 165.7

SITUATION
CE MAY 2005

Given the equilateral hyperbola $xy = 8$:

PROBLEM 19 - 50
①

What is the length of the conjugate axis of the hyperbola?

A. 12 C. 8
B. 16 D. 4

PROBLEM 19 - 51
②

How far apart are the vertices of the hyperbola?

A. 4 C. 16
B. 8 D. 12

PROBLEM 19 - 52
③

Determine the eccentricity of the hyperbola.

A. 1.414 C. 1.368
B. 1.732 D. 1.521

SITUATION
CE MAY 2005

Given the ellipse $16x^2 + 25y^2 = 400$.

PROBLEM 19 - 53
①

Compute its perimeter.

A. 32.653 C. 24.785
B. 28.448 D. 36.896

PROBLEM 19 - 54 ② Determine its second eccentricity.

A. 0.75
B. 0.82
C. 0.67
D. 0.6

PROBLEM 19 - 55 ③ What is the equation of its diameter bisecting the chords having equal slope of 1/5?

A. $5x - 16y = 0$
B. $5x + 16y = 0$
C. $16x + 5y = 0$
D. $16x - 5y = 0$

ANSWER SHEET

1. Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
2. Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
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60. Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ

Solutions to Set 19
Recent Board Exams 2

SOLUTION 19 - 1

Unit's digit = x; Ten's digit = y
Number = $10y + x$

SOLUTION 19 - 2

$4x + 100$

SOLUTION 19 - 3

$[8^x = 2^{y+2}]$ $\quad 2^{3x} = 2^{y+2}$

$3x = y + 2$

$y = 3x - 2 \quad$ → Eq. (1)

$[16^{3x-y} = 4^y]$ $\quad (4^2)^{3x-y} = 4^y$

$4^{6x-2y} = 4^y$

$6x - 2y = y;\ 6x = 3y$

$y = 2x \quad$ → Eq. (2)

$[y = y]$ $\quad 3x - 2 = 2x;\ x = 2$

SOLUTION 19 - 4

$7x^2 + (2k - 1)x - 3k + 2 = 0$

$A = 7;\ B = (2k - 1);\ C = 2 - 3k$

$$\text{Sum of roots} = -\frac{B}{A} = -\frac{2k-1}{7}$$

$$\text{Product of roots} = \frac{C}{A} = \frac{2-3k}{7}$$

$$\left[-\frac{B}{A} = \frac{C}{A}\right] \qquad -\frac{2k-1}{7} = \frac{2-3k}{7}$$

$-2k + 1 = 2 - 3k;\ k = 1$

SOLUTION 19 - 5

Let A by the sum of the first six numbers
x & $x + 4$ be the two numbers

$$\frac{A}{6} = 17;\ A = 102$$

$$\frac{A + x + (x + 4)}{8} = 21$$

$108 + 2x + 4 = 21(8);\ x = 28$

$x + 4 = 32$

Therefore, the two numbers are 28 & 32.

SOLUTION 19 - 6

Sum = 11 + 13 + 15 + ... + 497 + 499
First term, $a_1 = 11$
Common difference, $d = 2$
Last term, $a_n = 499$

$[a_n = a_1 + (n - 1)d]$ $\quad 499 = 11 + (n - 1)2$
$n = 245$

$\left[\text{Sum} = \frac{n}{2}(a_1 + a_n)\right]$ $\quad \text{Sum} = \frac{245}{2}(11 + 499) = 62{,}475$

SOLUTION 19 - 7

Terms in A.P. = 3, 5, 7, ...; $a_1 = 3, d = 2$

$\text{Sum} = \frac{n}{2}[2a_1 + (n - 1)d]$; $\quad 2600 = \frac{n}{2}[2(3) + (n - 1)(2)]$
$5200 = n(4 + 2n)$
$n^2 + 2n - 2600 = 0$; $\quad n = 50$

SOLUTION 19 - 8

$[a_n = a_1\, r^{n-1}]$ $\quad -1 = 27\, r^{4-1}$
$r = -1/3$

$[a_3 = a_1\, r^{3-1}]$ $\quad a_3 = 27(-1/3)^2 = 3$

SOLUTION 19 - 9
SOLUTION 19 - 10
SOLUTION 19 - 11

$a_4 = 6$; $\quad a_{10} = 384$

$a_n = a_m\, r^{n-m}$
$a_{10} = a_6\, r^{10-6}$; $\quad 384 = 6\, r^{10-4}$
$r^6 = 64$; $\quad r = 2$ → Common ratio

$a_4 = a_1\, r^{4-1}$; $\quad 6 = a_1\, 2^3$
$a_1 = 0.75$ → First term

$a_7 = a_4\, r^{7-4}$
$a_7 = 6 \times 2^3 = 48$

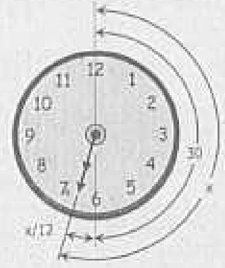
SOLUTION 19 - 12

Length of candle *A* at any time, $L_A = L - (L/8)t$
Length of candle *B* at any time, $L_B = L - (L/4)t$

$[L_A = 2L_B]$ $\quad L - (L/8)t = 2[L - (L/4)t]$
$L - Lt/8 = 2L - Lt/2$
$1 - \frac{1}{8}t = 2 - \frac{1}{2}t$
$\frac{3}{8}t = 1$; $t = \frac{8}{3}$ hours = **2 hrs & 40 min**

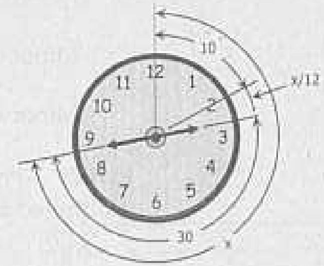
SOLUTION 19 - 13

$x = 30 + x/12$
$x = 32.727$ min
x = **32 min & 43.64 seconds**



SOLUTION 19 - 14

$x = 10 + x/12 + 30$
$\frac{11}{12}x = 40$
x = **43.636 min**



SOLUTION 19 - 15

$150(70\%) + x(100\%) = (150 + x)(85\%)$
$10{,}500 + 100x = 12{,}750 + 85x$
$15x = 2250$; $\quad x = 150$ ounces

SOLUTION 19 - 16

$P(n, r) = P(9, 3) = \frac{9!}{(9-3)!} = 504$

SOLUTION 19 - 17

Probability that the first draw in red = 5/8
Probability that the second draw in red = 4/7

Probability that the both balls drawn are red = (5/8)(4/7) = **0.357**

SOLUTION 19 - 18

The possibilities are O, N-O, N-N-O
O = probability that the door can be opened
N = probability that the door cannot be opened

O: 1/9
N-O: 8/9 × 1/8 = 1/9
N-N-O: 8/9 × 7/8 × 1/ 7 = 7/63

Probability = 1/9 + 1/9 + 7/63 = 0.333

SOLUTION 19 - 19

$i = \sqrt{-1}$; $i^2 = -1$

$(1 + i^2)^{10} = (1 - 1)^{10} = 0^{10} = 0$

SOLUTION 19 - 20
SOLUTION 19 - 21
SOLUTION 19 - 22

Part 1: The minor of an entry a_{ij} is the determinant of a submatrix resulting from the elimination of the single row i and the single column j.

$$\begin{Bmatrix} 2 & 5 & 1 \\ 3 & 1 & 0 \\ 6 & 7 & 4 \end{Bmatrix}$$

5 is at row 1 ($i = 1$) column 2 ($j = 2$)

$$\text{Minor of } 5 = \begin{vmatrix} 3 & 0 \\ 6 & 4 \end{vmatrix}$$

Minor of 5 = 3(4) - 6(0) = **12**

Part 2: The cofactor of an entry a_{ij} is the minor of a_{ij} multiplied by $(-1)^{i+j}$.

$$\begin{Bmatrix} 2 & 5 & 1 \\ 3 & 1 & 0 \\ 6 & 7 & 4 \end{Bmatrix}$$

3 is at row 2 column 1 ($i = 2$, $j = 1$)

$$\text{Cofactor of } 3 = (-1)^{2+1}\begin{vmatrix} 5 & 1 \\ 7 & 4 \end{vmatrix} = -1[5(4) - 7(1)]$$

Cofactor of 3 = **-13**

Part 3:
Multiplication of matrix can be done only if the number of columns of the left-hand matrix is equal to the number of rows of the right-hand matrix. Multiplication is done by multiplying the elements in each left-hand matrix row by the entries in each right-hand matrix column, adding the products, and placing the sum at the intersection point of the participating row and column.

$$A \times B = \begin{Bmatrix} 2 & 5 & 1 \\ 3 & 1 & 0 \\ 6 & 7 & 4 \end{Bmatrix} \times \begin{Bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{Bmatrix}$$

$$A \times B = \begin{Bmatrix} 2\times1+5\times0+1\times0 & 2\times0+5\times1+1\times0 & 2\times0+5\times0+1\times1 \\ 3\times1+1\times0+0\times0 & 3\times0+1\times1+0\times0 & 3\times0+1\times0+0\times1 \\ 6\times1+7\times0+4\times0 & 6\times0+7\times1+4\times0 & 6\times0+7\times0+4\times1 \end{Bmatrix}$$

$$A \times B = \begin{Bmatrix} 2 & 5 & 1 \\ 3 & 1 & 0 \\ 6 & 7 & 4 \end{Bmatrix}$$

Note: Matrix B is an identity matrix. A matrix multiplied to it remains the same. Thus $A \times B = A$

SOLUTION 19 - 23

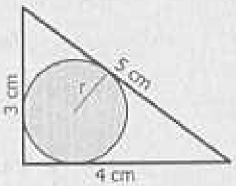
$\sin A = 4/5$; $A = 53.13° = 233.13°$ (in third quadrant)
$\sin B = 7/25$; $B = 16.26° = 163.74°$ (in second quadrant)

$\sin(A + B) = \sin(233.13° + 163.74°) = 0.6 = 3/5$

SOLUTION 19 - 24

$A_T = ½(3)(4) = 6\text{ m}^2$
$s = \frac{3+4+5}{2} = 6$
$[A_T = r\,s]$ $16 = r(6)$
$r = 1\text{ m}$

$[A_c = \pi r^2]$ $A_c = \pi(1)^2 = \pi\text{ m}^2$

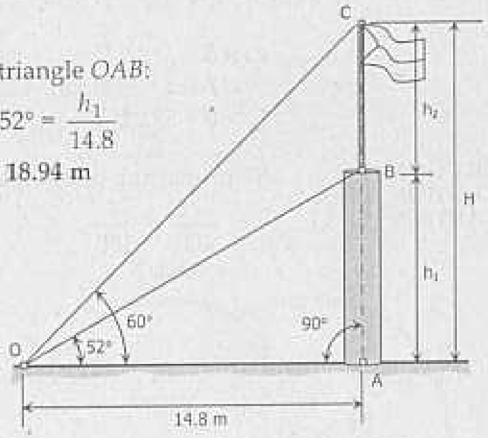


SOLUTION 19 - 25
SOLUTION 19 - 26
SOLUTION 19 - 27

Part 1:
In right triangle OAB:

$\tan 52° = \dfrac{h_1}{14.8}$

$h_1 =$ **18.94 m**



Part 2:
In right triangle OAC:

$\tan 60° = \dfrac{H}{14.8}$; $H = 25.63\text{ m}$

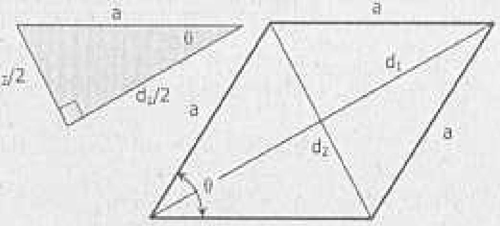
$h_2 = H - h_1 = 25.63 - 18.94 = 6.69\text{ m}$

Part 3:

In triangle OAC:

$$\cos 60° = \frac{14.8}{OC};\ OC = \mathbf{29.6\ m}$$

SOLUTION 19 - 28
SOLUTION 19 - 29
SOLUTION 19 - 30



Part 1:

$A = ½\, d_1\, d_2$; $132 = ½(d_1)(12)$

d_1 = **22 inches**

Part 2:

$a = \sqrt{(12/2)^2 + (22/2)^2}$; a = **12.53 inches**

Part 3:

Area = $a^2 \sin θ$; $132 = (12.53)^2 \sin θ$

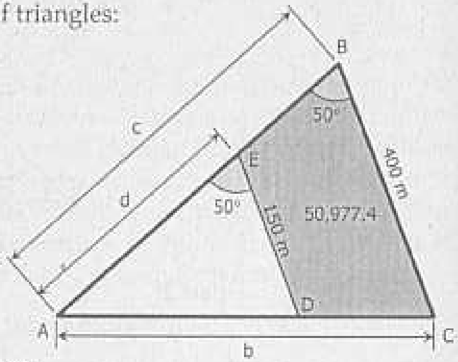
$θ$ = **57.22°**

SOLUTION 19 - 31
SOLUTION 19 - 32
SOLUTION 19 - 33

By proportion of triangles:

$$\frac{c}{400} = \frac{d}{150}$$

$c = 8d/3$



$A_{BCDE} = A_{ABC} - A_{ADE}$

$50{,}977.4 = ½(400)(c) \sin 50° - ½(150)(d) \sin 50°$

$50{,}977.4 = 200(8d/3) \sin 50° - 75\, d \sin 50°$

$50{,}977.4 = 351.1037d$

$d = 145.19$ m; $c = 387.18$ m

$A_{ABC} = ½(400)(387.18) \sin 50° = \mathbf{59{,}319.42\ m^2}$

$A_{ADE} = ½(150)(145.19) \sin 50° = \mathbf{8{,}341.65\ m^2}$

Angle C:

Solve for side b by cosine law:

$b^2 = 400^2 + 387.18^2 - 2(400)(387.18) \cos 50°$

$b = 332.88$ m

Solve for angle C by sine law:

$$[\frac{c}{\sin C} = \frac{b}{\sin B}] \qquad \frac{387.18}{\sin C} = \frac{332.88}{\sin 50°}$$

$\sin C = 0.891$; $C = \mathbf{63°}$

ANOTHER SOLUTION:

Since triangle ADE is similar to triangle ABC:

$$\frac{A_{ADE}}{A_{ABC}} = \frac{150^2}{400^2};\ A_{ABC} = \tfrac{64}{9} A_{ADE}$$

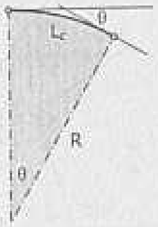
$A_{BCDE} = A_{ABC} - A_{ADE}$

$50{,}977.4 = \tfrac{64}{9} A_{ADE} - A_{ADE}$

$A_{ADE} = \mathbf{8{,}341.756\ m^2}$; $A_{ABC} = \mathbf{59{,}319.156\ m^2}$

SOLUTION 19 - 34

$L_c = 300$ m; $θ = 30°$



$$L_c = \frac{πRθ}{180°}$$

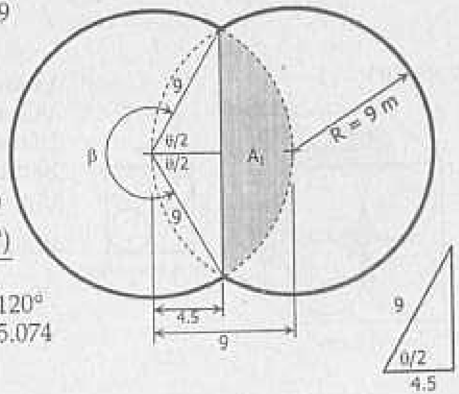
$$300 = \frac{πR(30°)}{180°}$$

R = **572.96 m**

SOLUTION 19 - 35
SOLUTION 19 - 36
SOLUTION 19 - 37

$\cos(θ/2) = 4.5/9$

$θ = 120°$



$$A_1 = \frac{πR^2θ}{360°} - ½\, R^2 \sin θ$$

$$A_1 = \frac{π(9)^2(120°)}{360°} - ½(9)^2 \sin 120°$$

$A_1 = 84.8230 - 35.074$

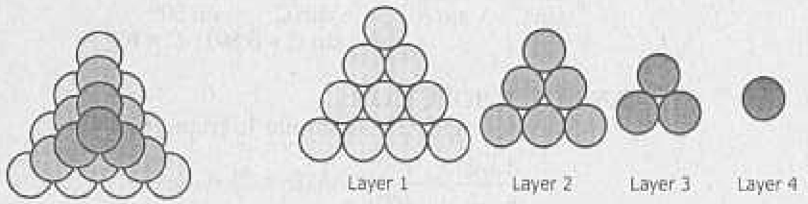
$A_1 = 49.749\ \mathrm{m}^2$

Common area = $2A_1$ = 99.498 m^2

Area of water surface = $2 \times \pi R^2 - 2A_1$
Area of water surface = $2 \times \pi(9)^2 - 99.498$ = **409.44 m^2**

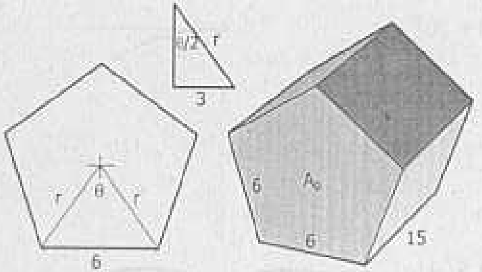
$$\text{Perimeter} = 2 \times \frac{\pi R\beta}{180^\circ} = 2 \times \frac{\pi(9)(360^\circ - 120^\circ)}{180^\circ}$$
Perimeter = **75.398 m**

SOLUTION 19 - 38



Total number of spheres = 10 + 6 + 3 + 1 = **20 spheres**

SOLUTION 19 - 39

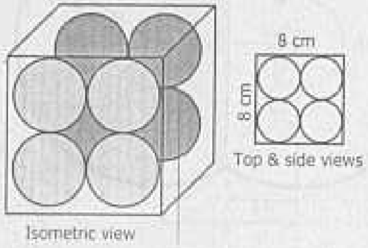


$\theta = 360^\circ/5 = 72^\circ$
$\sin(\theta/2) = 3/r$
$\sin 36^\circ = 3/r$
r = 5.1039 cm

$A_b = \frac{1}{2} r^2 \sin\theta \times 5$
$A_b = \frac{1}{2}(5.1039)^2 \sin 72^\circ \times 5$
A_b = 61.9372 cm^2

Volume = $A_b \times 15$
Volume = **929.06 cm^3**

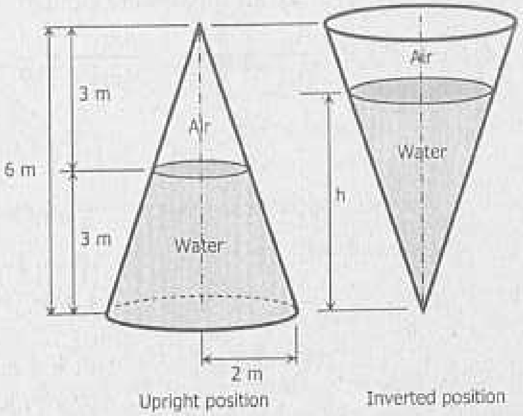
SOLUTION 19 - 40



Volume of container = 8^3
Volume of container = 512 cm^3
Diameter of balls = 8/2 = 4 cm
Radius = 2 cm
Volume of 8 balls = $\frac{4}{3}\pi(2)^3 \times 8$
Volume of 8 balls = 268.0826 cm^3

Volume of water = 512 - 268.0826
Volume of water = **243.92 cm^3**

SOLUTION 19 - 41
SOLUTION 19 - 42
SOLUTION 19 - 43



$V_{cone} = \frac{1}{3}\pi(2)^2(6) = 8\pi$ m^3

In upright position,
$$\frac{V_{cone}}{V_{air}} = \frac{6^3}{3^3};\quad \frac{8\pi}{V_{air}} = 8$$
$V_{air} = \pi$ m^3

$V_{water} = V_{cone} - V_{air} = 8\pi - \pi = 7\pi$ m^3
V_{water} = **21.991 m^3**

In inverted position:
$$\frac{V_{water}}{V_{cone}} = \frac{h^3}{6^3};\quad \frac{7\pi}{8\pi} = \frac{h^3}{6^3};\ h = 5.7388 \text{ m}$$

Weight of water = $\gamma_w V_{water}$ = 9800 × 21.991
Weight of water = 215,511.8 N
Weight of water = 215,511.8 N × (1 kg/9.81 N)
× (1 quintal/100 kg)
Weight of water = **219.686 quintals**

SOLUTION 19 - 44

Since all spheres are similar:

$$\left[\frac{A_{s1}}{A_{s2}} = \frac{4}{9}\right] \qquad \frac{4\pi r_1^2}{4\pi r_2^2} = \frac{4}{9}$$

$$\frac{r_1}{r_2} = \frac{2}{3};\ r_2 = \tfrac{3}{2} r_1$$

$$[V_1 + V_2 = 100\pi] \qquad \tfrac{4}{3}\pi r_1^3 + \tfrac{4}{3}\pi r_2^3 = 100\pi$$

$$r_1^3 + \left[\tfrac{3}{2} r_1\right]^3 = 75$$

$$4.375 r_1^3 = 75;\ r_1 = 2.578 \text{ cm}$$

$$[V_1 = \tfrac{4}{3}\pi r_1^3] \qquad V_1 = \tfrac{4}{3}\pi (2.578)^3$$

$$V_1 = \mathbf{71.81\ cm^3}$$

SOLUTION 19 - 45

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d = \sqrt{(-4-3)^2 + (-7-7)^2} = \mathbf{15.65\ units}$$

SOLUTION 19 - 46

$x^2 = 8y$; $a = 2$ & $LR = 8$

Area $= \tfrac{2}{3}(a)(LR) = \tfrac{2}{3}(2)(8) = \mathbf{10.667}$

SOLUTION 19 - 47
SOLUTION 19 - 48
SOLUTION 19 - 49

$9x^2 + 16y^2 - 144 = 0;\ 9x^2 + 16y^2 = 144$

$$\frac{x^2}{4^2} + \frac{y^2}{3^2} = 1;\ a = 4,\ b = 3$$

Part 1:

Circumference of ellipse:

$$C = 2\pi\sqrt{\frac{a^2+b^2}{2}} = 2\pi\sqrt{\frac{4^2+3^2}{2}} = 22.214$$

Length of arc in the first quadrant $= C/4 =$ **5.55 units**

Part 2:

$9x^2 + 16y^2 - 144 = 0$

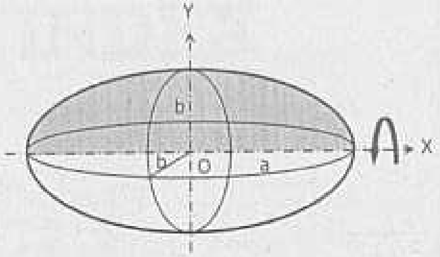
Differentiate with respect to x:

$$18x + 32y\,y' = 0$$

$$y' = \text{slope of chord} = 3$$

$$18x + 32y(3) = 0$$

$$\mathbf{6x + 32y = 0}$$



Part 3:

The resulting solid is a *prolate spheroid.*

$$V = \tfrac{4}{3}\pi ab^2$$

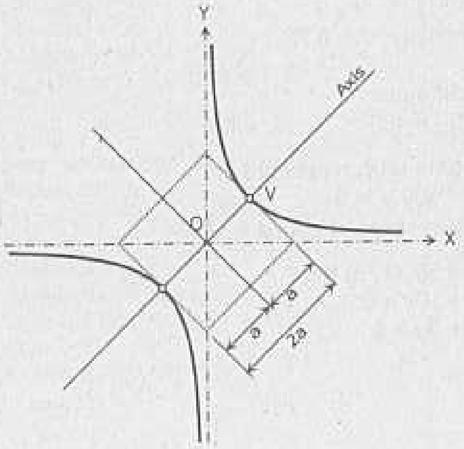
$$V = \tfrac{4}{3}\pi(4)(3)^2$$

$$V = \mathbf{150.8\ cu.\ units}$$

SOLUTION 19 - 50
SOLUTION 19 - 51
SOLUTION 19 - 52

$$xy = 8;\ y = \frac{8}{x} \quad \text{or} \quad x = \frac{8}{y}$$

When $x \to \infty$, $y \to 0$, and when $y \to \infty$, $x \to 0$. This means that the asymptotes of the hyperbola are the X- and Y-axes and $a = b$.



At the vertex, $x = y$

$xy = 8$

$x(x) = 8;\ x = \sqrt{8}$ and $y = \sqrt{8}$

$V(\sqrt{8}, \sqrt{8})$

$$a = \sqrt{\sqrt{8}^2 + \sqrt{8}^2}$$

$a = 4$ and $b = 4$

Part 1: Length of conjugate axis $= 2b = 8$

Part 2: Distance between vertices $= 2a = 8$

Part 3: $c^2 = a^2 + b^2 = 4^2 + 4^2$

$c = 4\sqrt{2}$

$$\text{Eccentricity, } e = \frac{c}{a} = \frac{4\sqrt{2}}{4} = \sqrt{2} = 1.414$$

SOLUTION 19 - 53
SOLUTION 19 - 54
SOLUTION 19 - 55

$16x^2 + 25y^2 = 400$

$\frac{x^2}{25} + \frac{y^2}{16} = 1$

$a = 5; \quad b = 4$

$a^2 = b^2 + c^2$

$5^2 = 4^2 + c^2$

$c = 3$

$$\text{Perimeter} = 2\pi\sqrt{\frac{a^2 + b^2}{2}}$$

$$\text{Perimeter} = 2\pi\sqrt{\frac{5^2 + 4^2}{2}}$$

Perimeter = **28.448 units**

$$\text{Second eccentricity, } e' = \frac{c}{b} = \frac{3}{4}$$

Second eccentricity, $e' =$ **0.75**

Equation of diameter:

$16x^2 + 25y^2 = 400$

Differentiate with respect to x:

$32x + 50y\,y' = 0$

$y' = \text{slope of chord} = 1/5.$

$32x + 50y(1/5) = 0$

$32x + 10y = 0$

$\mathbf{16x + 5y = 0}$

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